Bellcomm, Inc.

TR-68-310-1

A Compendium of the Moon's Motion and Geometry: 1966 through 1985

January 9, 1968

J. O. Cappellari, Jr.

Work performed for Manned Space Flight,
National Aeronautics and Space Administration,
Under Contract NASw-417

ABSTRACT

This report provides tabular and graphical information associated with the geometry and dynamics of the Earth-Moon-Sun system for the 1966-1985 time period. The graphical presentation of data makes possible a rapid scanning and approximate determination of those parameters which satisfy a given set of requirements.

The graphical results include inclination of the Moon's plane of motion, right ascension of the ascending node of the Moon's plane of motion, right ascension and declination of the Moon, Earth-Moon distance, radial and transverse velocity of the Moon, geocentric angular momentum of the Moon, semi-major axis of the Moon's orbit, argument of the Moon's perigee, eccentricity of the Moon's orbit, argument of the Moon's position, Moon-Earth-Sun angle, selenographic latitude and longitude of the Sun, selenographic latitude and longitude of the Earth (lunar libration) and translunar dihedral angles for launches from Cape Kennedy.

TABLE OF CONTENTS

A COMPENDIUM OF THE MOON'S MOTION AND GEOMETRY: 1966 THROUGH 1985

1.0 INTRODUCTION

The characteristics of lunar trajectories are strongly dependent upon the dynamics and geometry of the Earth-Moon-Sun system. Thus, effective lunar mission planning requires adequate information concerning the characteristics of this system.

Positional data are available for the Sun and Moon in tabular form (Reference 1) for a few years in the future, while digital velocity and positional data are available on magnetic tapes (References 2 and 3) up to the year 2000. However, there is much additional information that can be derived from the data available, and, in addition, it is useful for mission planning and preliminary trajectory selection purposes to have pertinent data available in graphical form. The purpose of this report is to provide a number of tabular and graphical representations associated with the geometry and dynamics of the Earth-Moon-Sun system for the 1965-1985 period. In this form the data make possible a rapid scanning and approximate determination of those parameters which satisfy a given set of mission requirements.

The tabular quantities presented (Appendix A) include Julian Date-Gregorian Date conversions and Day of Month - Day of Year conversions. The graphical results (Appendix B) include inclination of the Moon's plane of motion, right ascension of the ascending node of the Moon's plane of motion, right ascension and declination of the Moon, Earth-Moon distance, radial and transverse velocity of the Moon, geocentric angular momentum of the Moon, semi-major axis of the Moon's osculating orbit, osculating argument of the Moon's perigee, eccentricity of the Moon's osculating orbit, argument of the Moon's position, Moon-Earth-Sun angle, selenographic latitude and longitude of the Sun, selenographic latitude of the Earth (libration in **latitude)**, selenographic longitude of the Earth (libration in longitude), and translunar dihedral angles for launch azimuths of 72°, 90°, and 108° from Cape Kennedy. Each graph includes data for a complete year.

2.0 DATA SOURCE

The primary source of the data presented is the set of JPL Ephemeris Tapes (E9510, E9511 and E9512) described in References 2 and 3.* These tapes contain position and velocity data for the planets and Moon, as well as nutation and nutation rate, for the 50 year period beginning December 30, 1949 and ending January 5, 2000. On these tapes lunar positions and velocities are referred to a geocentric equatorial rectangular reference frame associated with the mean

*A new set of ephemeris tapes, including an improved lunar ephemeris, has been released by JPL recently (References 11 and 12). However, the results of this report are not affected within the accuracy of the graphical presentations.

equator and equinox of 1950.0 = Julian Date 2433282.423 (Figure 1). The position and velocity of the Earth-Moon barycenter (as well as the other planets) are referred to the heliocentric equatorial rectangular frame associated with the mean equator and equinox of 1950.0.

3.0 REFERENCE COORDINATE SYSTEMS AND CONSTANTS

Three basic reference coordinate systems are utilized in this report. The first two are the geocentric equatorial (Reference 4) and geocentric ecliptic rectangular reference frames associated with the mean equator and equinox of date (Figure 2), i.e., the mean equator and equinox corresponding to the time for which data is being read from the tape. The mean equator and equinox of date differ from the true equator and equinox of date by a small amount due to nutation (References 6 and 7).

The third basic reference system (Figure 3) is a selenocentric equatorial rectangular, or selenographic, reference frame (Reference 4). This system is particularly useful in defining the librations of the Moon and the position of the subsolar point on the lunar surface.

The value for μ, the Earth-Moon mass ratio, is taken as 81.3015 and the modified gravitational parameter (GM of the Earth plus GM of the Moon) as 1.4249679×10^{16} ft^3/sec^2. These values are consistent with the standard astrodynamic constants adopted for Project Apollo (Reference 5).

4.0 ACKNOWLEDGEMENT

Mr. R. D. Tigner made a substantial contribution to the generation of the data used in this report.

J. O. Cappellari, Jr.

2011-JOC-vh

J. O. Cappellari, Jr.

REFERENCES

1. The American Ephemeris and Nautical Almanac, U. S. Government Printing Office, Washington, D. C., 1968.

2. Peabody, P. R., Scott, J. F., and Orozco, E. G., "Users' Description of JPL Ephemeris Tapes", Technical Report No. 32-580, Jet Propulsion Laboratory, Pasadena, California, March 2, 1964.

3. Peabody, P. R., Scott, J. F., and Orozco, E. G., "JPL Ephemeris Tapes E9510, E9511, and E9512," Technical Memorandum No. 33-167, Jet Propulsion Laboratory, Pasadena, California, March 2, 1964.

4. "Project Apollo Coordinate System Standards," SE 008-001-1, Office of Manned Space Flight, June, 1965.

5. "Natural Environment and Physical Standards for the Apollo Program," M-D E 8020.008B, SE 015-001-1, Office of Manned Space Flight, April, 1965.

6. Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac, Her Majesty's Stationery Office, London, 1961.

7. Woolard, E. W., and Clemence, G. M., Spherical Astronomy, Academic Press, New York, 1966.

8. Roy, A. E., The Foundations of Astrodynamics, The Macmillan Company, New York, 1965.

9. Roth, H. L., and Escobal, P. R., "Transformations Involving the Selenographic Coordinate System," Summary Technical Report for Task MSC/STL A-42, 3400-6045-RU000, June 14, 1965.

10. Hayn, F., Enzyklopädié der Mathematischen Wissenschaften, Volume 6, Part 2, Chapter 20a, 1923.

11. Mulholland, J. D., and Block, N., "JPL Lunar Ephemeris Number 4," Technical Memorandum No. 33-346, Jet Propulsion Laboratory, Pasadena, California, August 1, 1967.

12. Devine, C. J., "JPL Development Ephemeris Number 19," Technical Report No. 32-1181, Jet Propulsion Laboratory, Pasadena, California, November 15, 1967.

13. Woolston, D. S., "Declination, Radial Distance, and Phases of the Moon for the Years 1961 to 1971 for Use in Trajectory Considerations", NASA TN D-911, August, 1961.

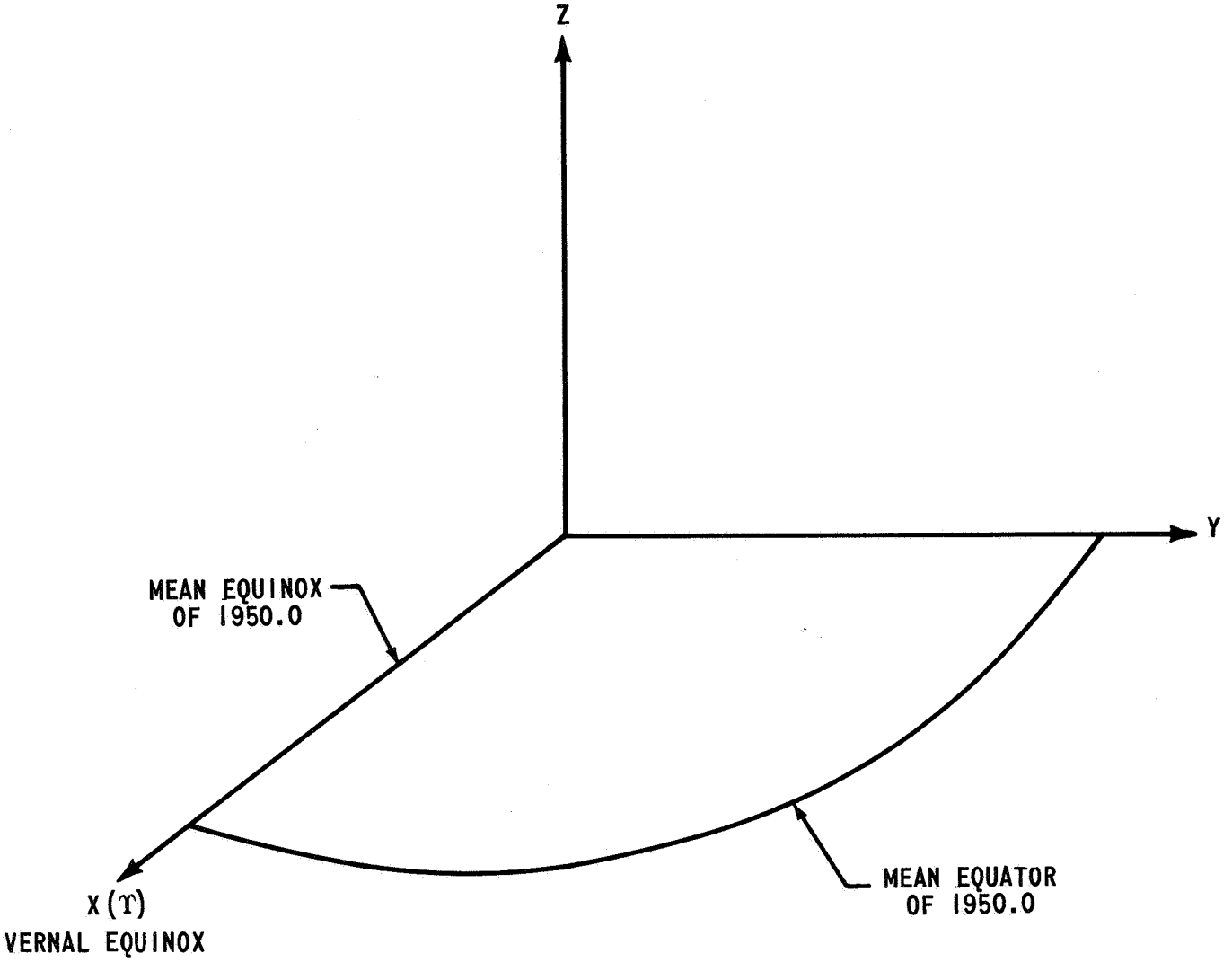


FIGURE 1 - GEOCENTRIC EQUATORIAL RECTANGULAR REFERENCE FRAME - MEAN EQUATOR AND EQUINOX OF 1950.0

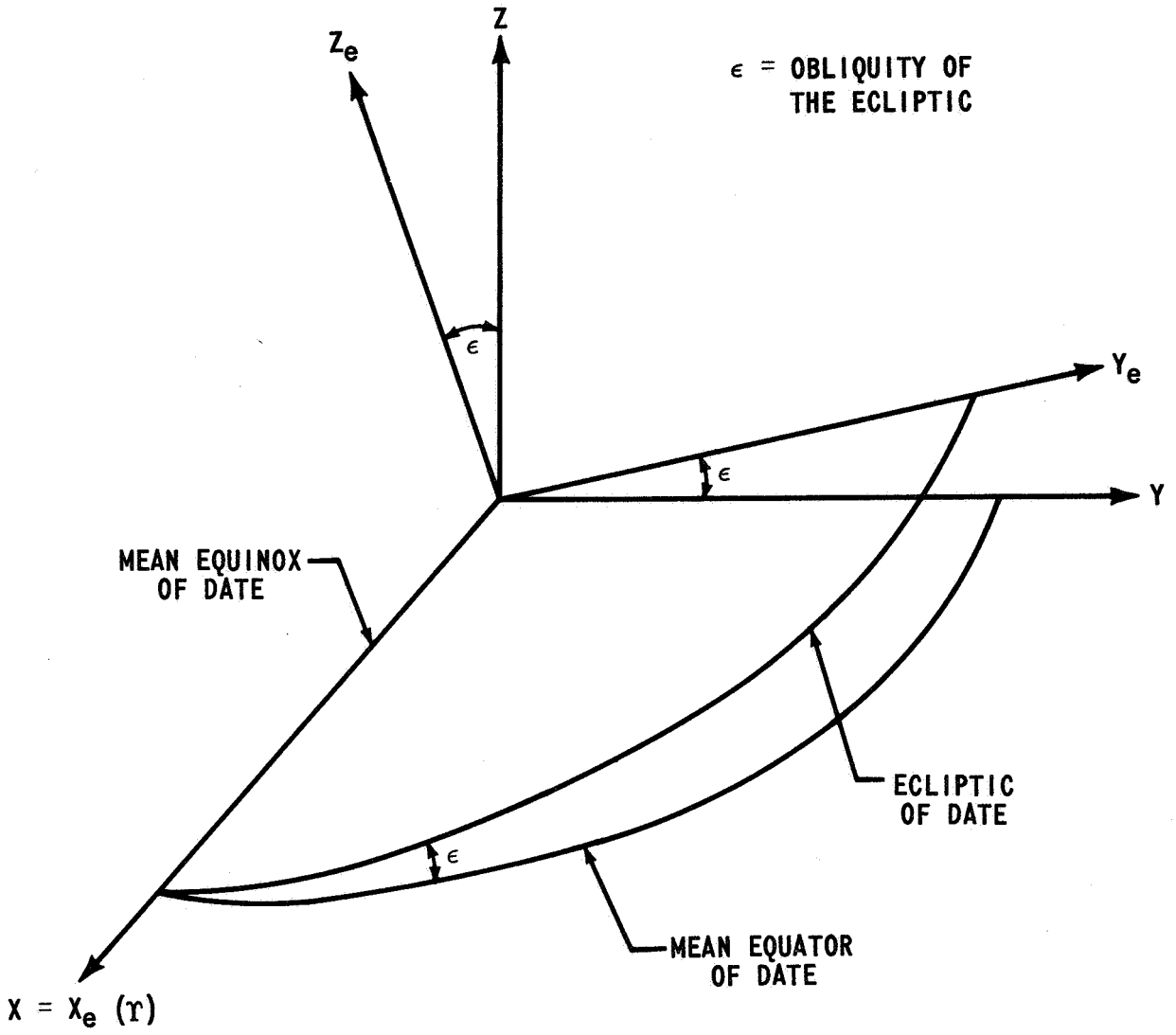


FIGURE 2 - GEOCENTRIC EQUATORIAL AND ECLIPTIC RECTANGULAR REFERENCE FRAMES - MEAN EQUATOR AND EQUINOX OF DATE

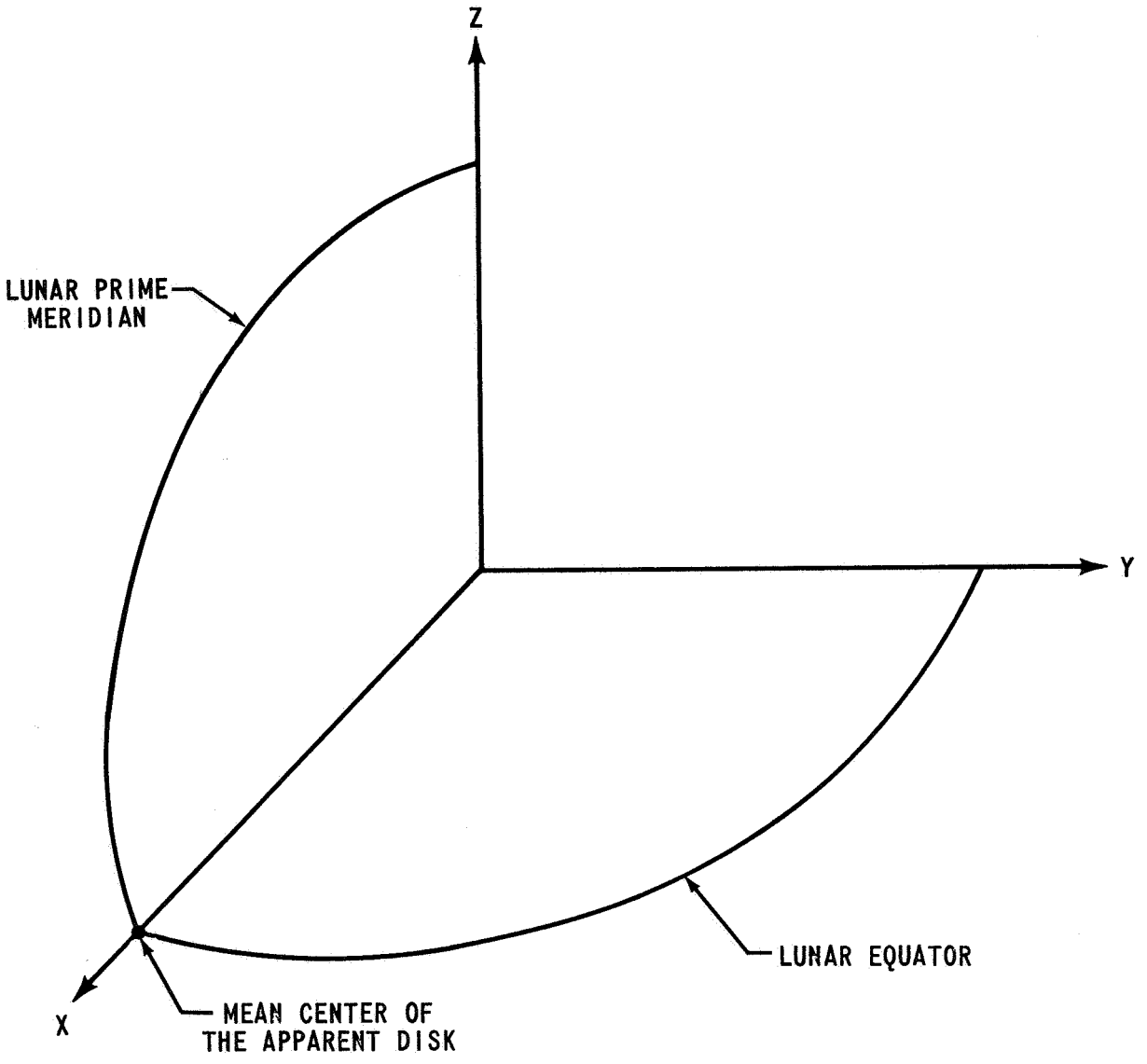


FIGURE 3 - SELENOCENTRIC EQUATORIAL RECTANGULAR (SELENOGRAPHIC) REFERENCE FRAME

APPENDIX A

JULIAN DATE-GREGORIAN DATE AND DAY OF MONTH-DAY OF YEAR CONVERSIONS

Tables A-1 and A-2 supply conversions between the day of the year and the day of the month for nonleap years and leap years, respectively (Reference 13), while Table A-3 gives the relationship between the Julian Date and Gregorian Date (Reference 1). As an example, the 92nd day of the year corresponds to April 1 for leap years and April 2 for nonleap years. Similarly, 0^h U.T. (midnight at the beginning of the day) on April 1, 1968 corresponds to Julian Date 2439947.5.

TABLE A-1 NUMBERS CORRESPONDING TO DAY OF THE YEAR AFTER JANUARY 0

(NONLEAP YEARS)

Day of month	Day of year											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29		88	119	149	180	210	241	272	302	333	363
30	30		89	120	150	181	211	242	273	303	334	364
31	31		90		151		212	243		304		365

TABLE A-2 NUMBERS CORRESPONDING TO DAY OF THE YEAR AFTER JANUARY 0

(LEAP YEARS: 1968, 1972, 1976, 1980 and 1984)

Day of month	Day of year											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	1	32	61	92	122	153	183	214	245	275	306	336
2	2	33	62	93	123	154	184	215	246	276	307	337
3	3	34	63	94	124	155	185	216	247	277	308	338
4	4	35	64	95	125	156	186	217	248	278	309	339
5	5	36	65	96	126	157	187	218	249	279	310	340
6	6	37	66	97	127	158	188	219	250	280	311	341
7	7	38	67	98	128	159	189	220	251	281	312	342
8	8	39	68	99	129	160	190	221	252	282	313	343
9	9	40	69	100	130	161	191	222	253	283	314	344
10	10	41	70	101	131	162	192	223	254	284	315	345
11	11	42	71	102	132	163	193	224	255	285	316	346
12	12	43	72	103	133	164	194	225	256	286	317	347
13	13	44	73	104	134	165	195	226	257	287	318	348
14	14	45	74	105	135	166	196	227	258	288	319	349
15	15	46	75	106	136	167	197	228	259	289	320	350
16	16	47	76	107	137	168	198	229	260	290	321	351
17	17	48	77	108	138	169	199	230	261	291	322	352
18	18	49	78	109	139	170	200	231	262	292	323	353
19	19	50	79	110	140	171	201	232	263	293	324	354
20	20	51	80	111	141	172	202	233	264	294	325	355
21	21	52	81	112	142	173	203	234	265	295	326	356
22	22	53	82	113	143	174	204	235	266	296	327	357
23	23	54	83	114	144	175	205	236	267	297	328	358
24	24	55	84	115	145	176	206	237	268	298	329	359
25	25	56	85	116	146	177	207	238	269	299	330	360
26	26	57	86	117	147	178	208	239	270	300	331	361
27	27	58	87	118	148	179	209	240	271	301	332	362
28	28	59	88	119	149	180	210	241	272	302	333	363
29	29	60	89	120	150	181	211	242	273	303	334	364
30	30		90	121	151	182	212	243	274	304	335	365
31	31		91		152		213	244		305		366

TABLE A-3

JULIAN DAY NUMBER

DAYS ELAPSED AT GREENWICH NOON, A.D. 1966 – 1985

YEAR	JAN. 0	FEB. 0	MAR. 0	APR. 0	MAY 0	JUNE 0	JULY 0	AUG. 0	SEPT. 0	OCT. 0	NOV. 0	DEC. 0
1966	243 9126	9157	9185	9216	9246	9277	9307	9338	9369	9399	9430	9460
1967	9491	9522	9550	9581	9611	9642	9672	9703	9734	9764	9795	9825
1968	9856	9887	9916	9947	9977	*0008	*0038	*0069	*0100	*0130	*0161	*0191
1969	244 0222	0253	0281	0312	0342	0373	0403	0434	0465	0495	0526	0556
1970	0587	0618	0646	0677	0707	0738	0768	0799	0830	0860	0891	0921
1971	244 0952	0983	1011	1042	1072	1103	1133	1164	1195	1225	1256	1286
1972	1317	1348	1377	1408	1438	1469	1499	1530	1561	1591	1622	1652
1973	1683	1714	1742	1773	1803	1834	1864	1895	1926	1956	1987	2017
1974	2048	2079	2107	2138	2168	2199	2229	2260	2291	2321	2352	2382
1975	2413	2444	2472	2503	2533	2564	2594	2625	2656	2686	2717	2747
1976	244 2778	2809	2838	2869	2899	2930	2960	2991	3022	3052	3083	3113
1977	3144	3175	3203	3234	3264	3295	3325	3356	3387	3417	3448	3478
1978	3509	3540	3568	3599	3629	3660	3690	3721	3752	3782	3813	3843
1979	3874	3905	3933	3964	3994	4025	4055	4086	4117	4147	4178	4208
1980	4239	4270	4299	4330	4360	4391	4421	4452	4483	4513	4544	4574
1981	244 4605	4636	4664	4695	4725	4756	4786	4817	4848	4878	4909	4939
1982	4970	5001	5029	5060	5090	5121	5151	5182	5213	5243	5274	5304
1983	5335	5366	5394	5425	5455	5486	5516	5547	5578	5608	5639	5669
1984	5700	5731	5760	5791	5821	5852	5882	5913	5944	5974	6005	6035
1985	244 6066	6097	6125	6156	6186	6217	6247	6278	6309	6339	6370	6400

APPENDIX B

EXPLANATION AND USE OF GRAPHICAL DATA

This appendix contains graphical representations of a number of quantities which depend on the geometry and dynamics of the Moon's motion. The specific quantities graphed and a definition of each are presented in Table B-1; these quantities are also defined pictorially in Figures B-1a through B-1d. Osculating quantities are those quantities associated with the geocentric Keplerian orbit having the same position and velocity as the true orbit of the Moon at the instant of time being considered.

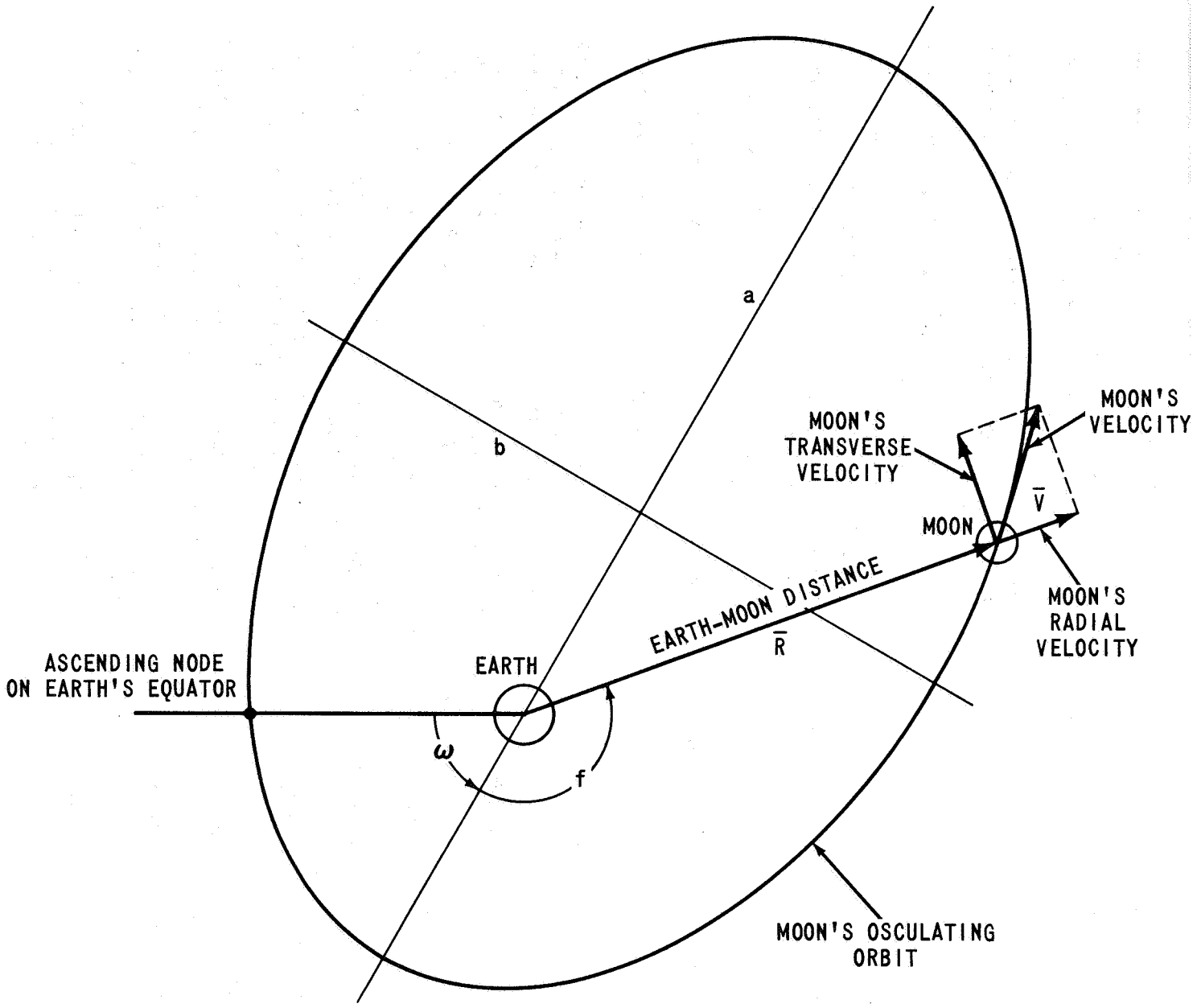
Two of the quantities presented, inclination of the Moon's osculating plane of motion (Figure B-2) and right ascension of the ascending node of the Moon's osculating plane of motion (Figure B-3), exhibit cyclic behavior with long periods. The remaining quantities are also cyclic in nature, but with short periods of approximately one month. Each of these parameters, sixteen in number, is depicted for a complete year on a single page; thus, there are 10 sections (years), with 16 pages of data per section.

The figure numbers reflect the year under consideration and the quantity being displayed. For example, Figure B1966-16 denotes quantity 16 (translunar dihedral angles) for the year 1966. For convenience, succeeding graphs of the same quantity present overlapping information, and related items have been paired together on opposite pages. These deliberately paired items include (a) right ascension and declination of the Moon, (b) lunar libration in latitude and libration in longitude, (c) selenographic longitude and latitude of the Sun, and (d) radial and transverse velocity of the Moon.

The abscissa in each figure represents the day of the year, measured from 0^h U.T. (Greenwich Mean Time) on January 1. Thus, the data point corresponding to any day of the year represents the value of the measured quantity at 24^h U.T. (midnight at the end of the day) on that day. Tables A-1 and A-2 in Appendix A give the conversions from day of year to day of month.

TABLE B-1 DEFINITIONS OF GRAPHICAL QUANTITIES

NO.	QUANTITY	DIMENSION	DEFINITION
1	Earth-Moon distance	feet	Linear distance from the center of the Earth to the center of the Moon
2	Right ascension of the Moon.	degrees	Angular distance measured eastward from the vernal equinox along the equator to the hour circle (meridian) of the Moon
3	Declination of the Moon	degrees	Angular distance of the Moon north (positive) or south (negative) from the equator
4	Radial velocity of the Moon	feet/second	Component of the Moon's velocity along the Earth-Moon line, positive away from the Earth
5	Transverse velocity of the Moon	feet/second	Component of the Moon's velocity perpendicular to the Earth-Moon line
6	Semimajor axis of the Moon's osculating orbit	feet	Instantaneous, or osculating, semi-major axis of the Moon's orbit
7	Geocentric angular momentum of the Moon	$(\text{feet})^2/\text{sec}$	Instantaneous geocentric angular momentum per unit mass of the Moon
8	Eccentricity of the Moon's osculating orbit		Instantaneous, or osculating, eccentricity of the Moon's orbit
9	Osculating argument of the Moon's perigee	degrees	Angular distance measured in the osculating plane of the Moon's motion from the ascending node on the Earth's equator to the osculating position of perigee
10	Argument of the Moon's position	degrees	Angular distance measured in the osculating plane of the Moon's motion from the ascending node on the Earth's equator to the instantaneous Earth-Moon line
11	Moon-Earth-Sun angle	degrees	Angular distance between the Earth-Moon and the Earth-Sun lines
12	Selenographic latitude of the Sun	degrees	Angular distance of the Moon-Sun line north (positive) or south (negative) from the lunar equator
13	Selenographic longitude of the Sun	degrees	Angular distance measured eastward from the lunar prime meridian along the equator to the meridian containing the Moon-Sun line
14	Selenographic latitude of the Earth (libration in latitude)	degrees	Angular distance of the Moon-Earth line north (positive) or south (negative) from the lunar equator
15	Selenographic longitude of the Earth (libration in longitude)	degrees	Angular distance measured eastward from the lunar prime meridian along the equator to the meridian containing the Moon-Earth line
16	Translunar dihedral angles	degrees	Dihedral angle between the translunar trajectory plane associated with a given launch azimuth (72°, 90° or 108°) and the osculating plane of the Moon's motion



$\bar{R} \times \bar{V}$ = GEOCENTRIC ANGULAR MOMENTUM OF MOON

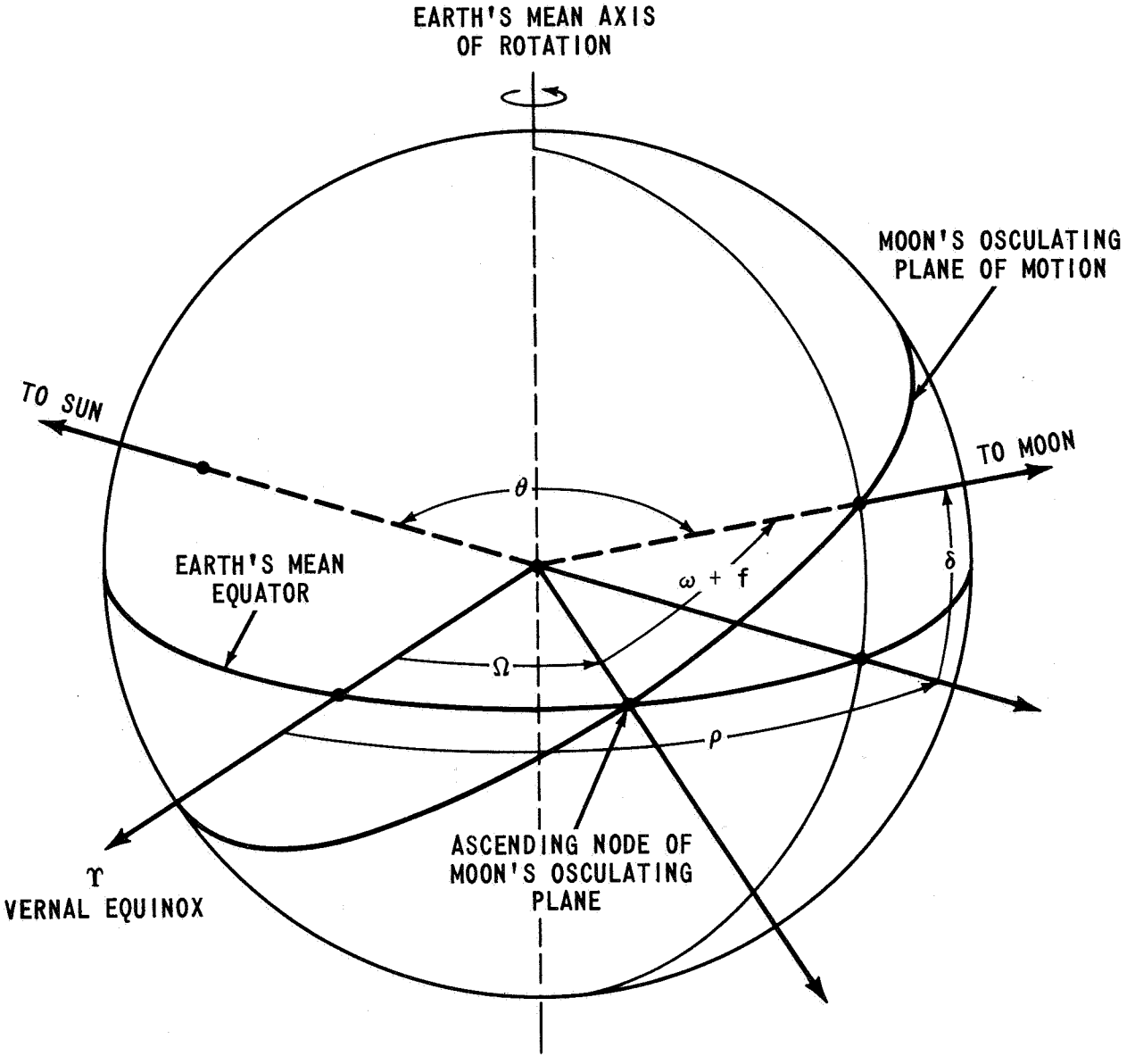
a = SEMI-MAJOR AXIS OF THE MOON'S OSCULATING ORBIT

$e = \sqrt{1 - \frac{b^2}{a^2}}$ = ECCENTRICITY OF THE MOON'S OSCULATING ORBIT

ω = OSCULATING ARGUMENT OF THE MOON'S PERIGEE

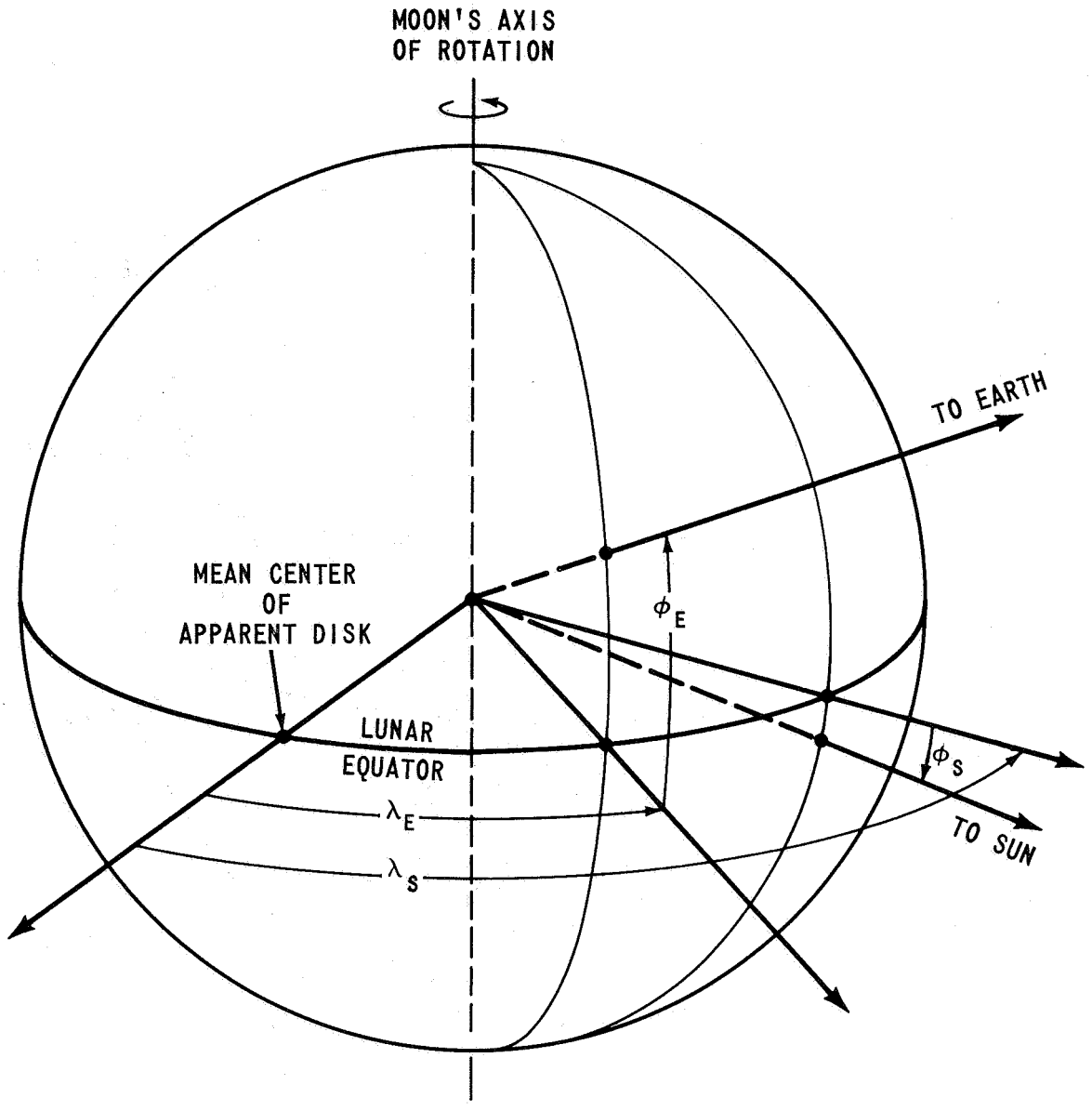
$\omega + f$ = ARGUMENT OF THE MOON'S POSITION

FIGURE B-1a - A PICTORIAL DEFINITION OF QUANTITIES PRESENTED



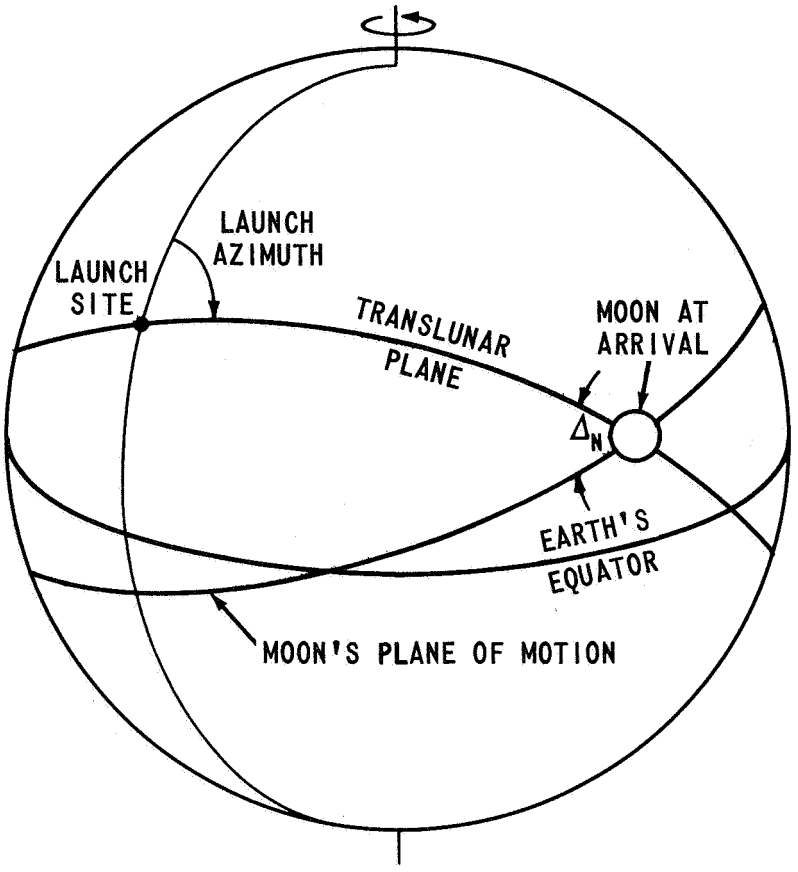
ρ = RIGHT ASCENSION OF THE MOON
δ = DECLINATION OF THE MOON
Ω = RIGHT ASCENSION OF THE ASCENDING NODE OF THE MOON'S OSCULATING PLANE OF MOTION
$\omega + f$ = ARGUMENT OF THE MOON'S POSITION
θ = MOON-EARTH-SUN ANGLE

FIGURE B-1b - A PICTORIAL DEFINITION OF QUANTITIES PRESENTED

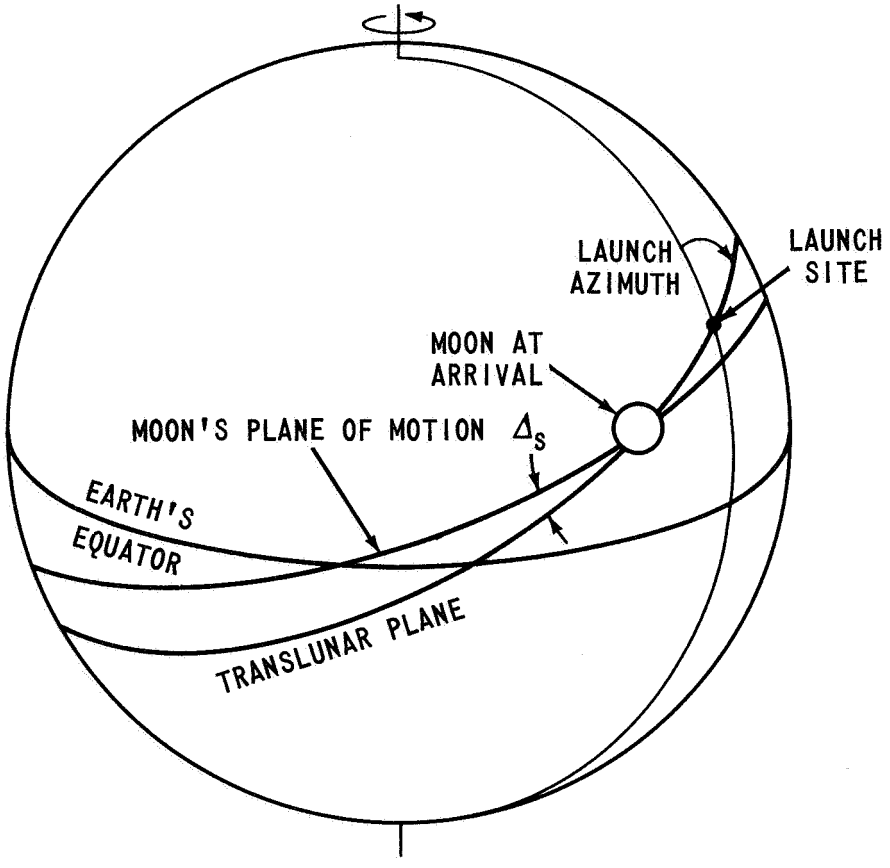


λ_E = SELENOGRAPHIC LONGITUDE OF THE EARTH
λ_S = SELENOGRAPHIC LONGITUDE OF THE SUN
ϕ_E = SELENOGRAPHIC LATITUDE OF THE EARTH
ϕ_S = SELENOGRAPHIC LATITUDE OF THE SUN

FIGURE B-Ic - A PICTORIAL DEFINITION OF QUANTITIES PRESENTED



Δ_N = TRANSLUNAR DIHEDRAL ANGLE (NORTH INJECTION)



Δ_S = TRANSLUNAR DIHEDRAL ANGLE (SOUTH INJECTION)

FIGURE B-1d - A PICTORIAL DEFINITION OF QUANTITIES PRESENTED

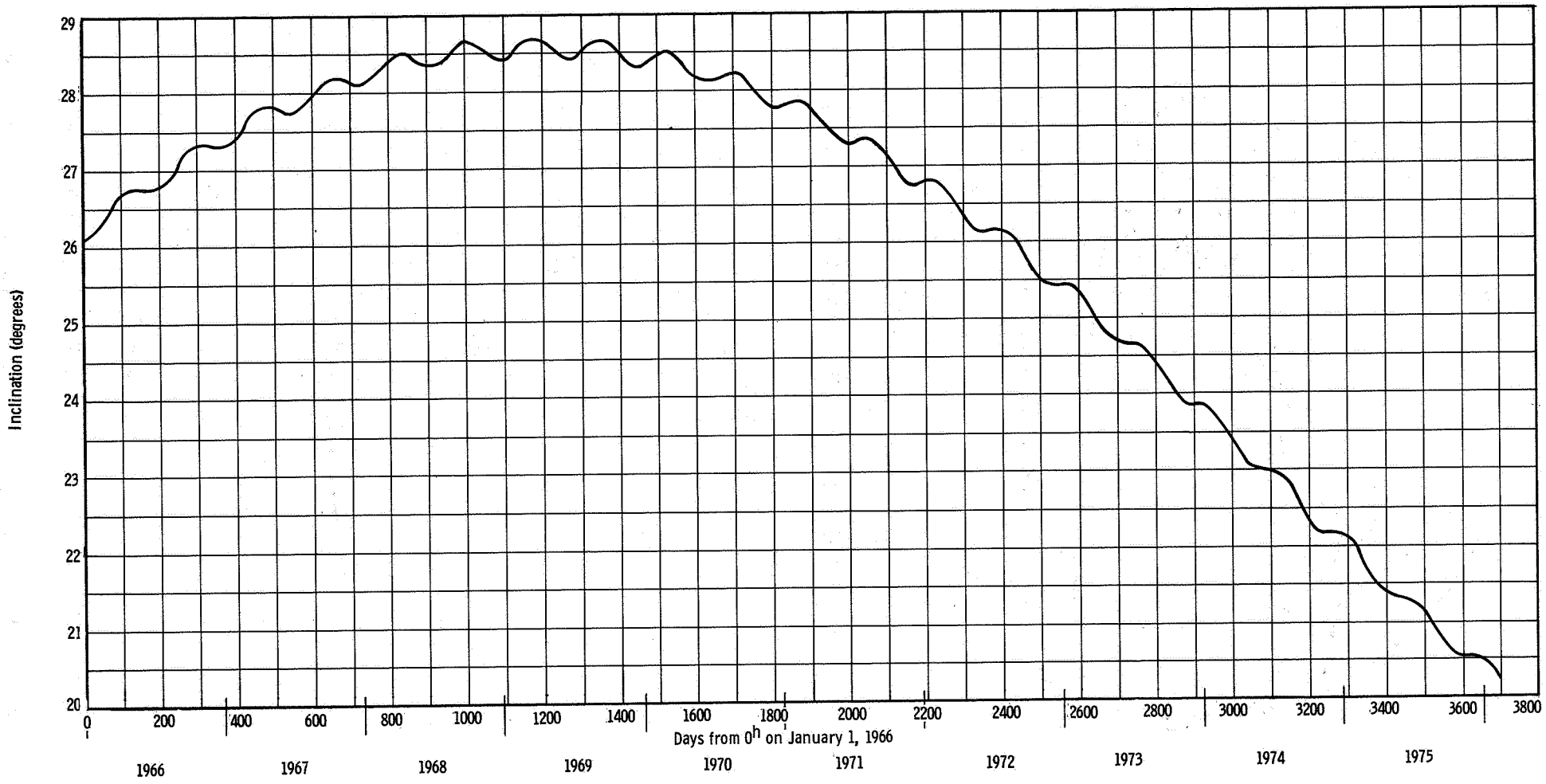


FIGURE B-2a INCLINATION OF THE MOON'S OSCULATING PLANE

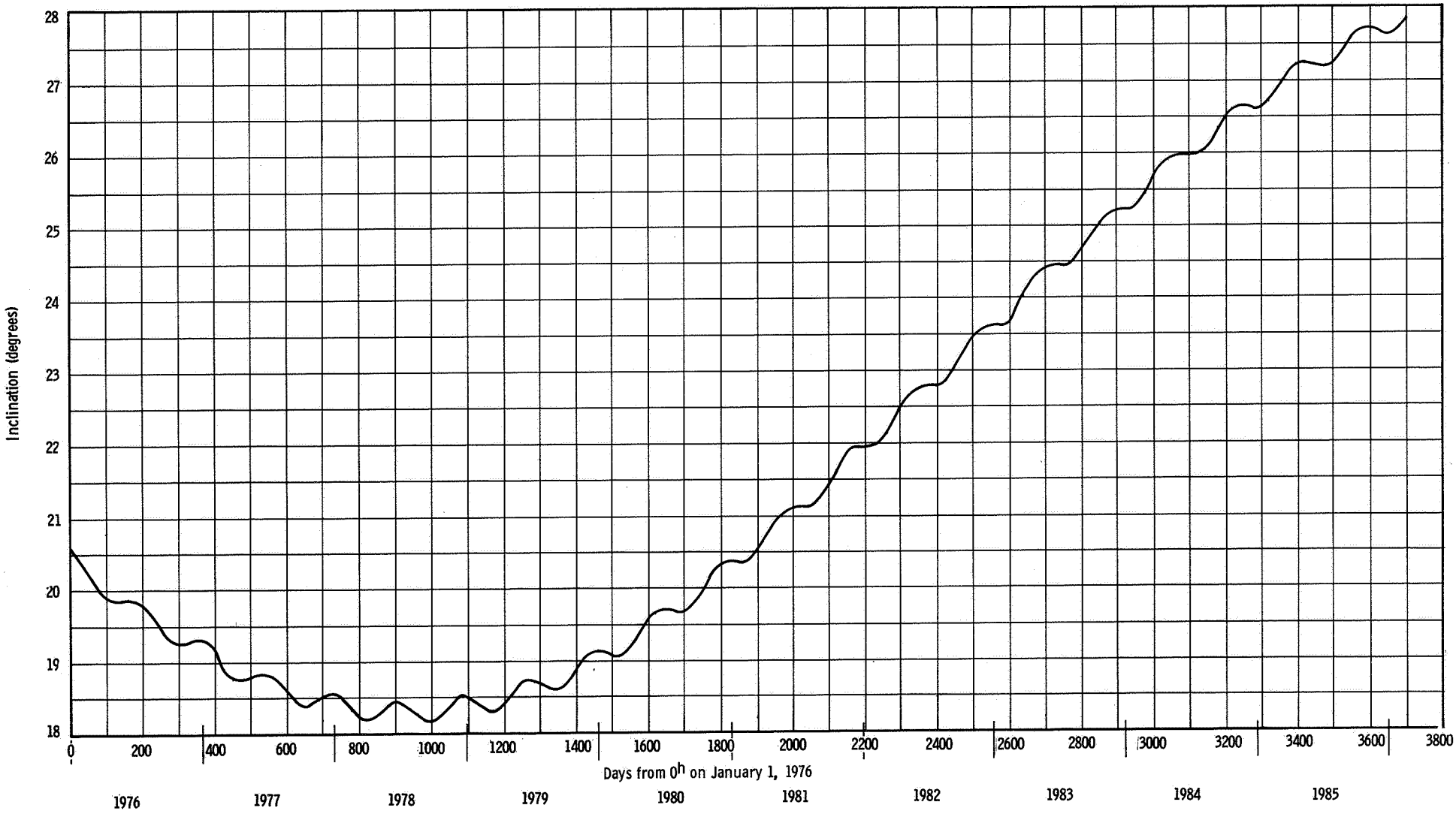


FIGURE B-2b INCLINATION OF THE MOON'S OSCULATING PLANE

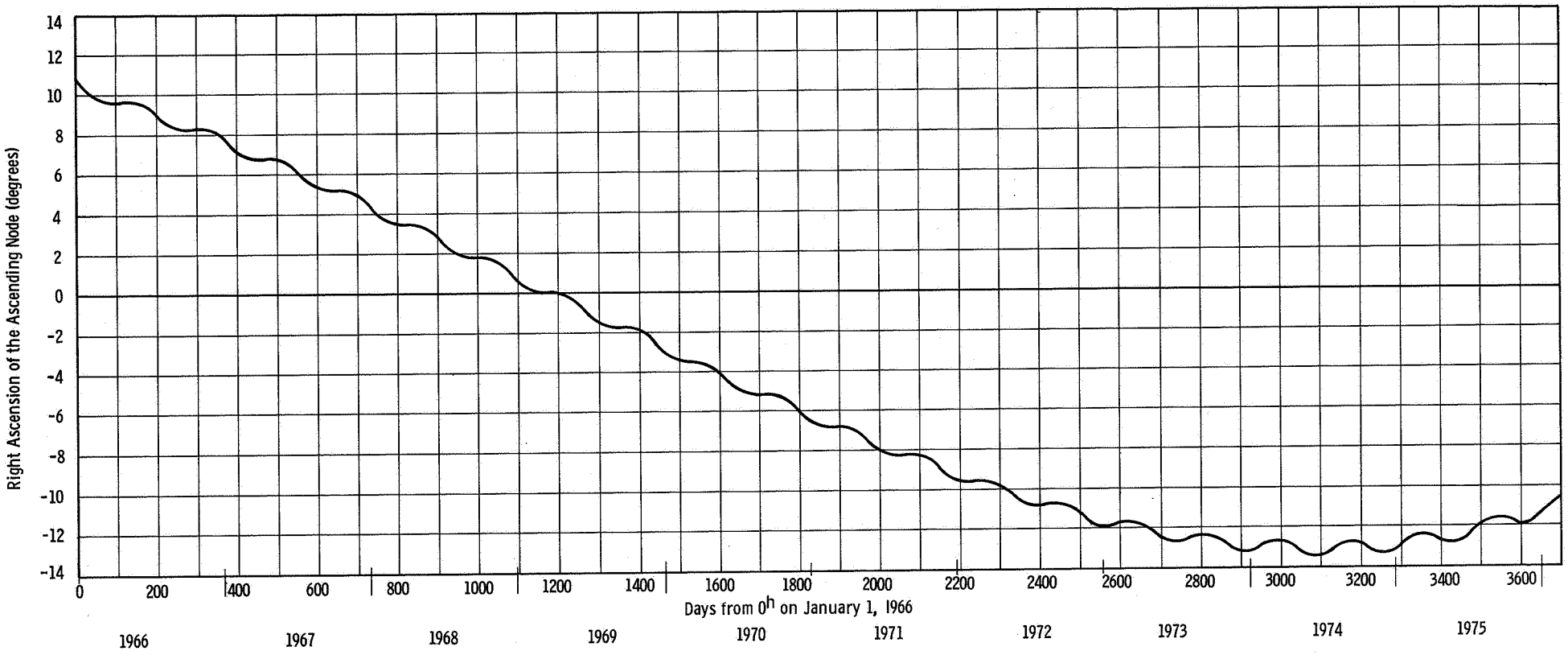


FIGURE B-3a RIGHT ASCENSION OF THE ASCENDING NODE OF THE MOON'S OSCULATING ORBIT

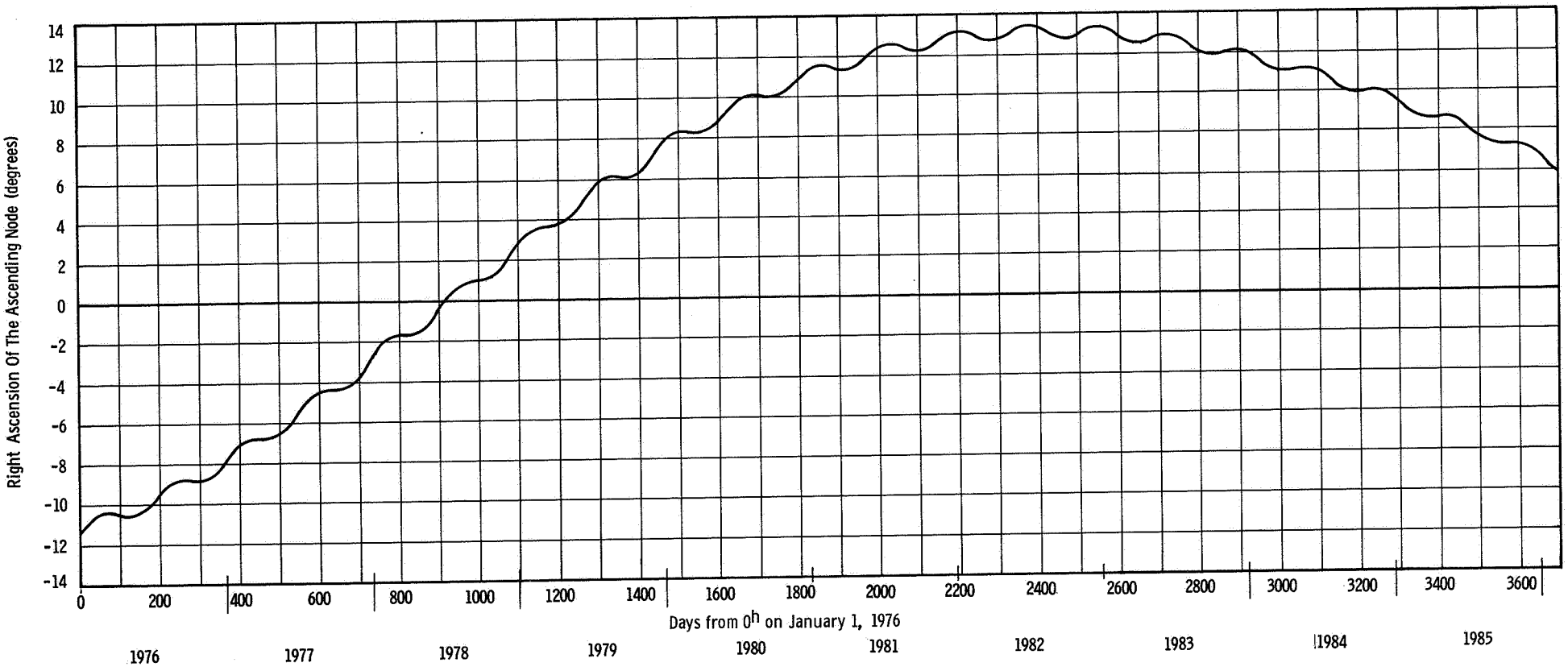


FIGURE B-3b RIGHT ASCENSION OF THE ASCENDING NODE OF THE MOON'S OSCULATING ORBIT

1966

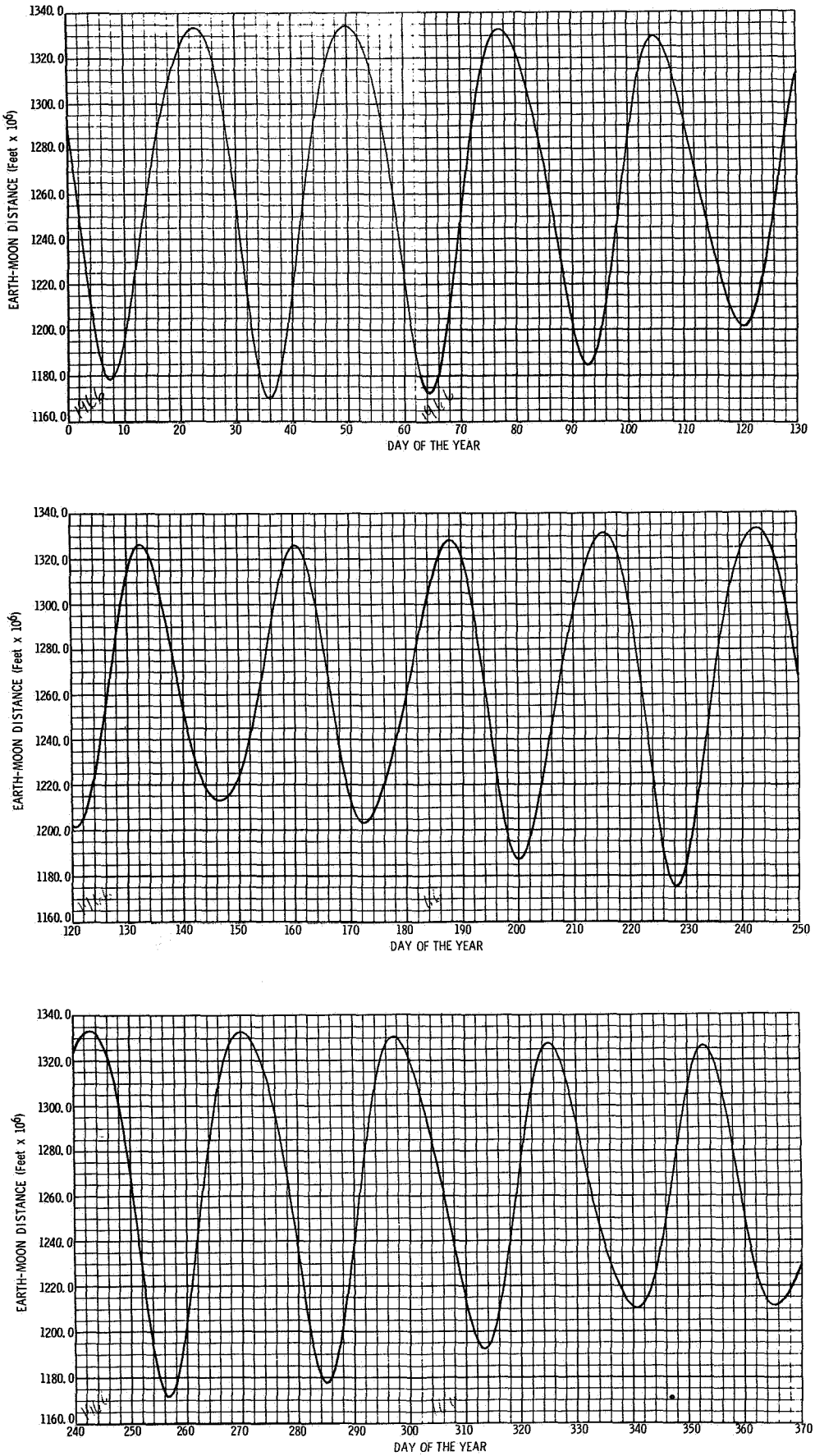
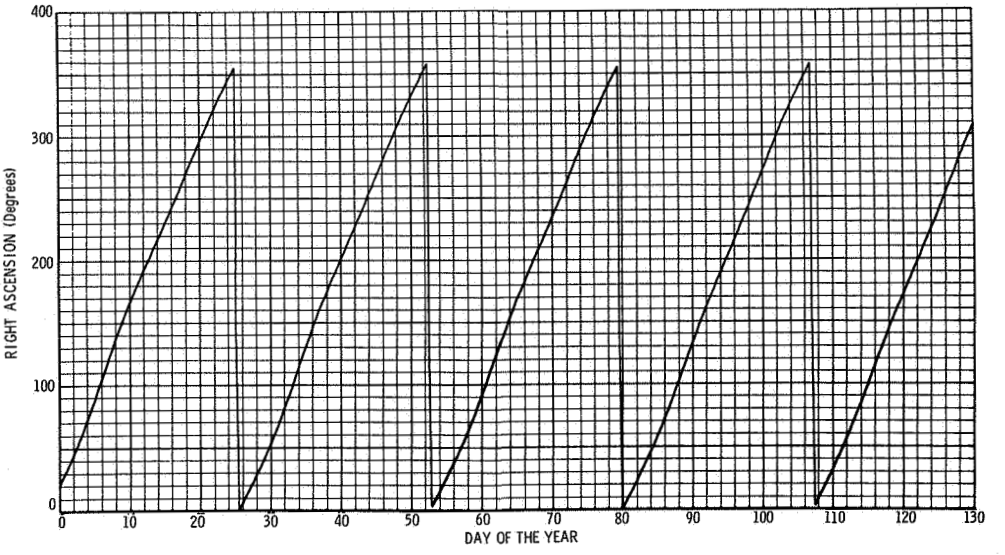
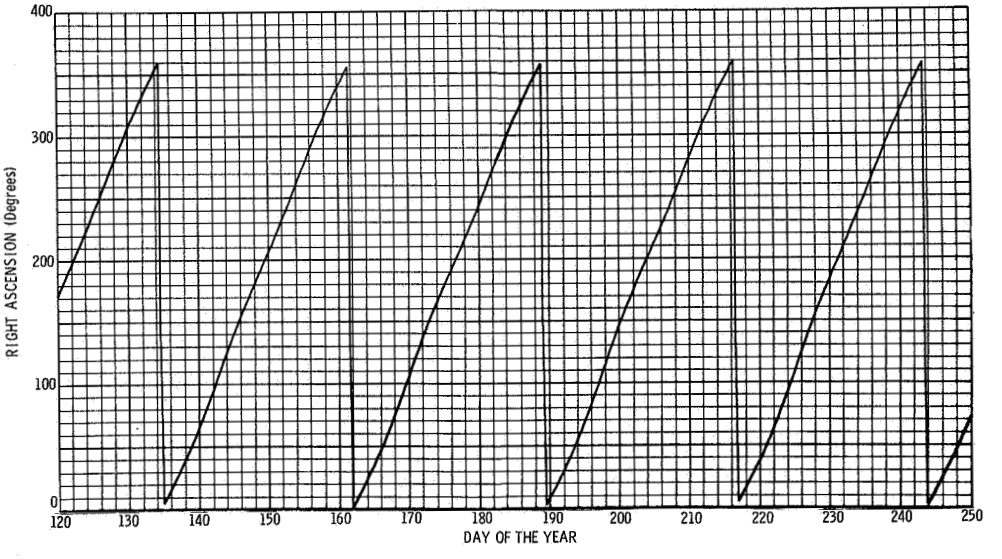


FIGURE B1966-1 EARTH-MOON DISTANCE





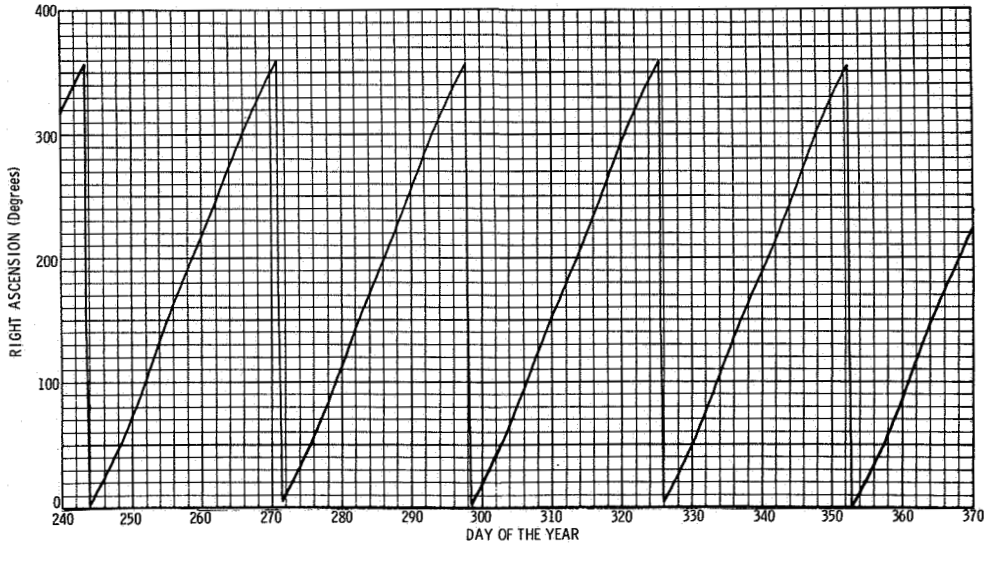


FIGURE B1966-2 RIGHT ASCENSION OF THE MOON

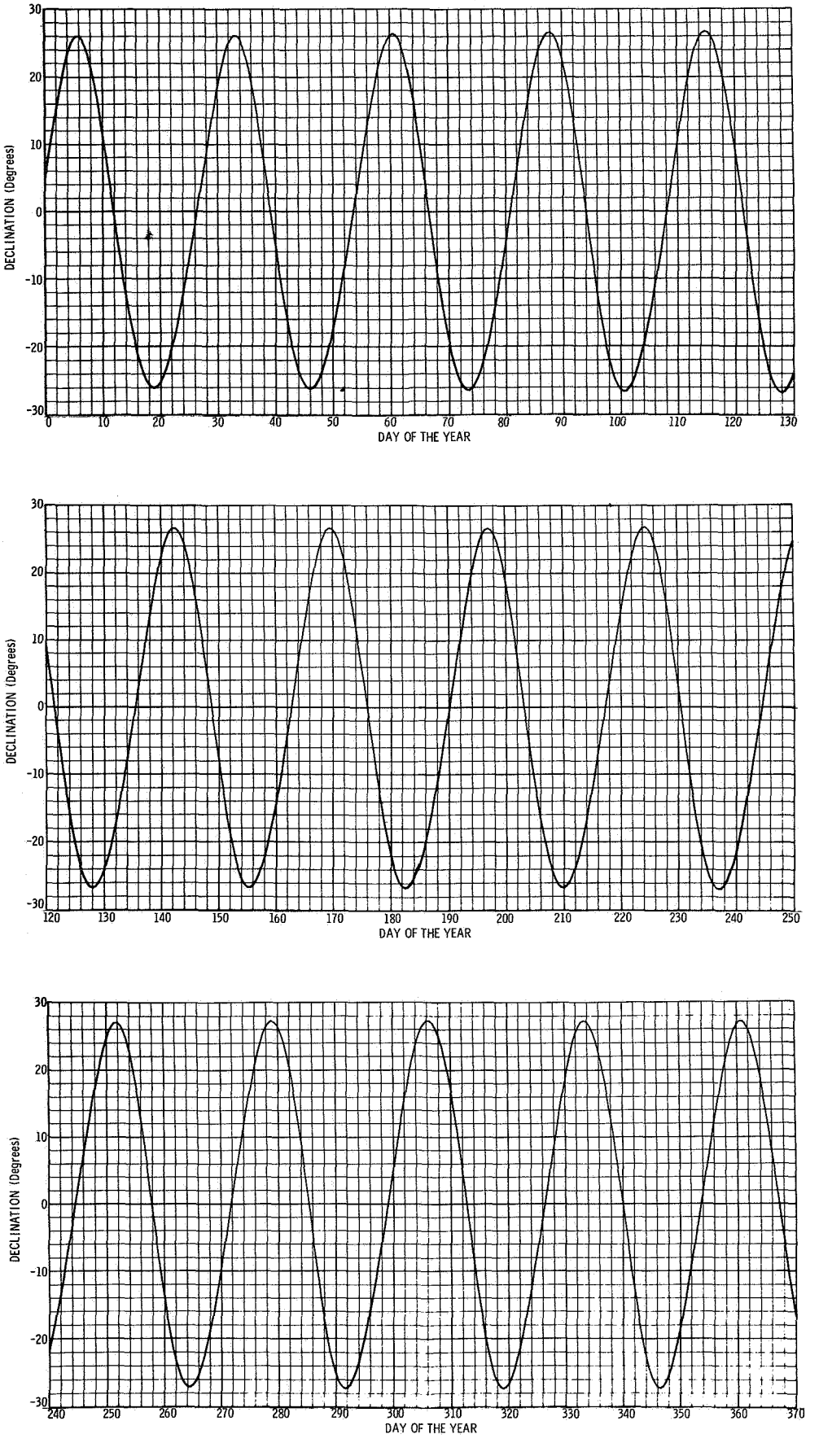


FIGURE B1966-3 DECLINATION OF THE MOON

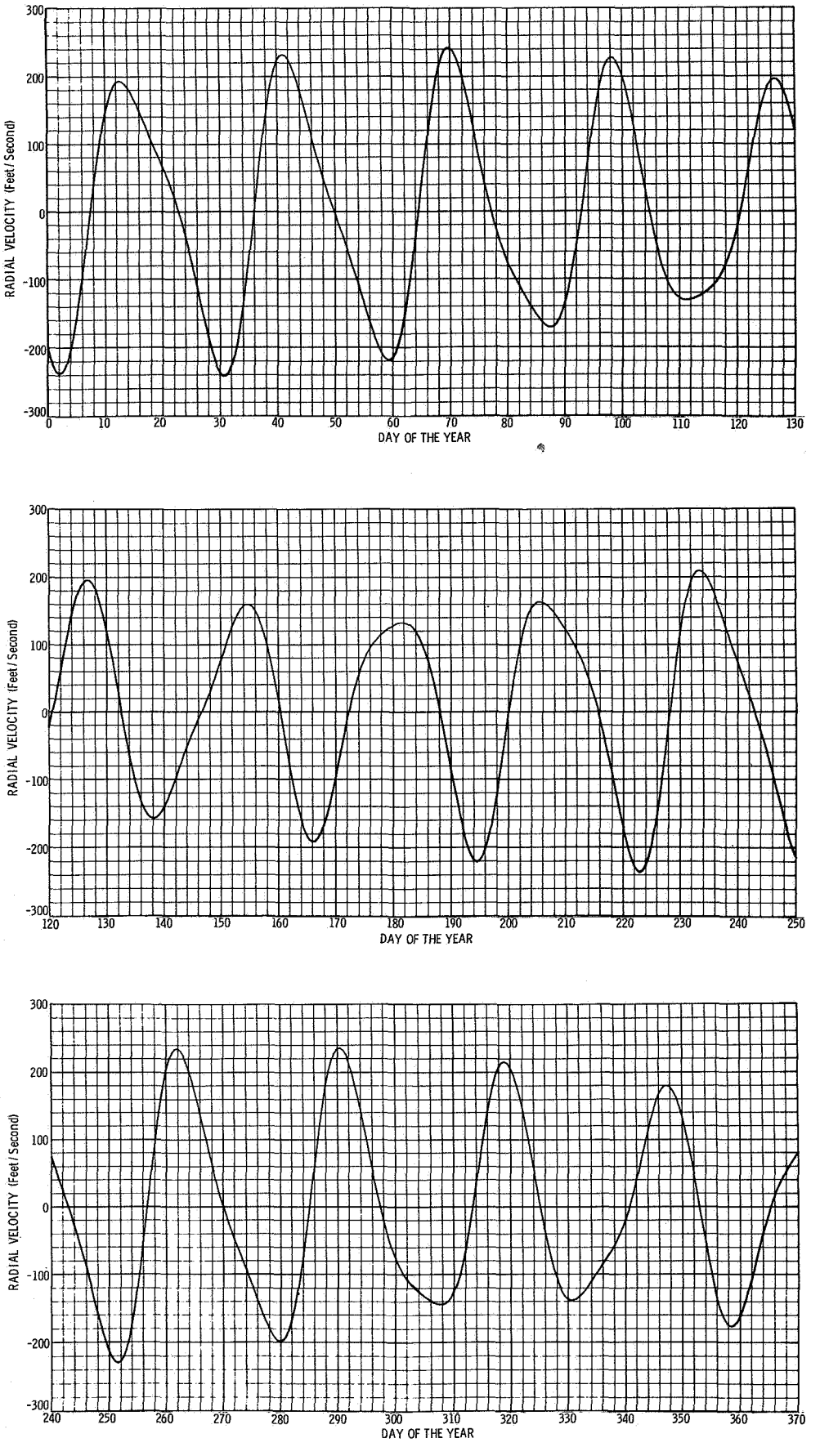
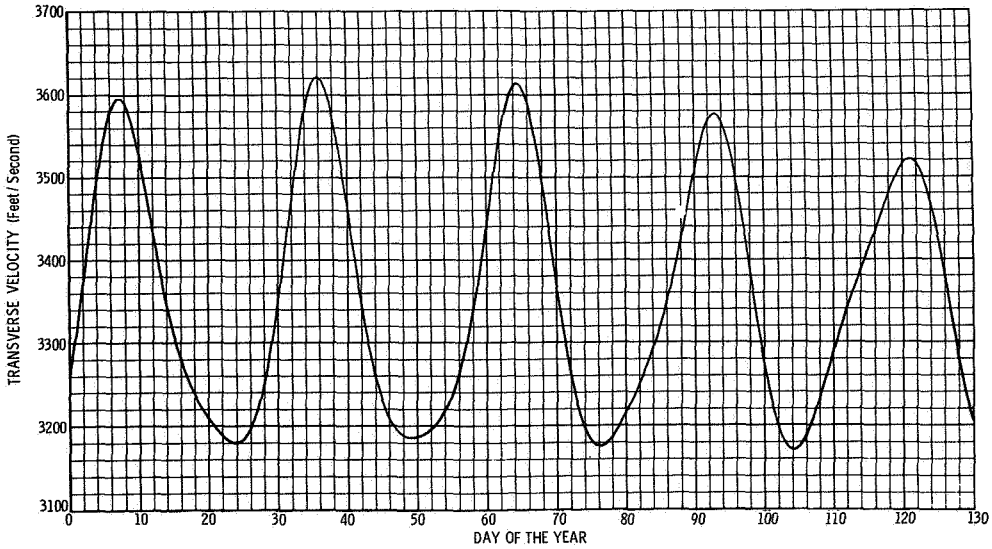
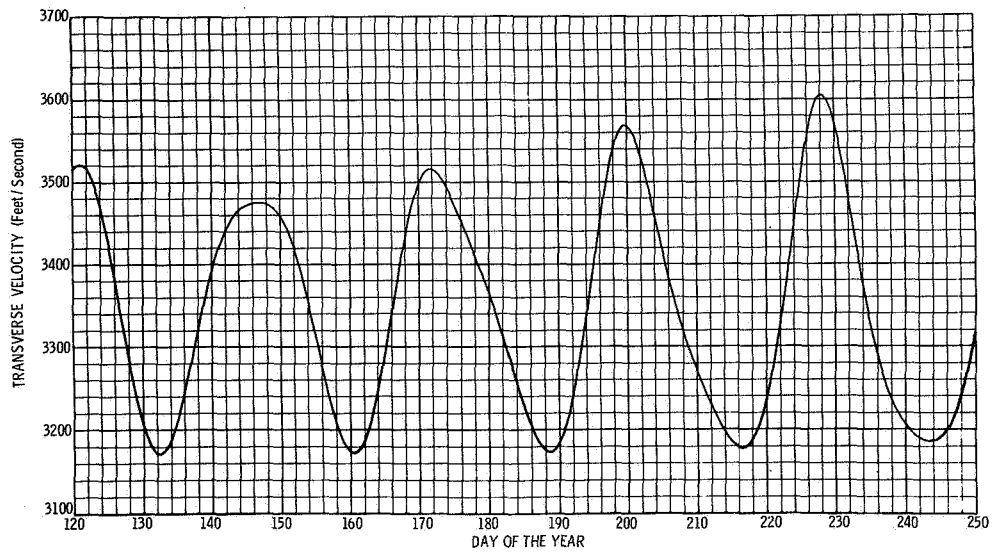


FIGURE B1966-4 RADIAL VELOCITY OF THE MOON





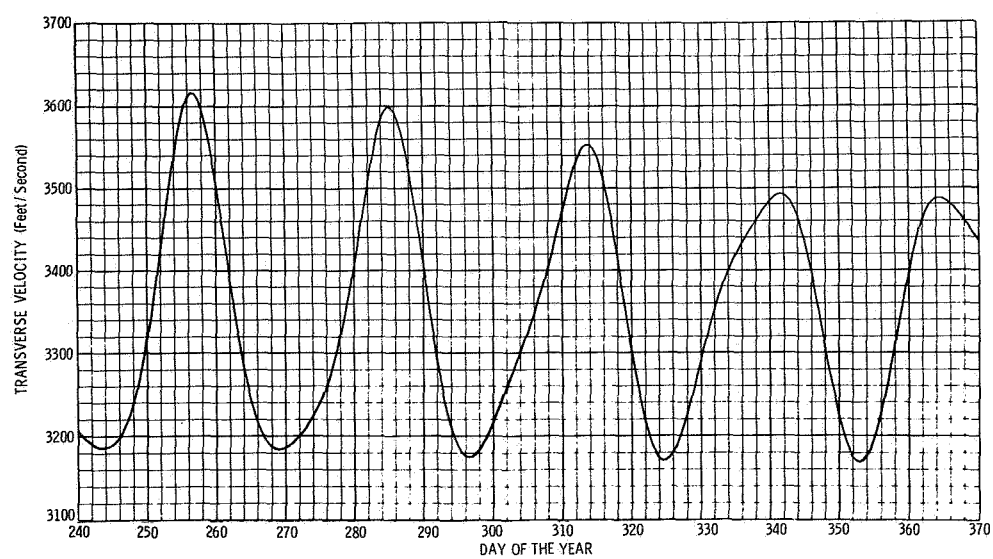
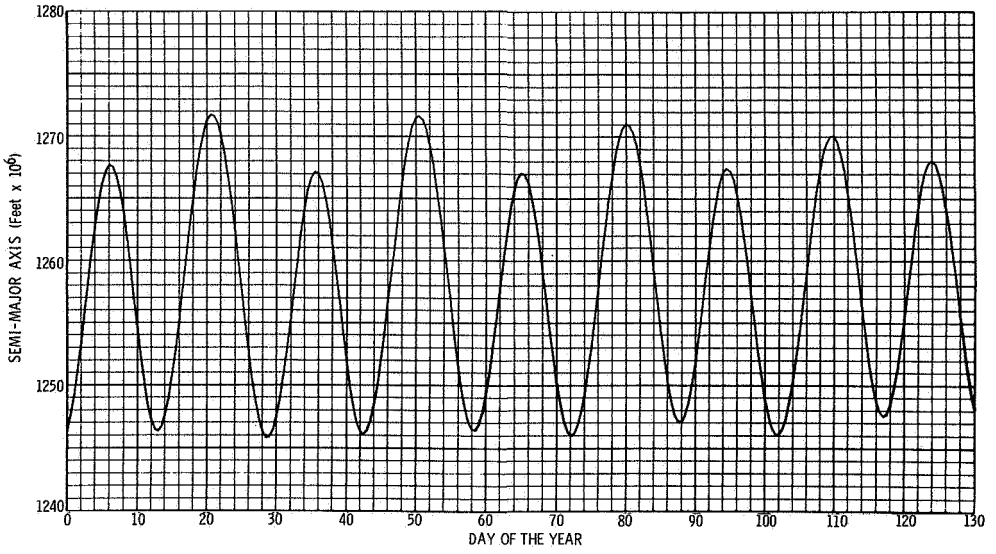
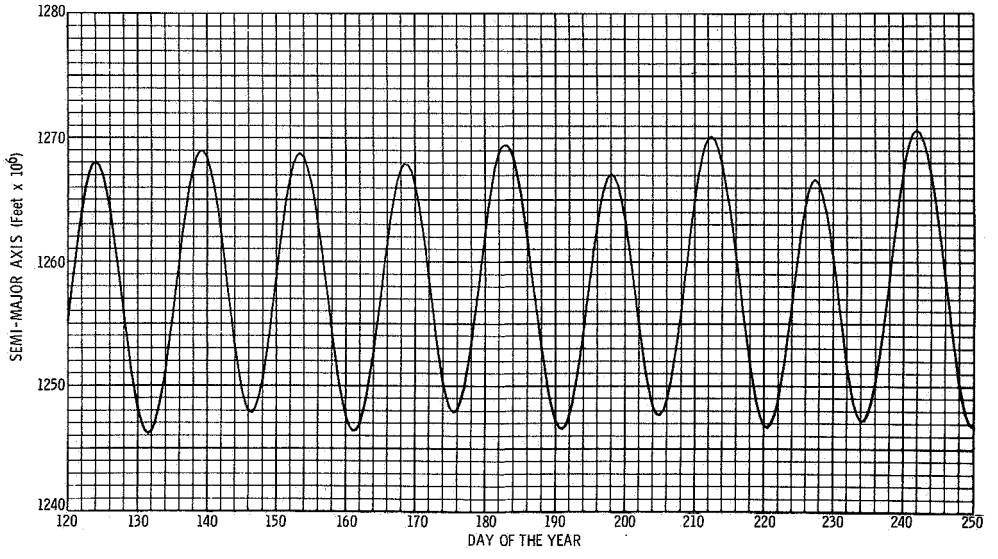


FIGURE B1966-5 TRANSVERSE VELOCITY OF THE MOON





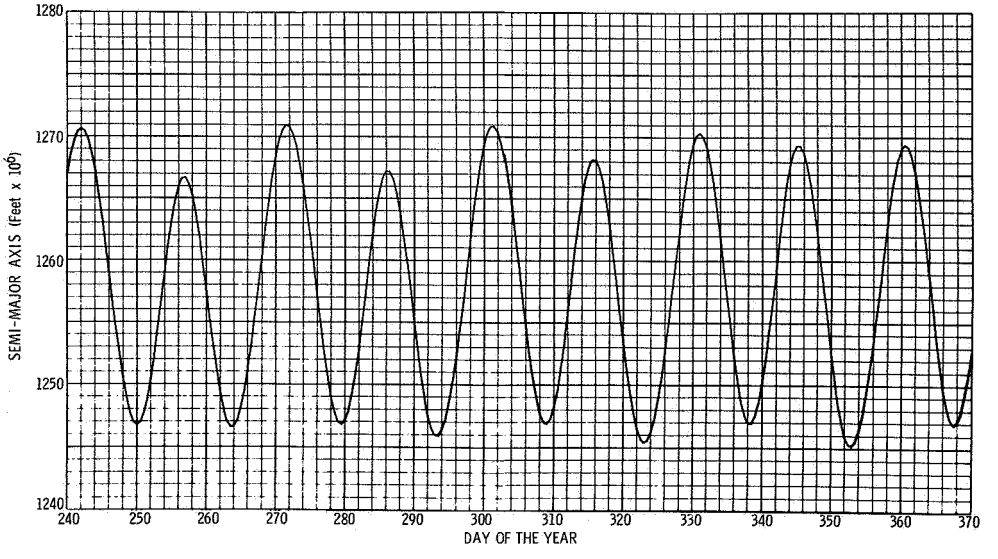


FIGURE B1966-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT

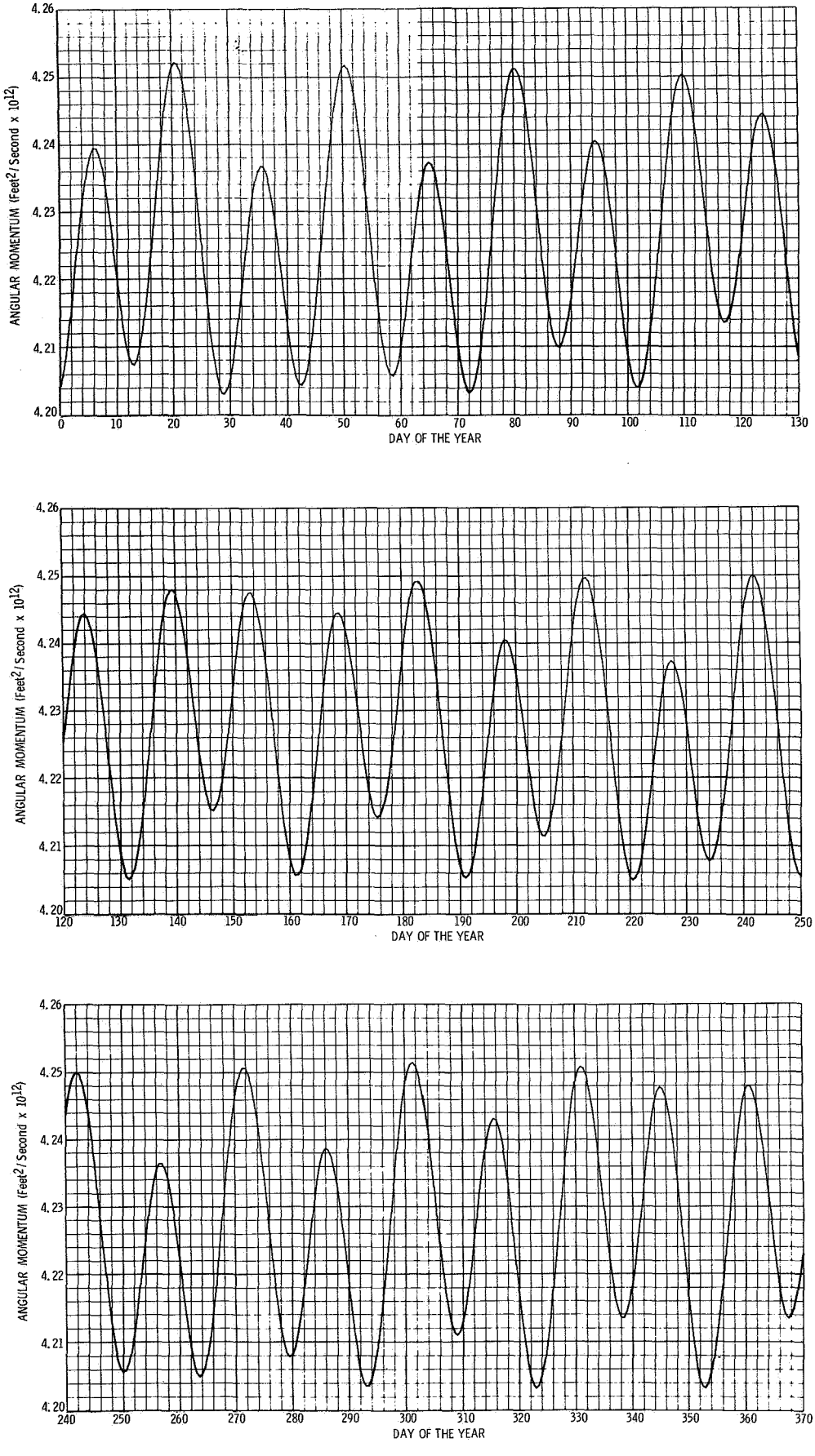
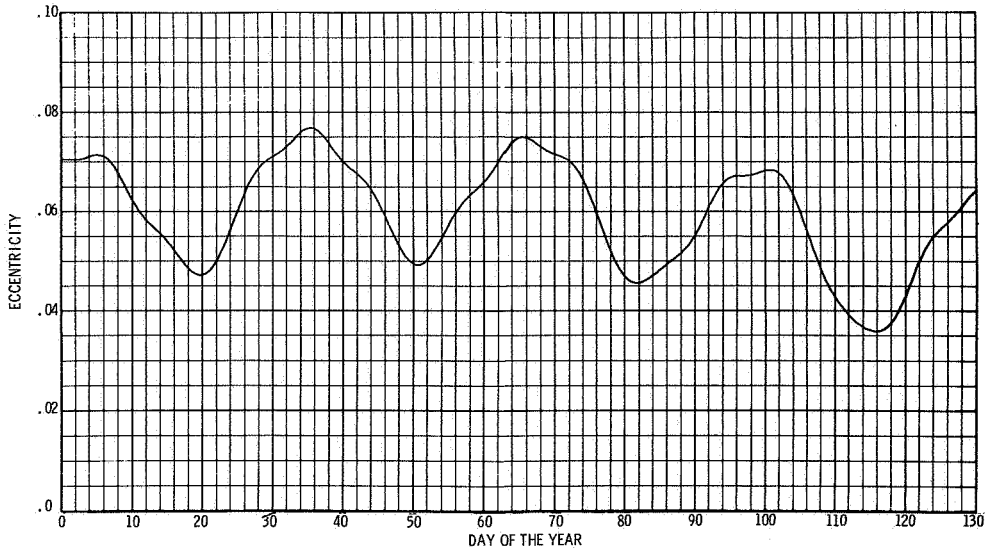
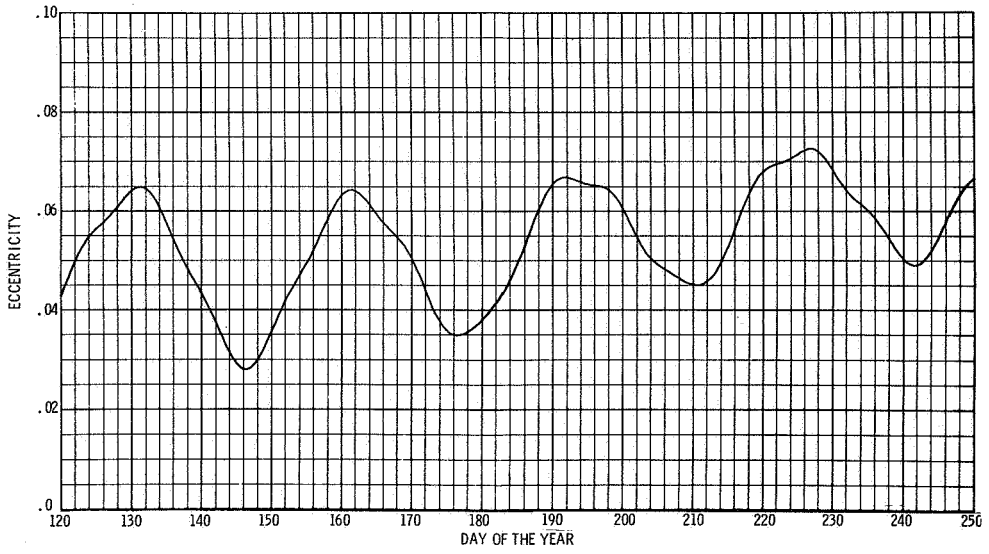


FIGURE B1966-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON





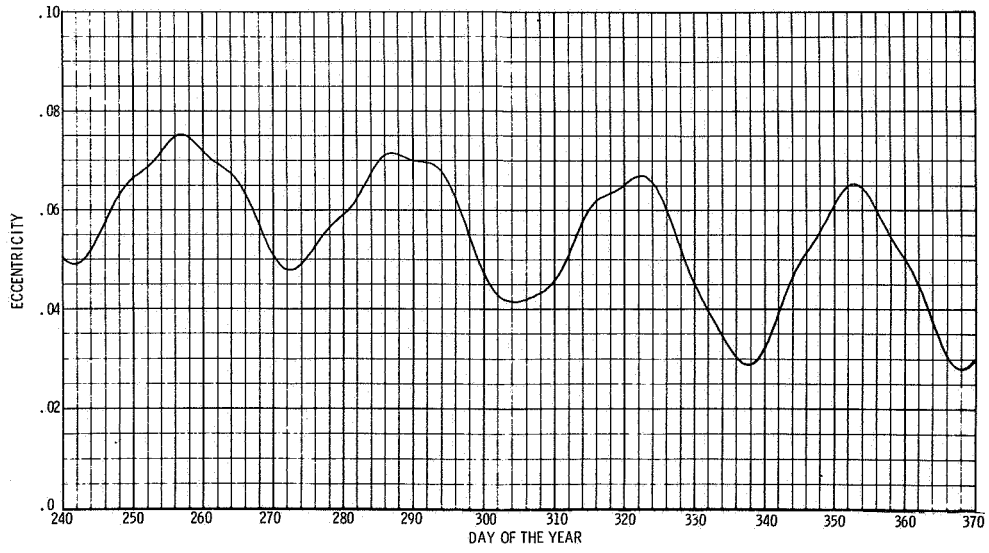
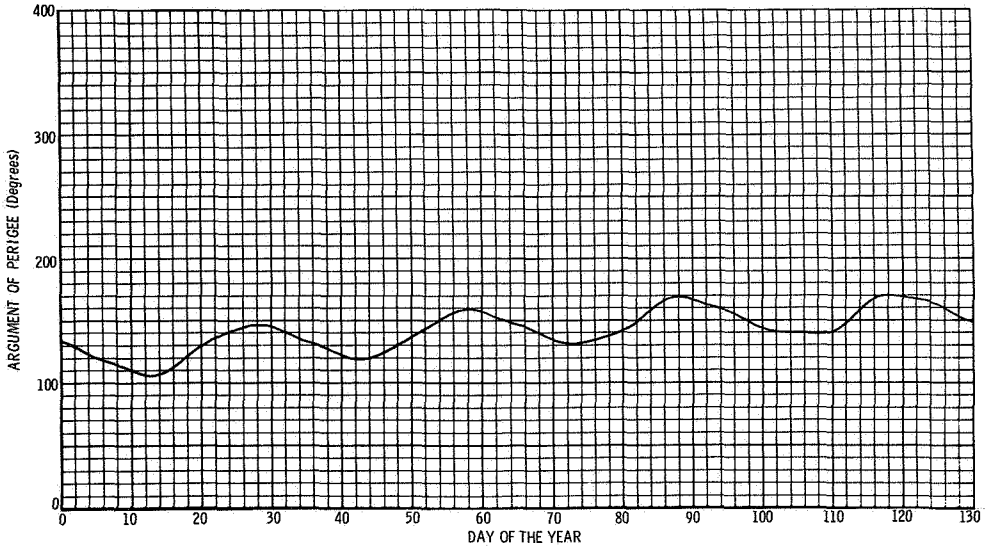
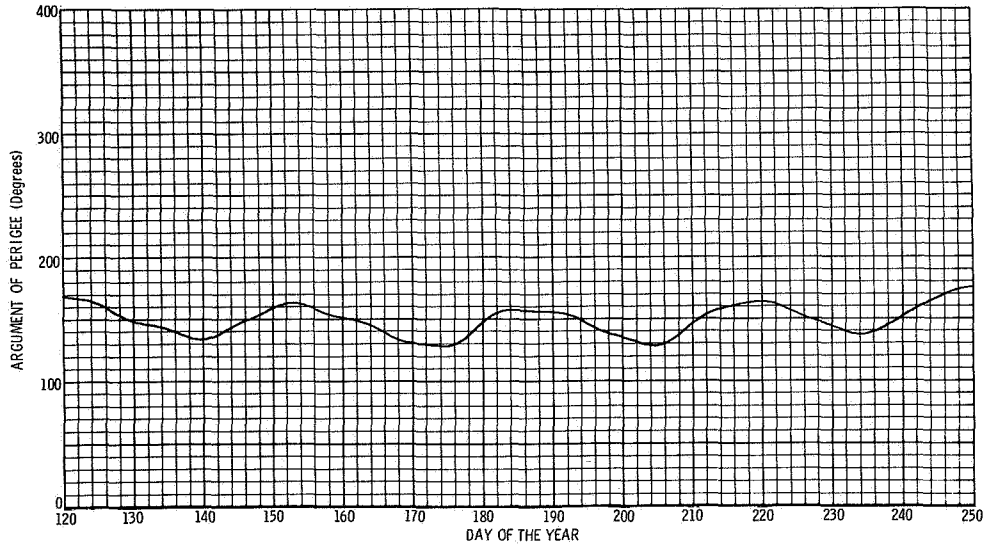


FIGURE B1966-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT





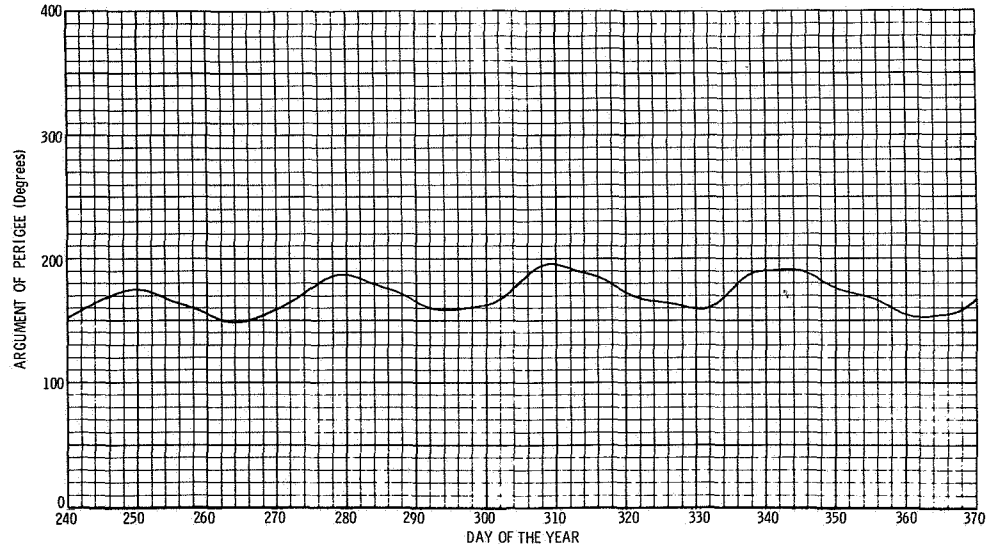
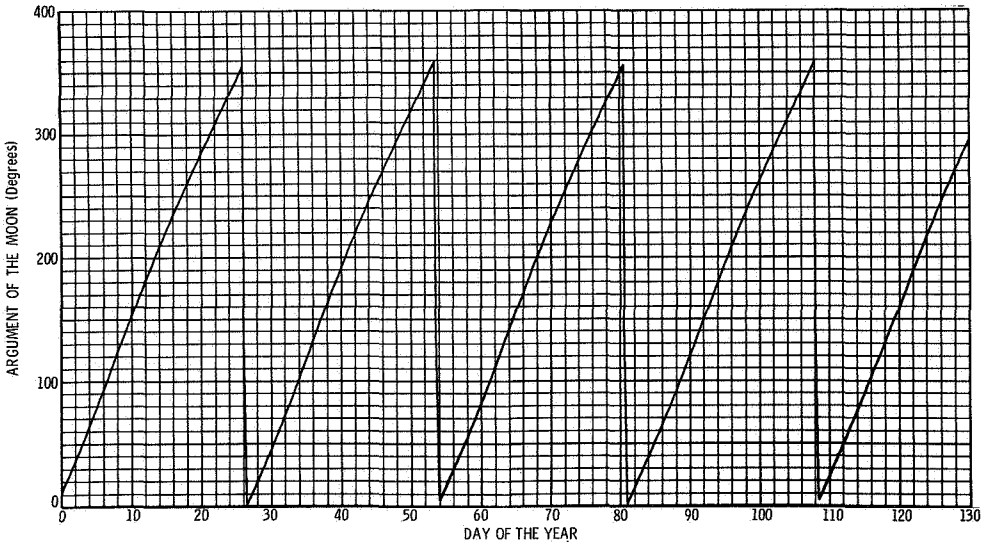
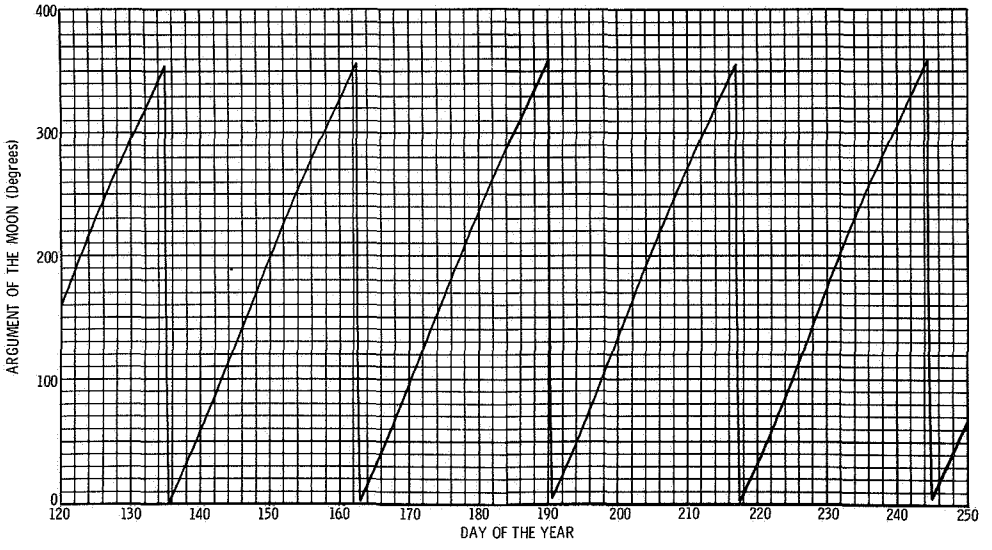


FIGURE B1966-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE





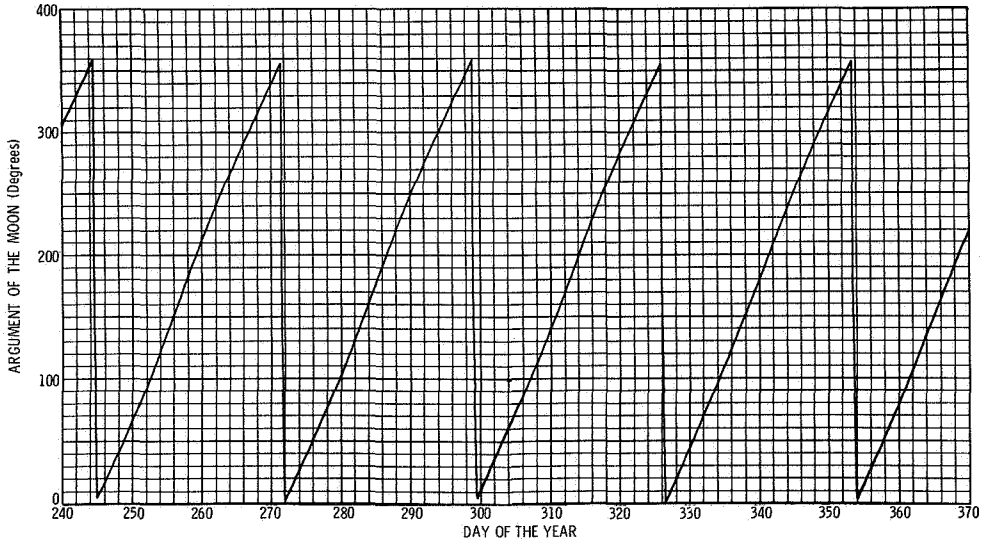
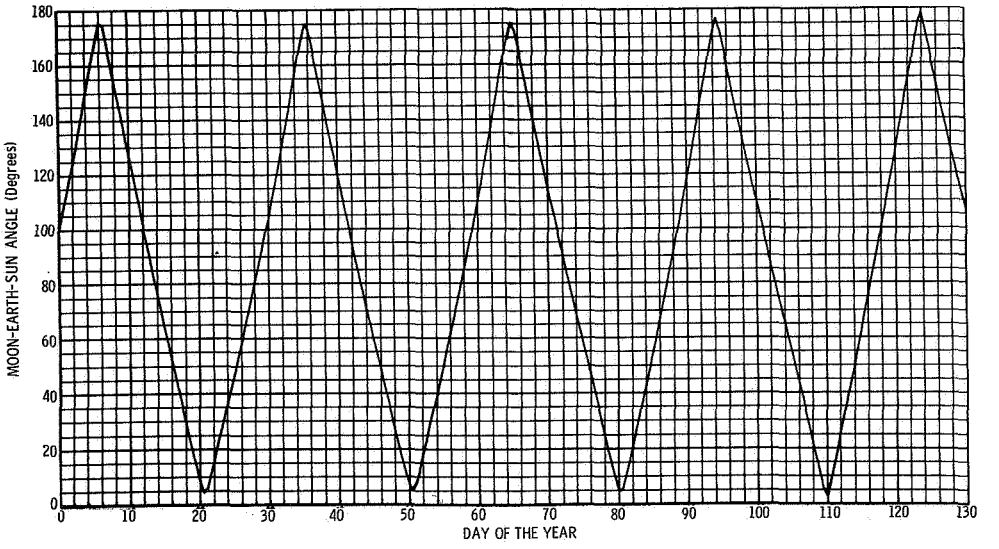
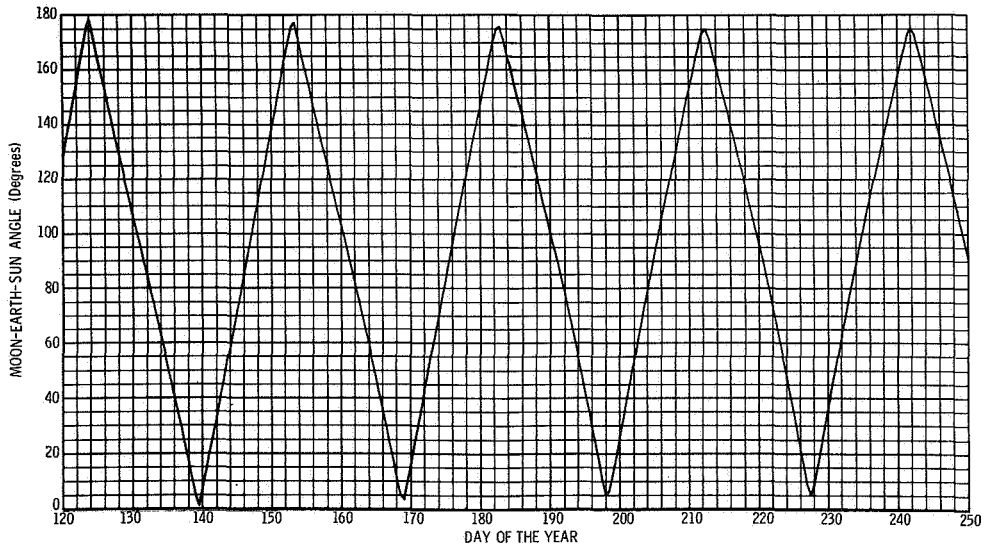


FIGURE B1966-10 ARGUMENT OF THE MOON'S POSITION





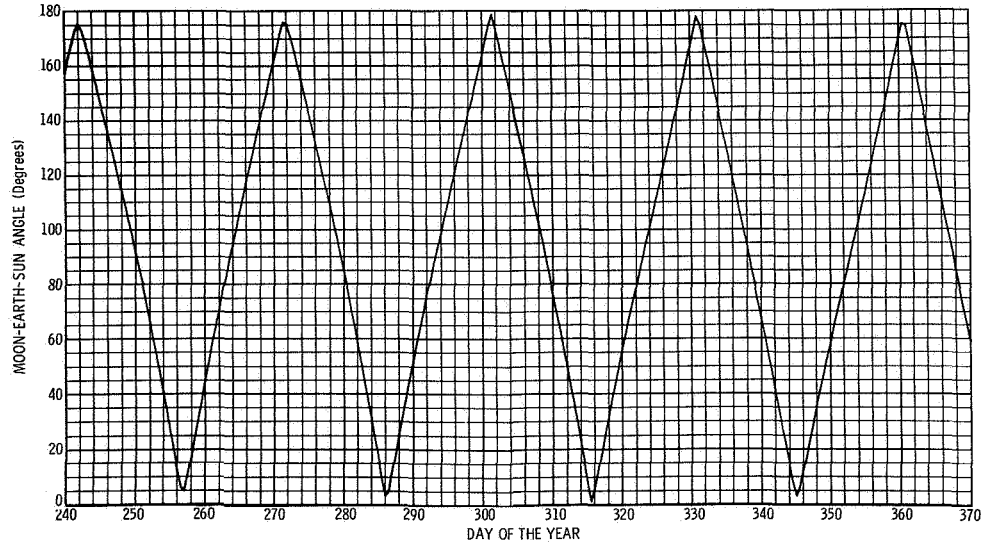
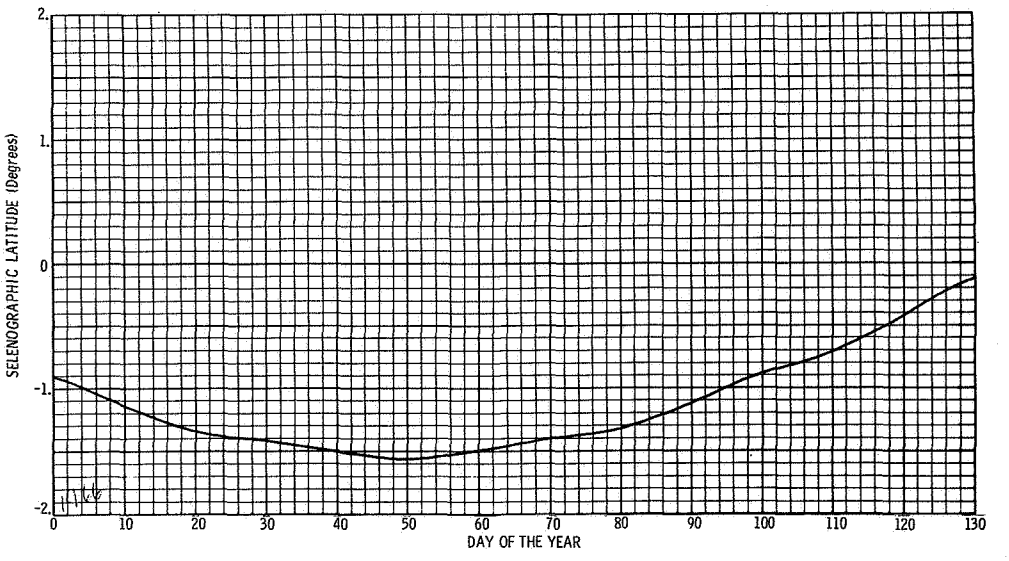
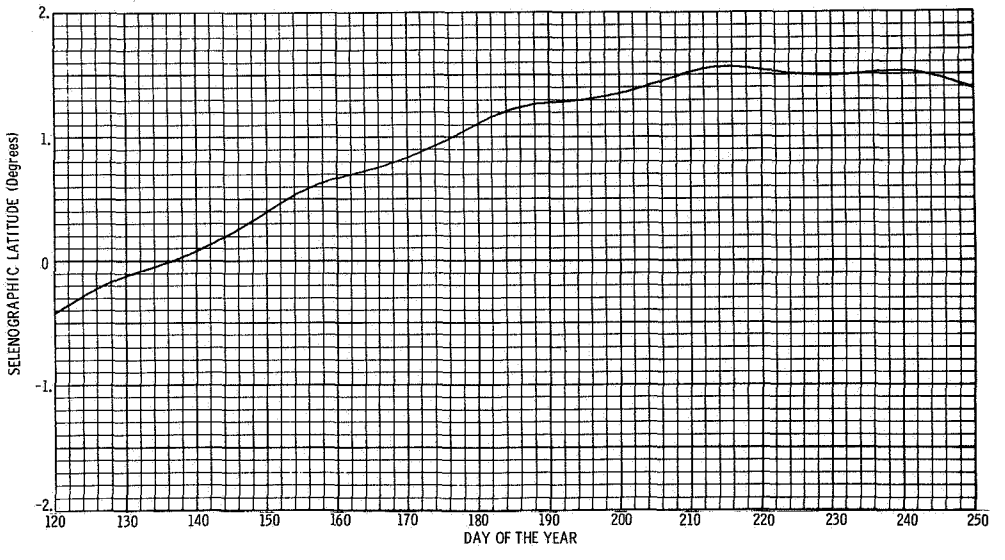


FIGURE B1966-11 MOON-EARTH-SUN ANGLE





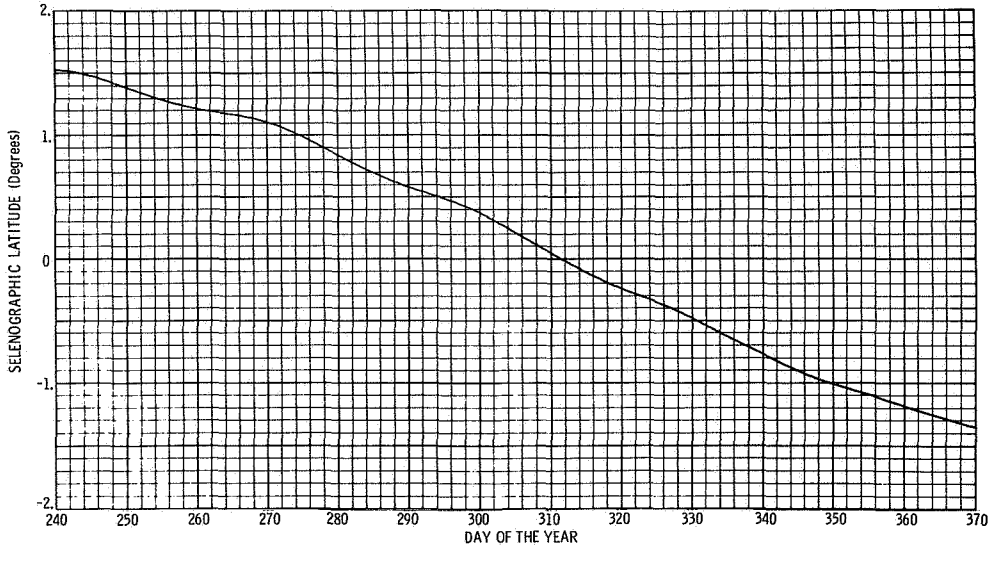


FIGURE B1966-12 SELENOGRAPHIC LATITUDE OF THE SUN

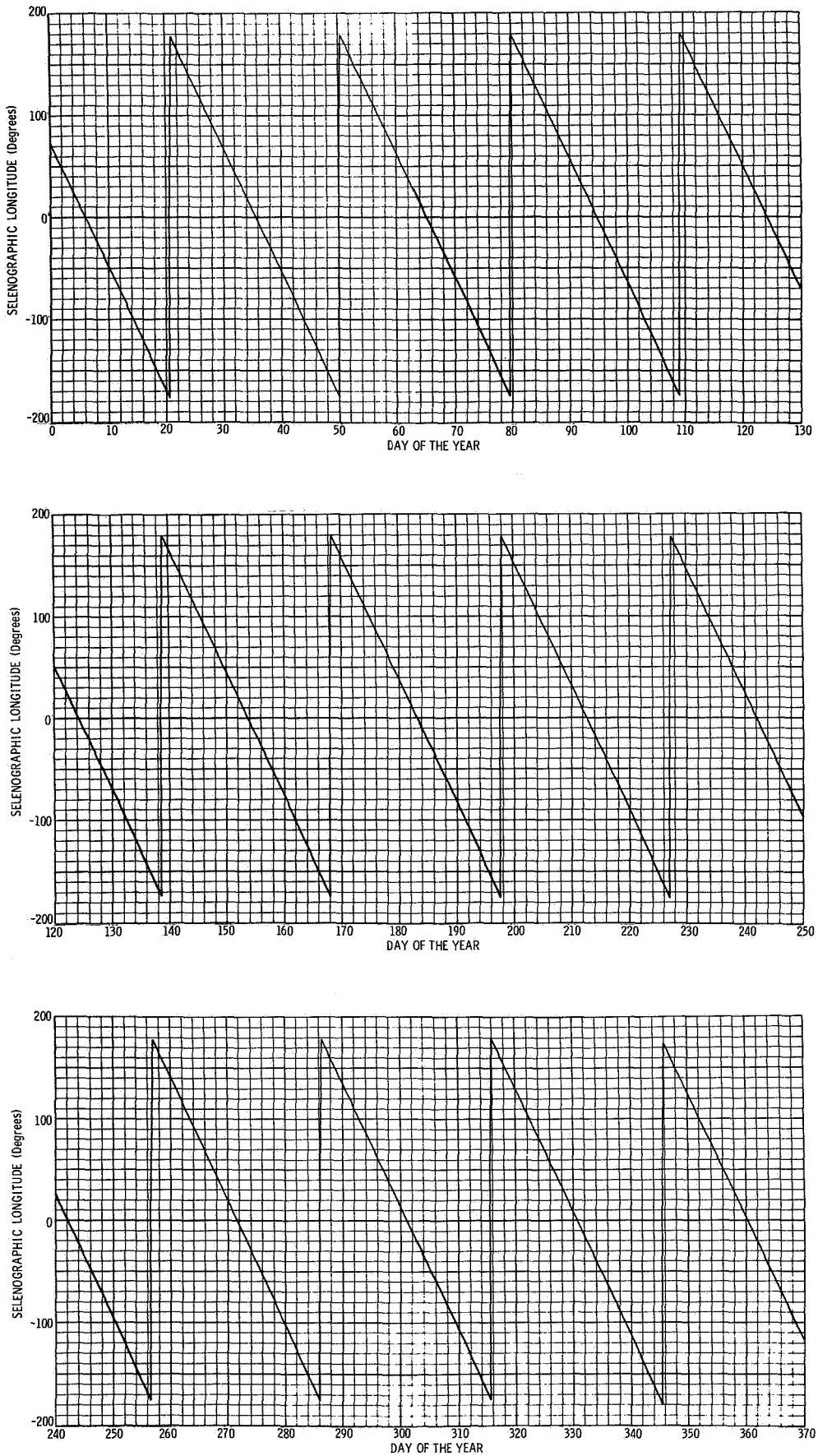
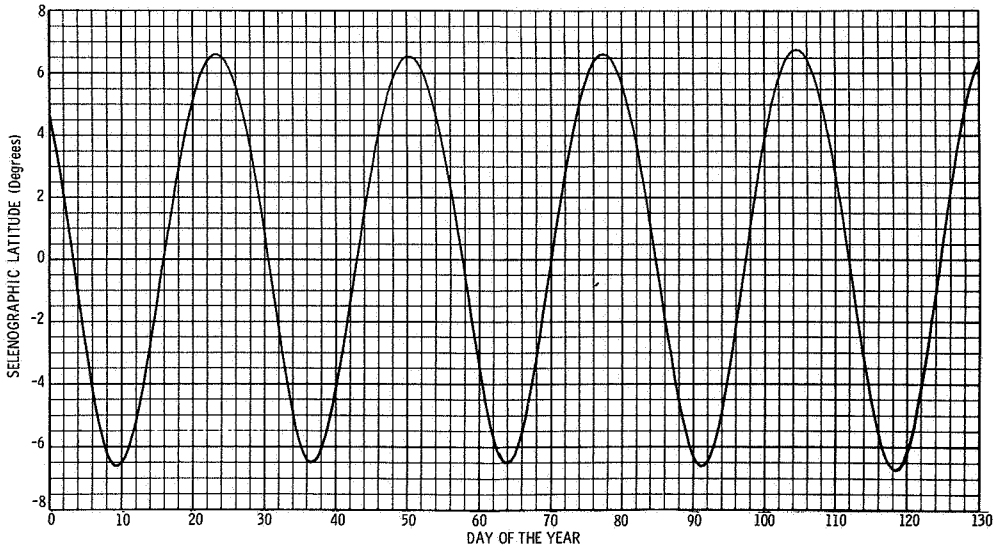
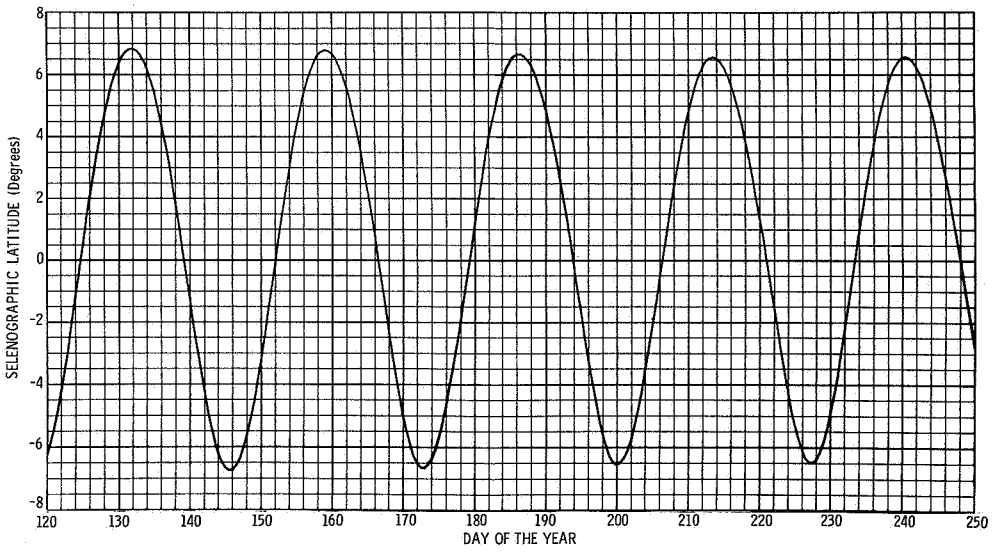


FIGURE B1966-13 SELENOGRAPHIC LONGITUDE OF THE SUN





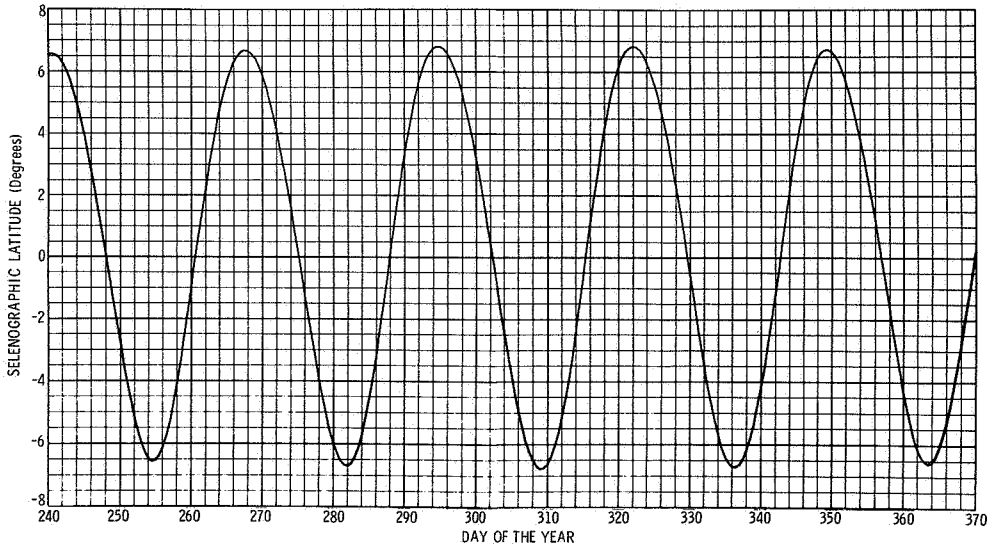


FIGURE B1966-14 SELENOGRAPHIC LATITUDE OF THE EARTH

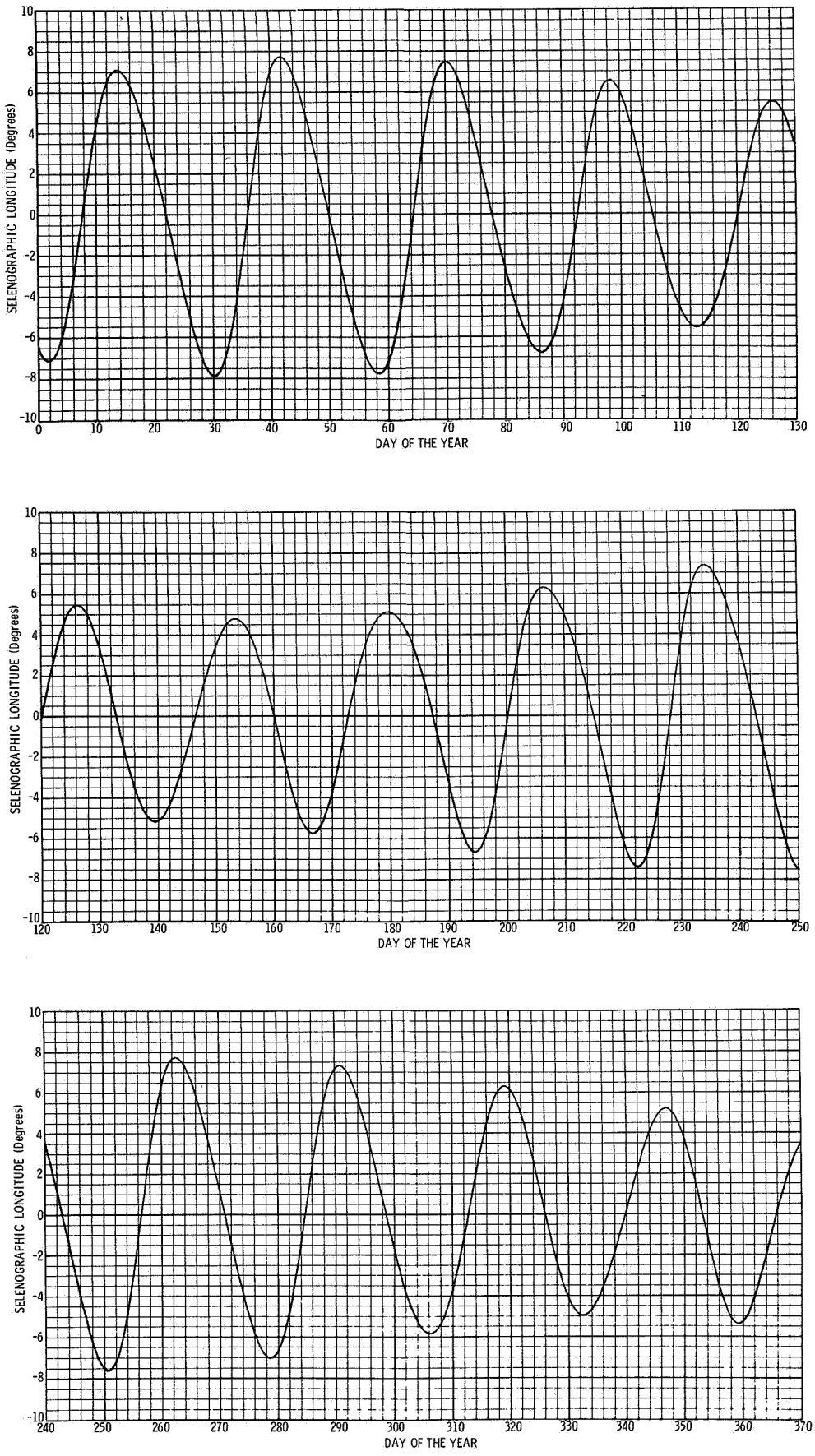


FIGURE B1966-15 SELENOGRAPHIC LONGITUDE OF THE EARTH

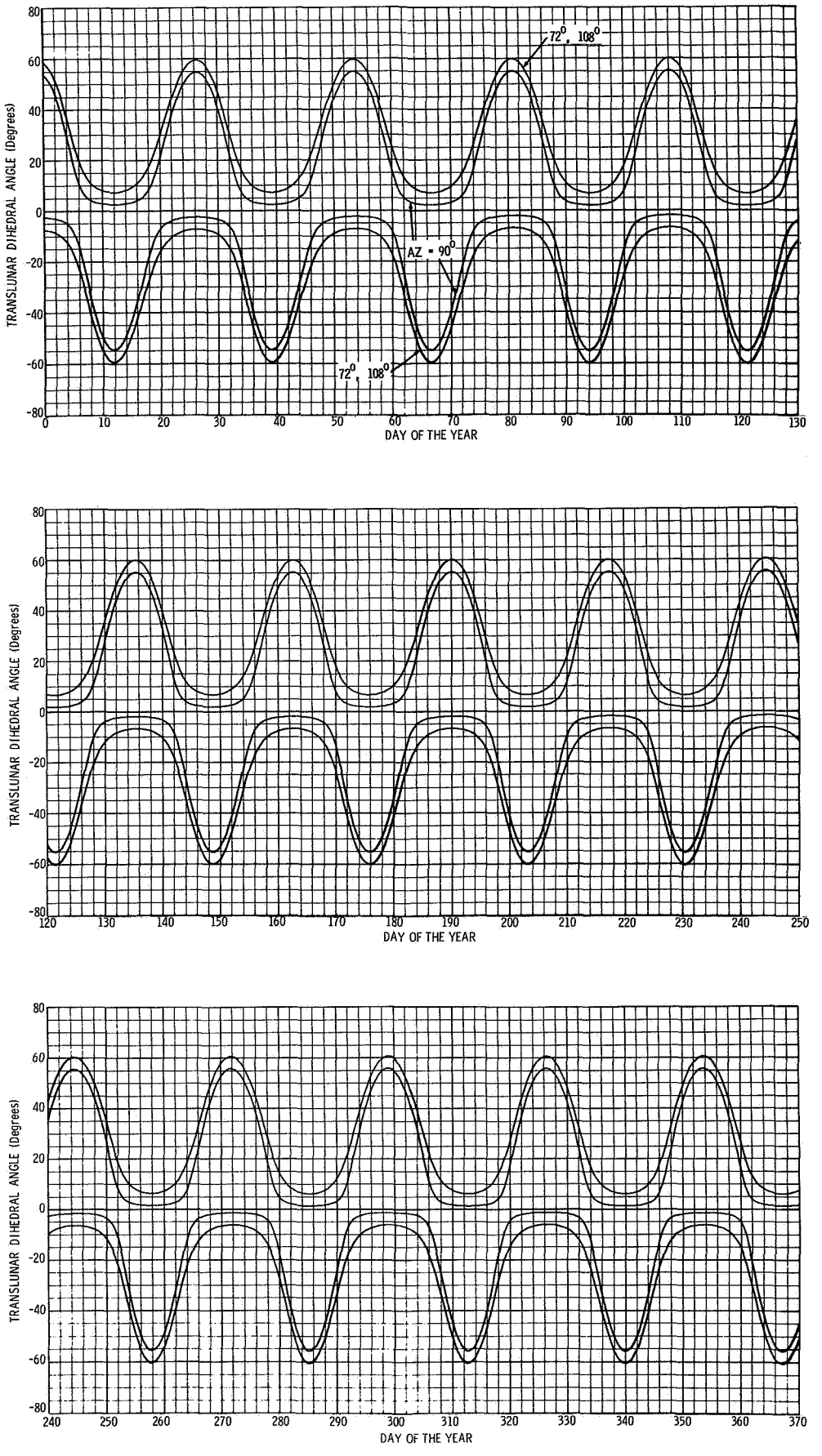
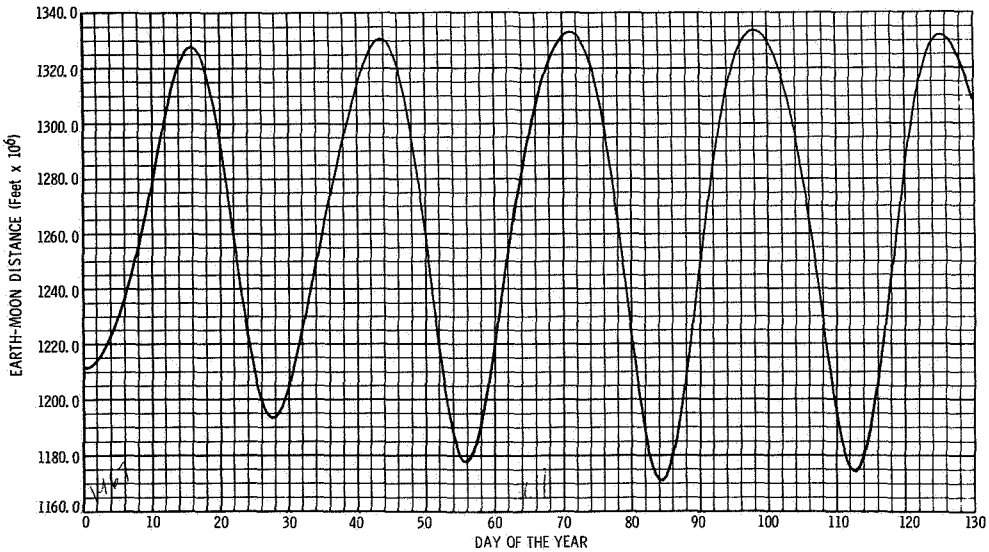
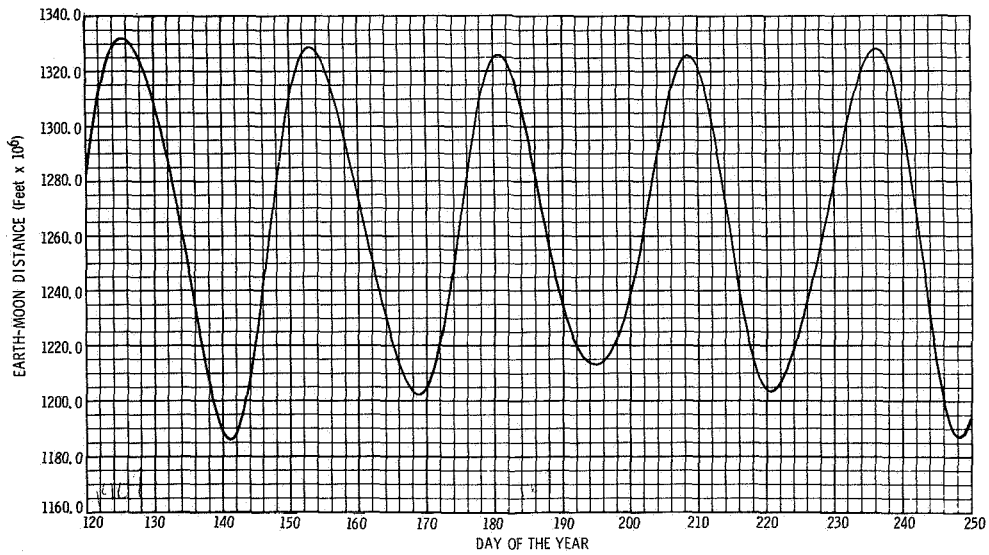


FIGURE B1966-16 TRANSLUNAR DIHEDRAL ANGLES

1967





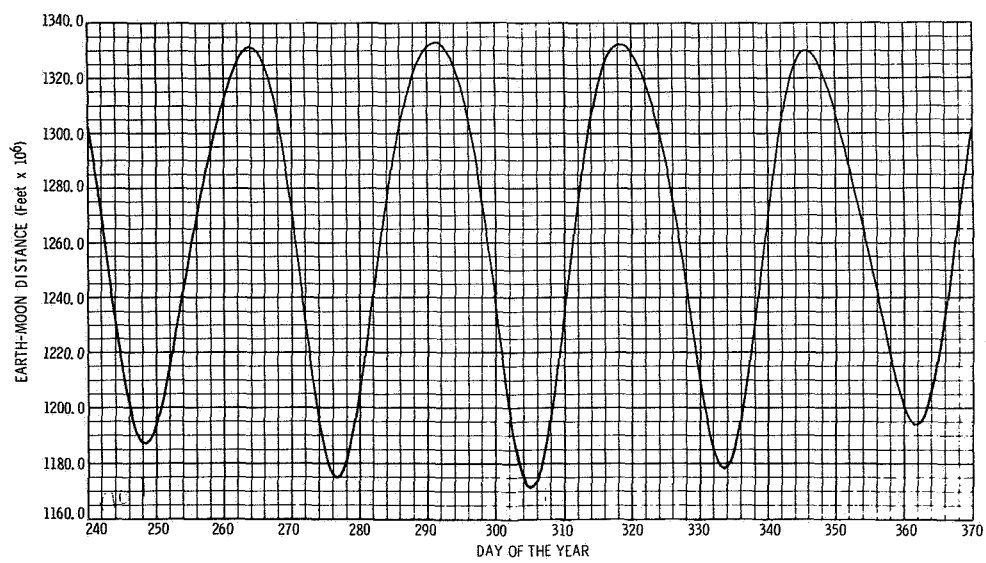
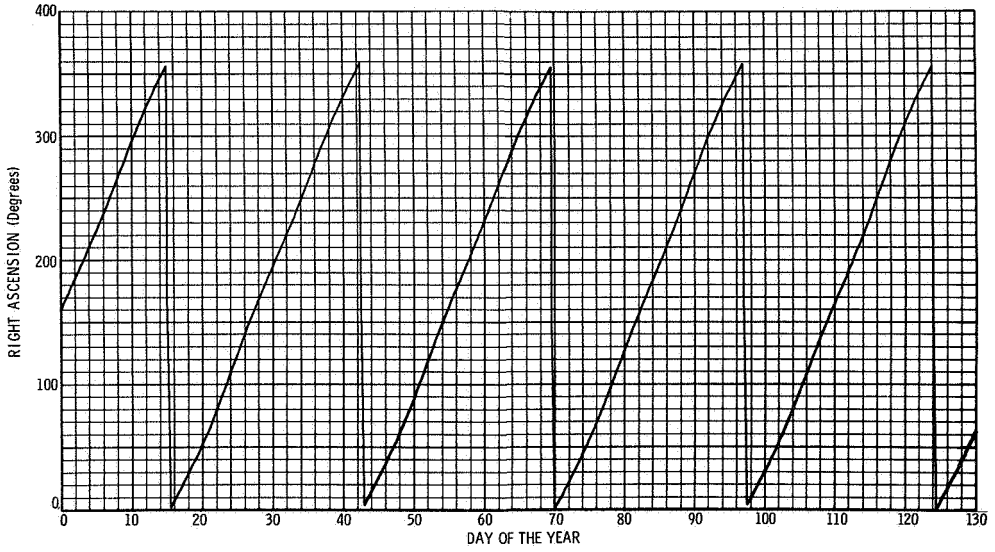
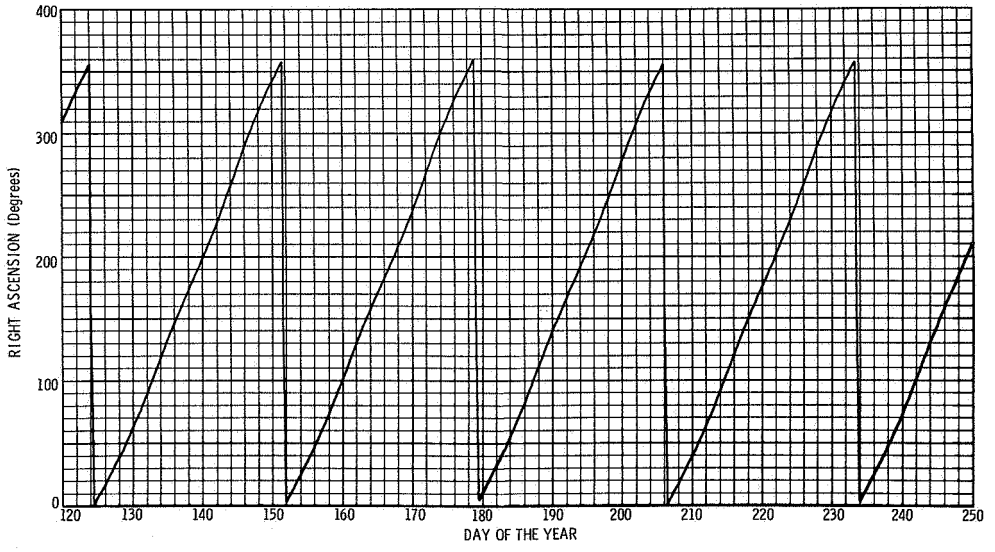


FIGURE B1967-1 EARTH-MOON DISTANCE





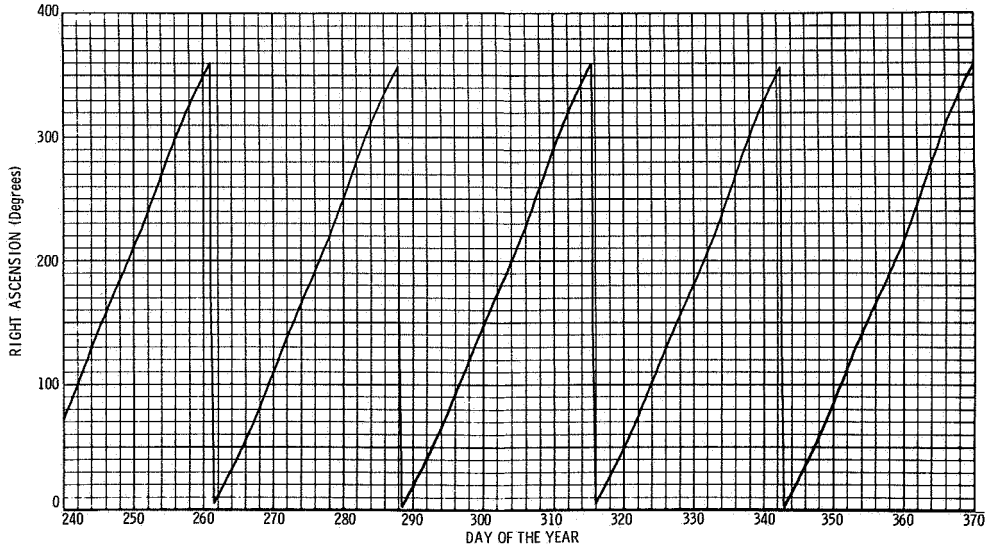
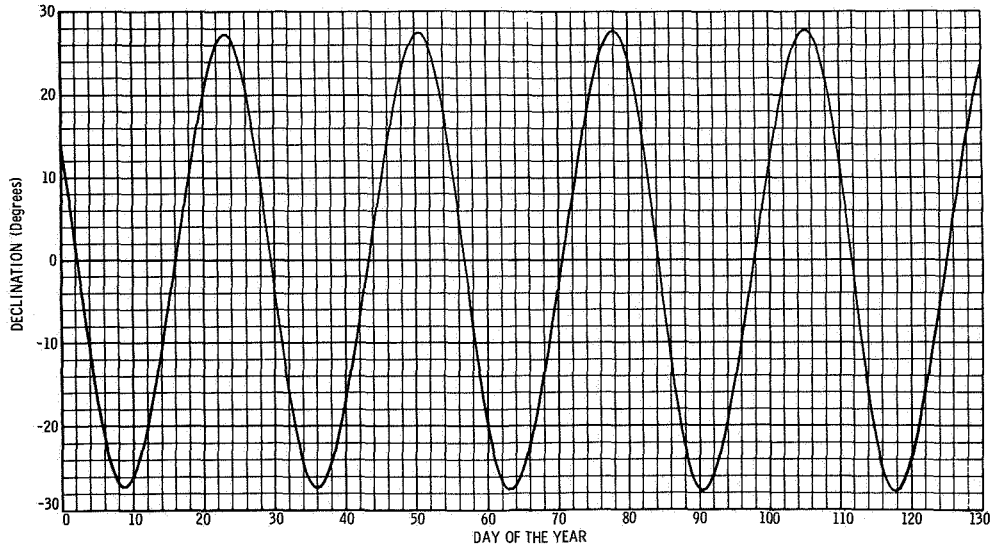
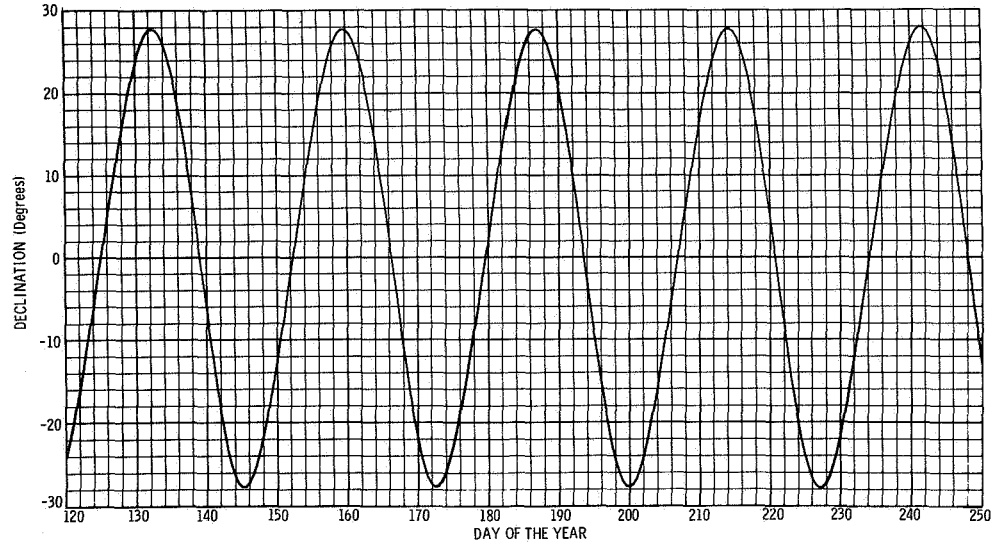


FIGURE B1967-2 RIGHT ASCENSION OF THE MOON





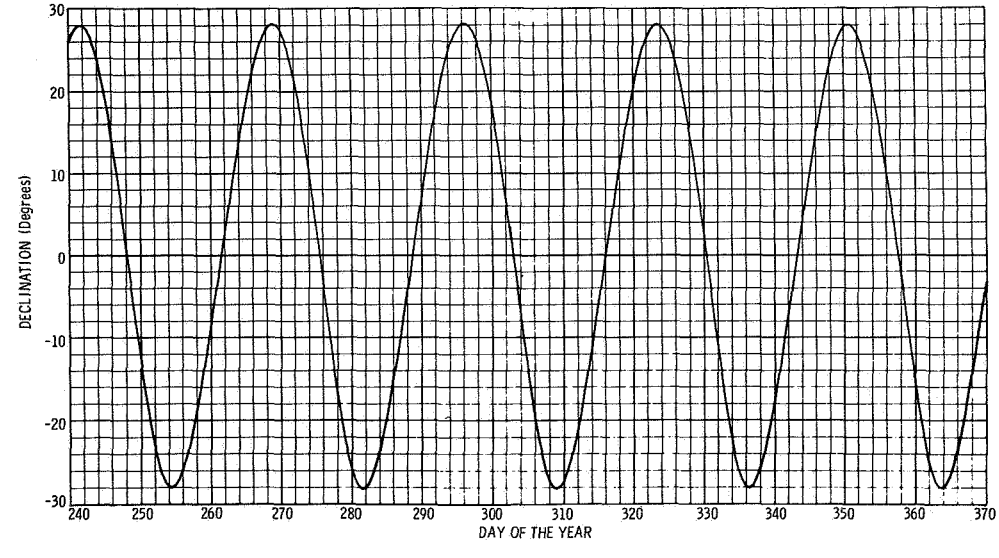
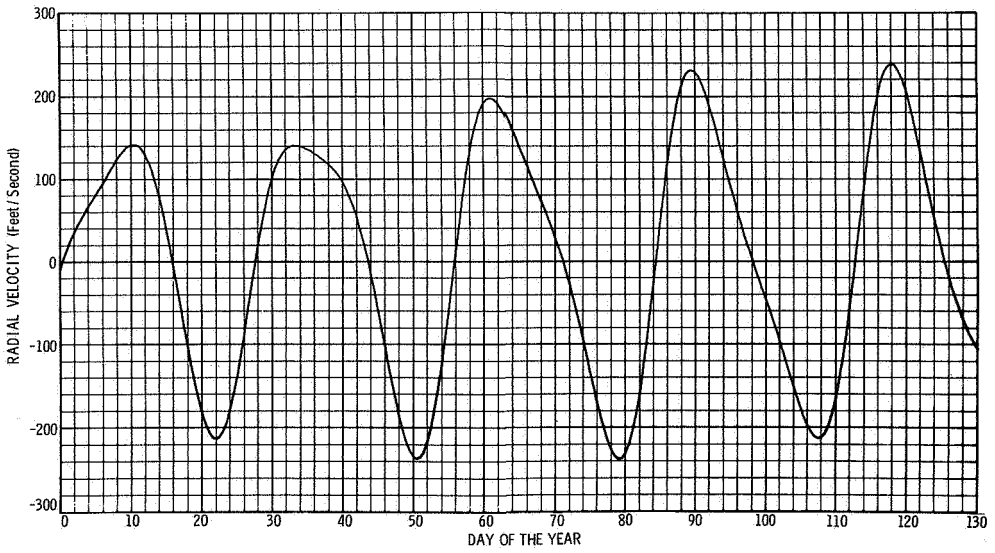
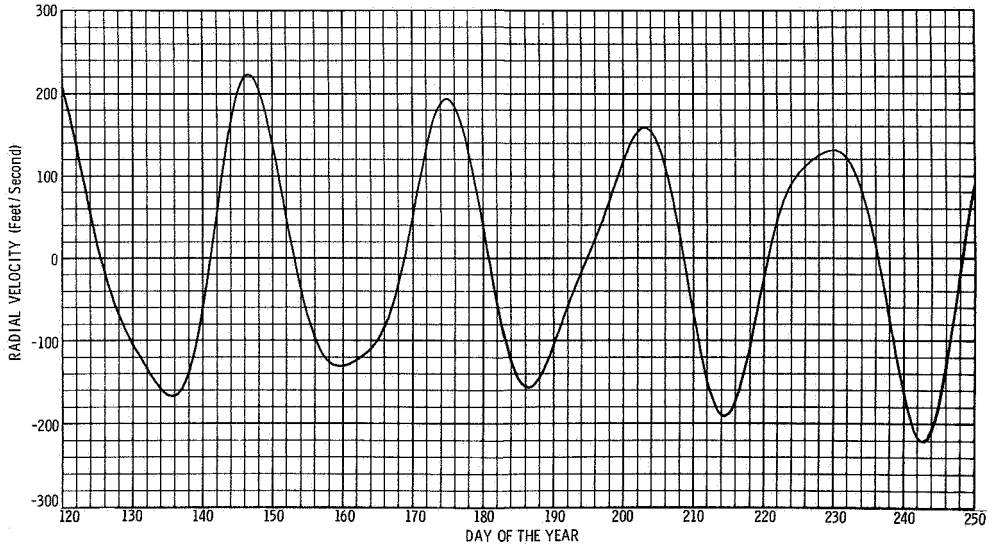


FIGURE B1967 -3 DECLINATION OF THE MOON





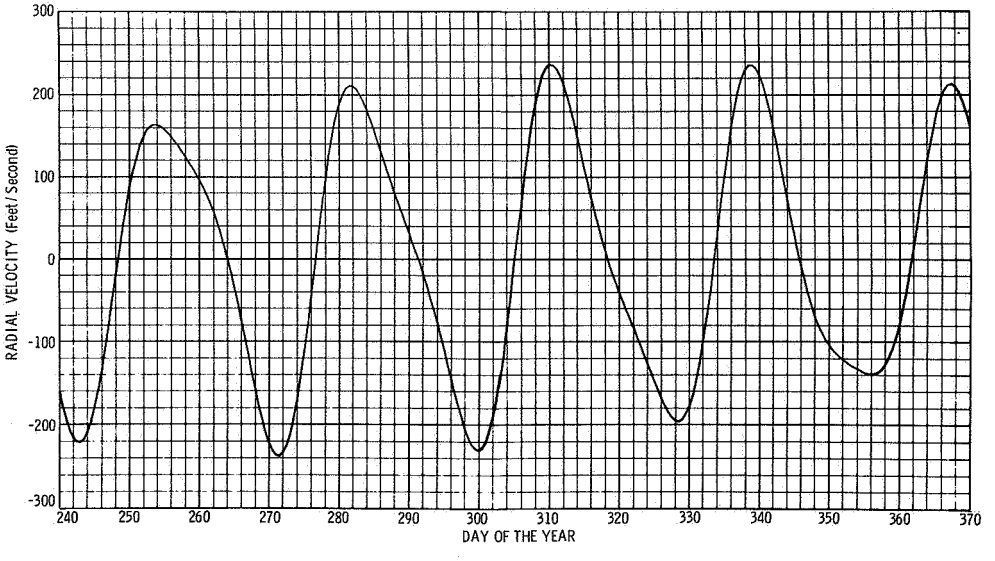
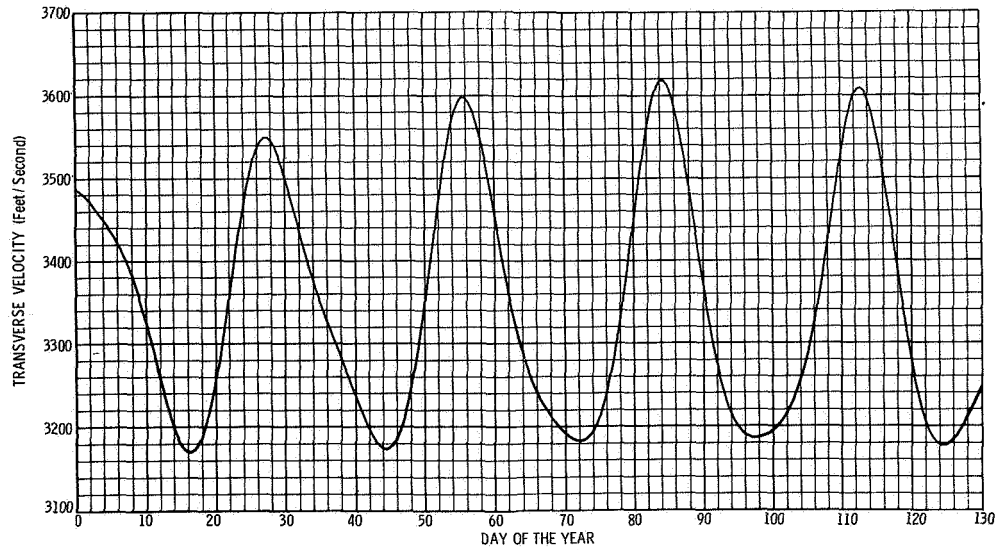
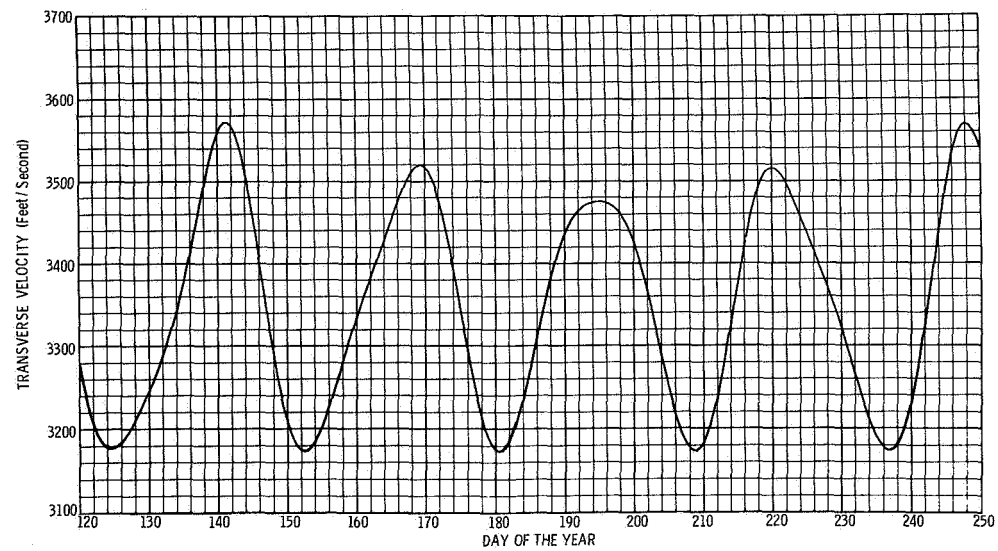


FIGURE B1967-4 RADIAL VELOCITY OF THE MOON





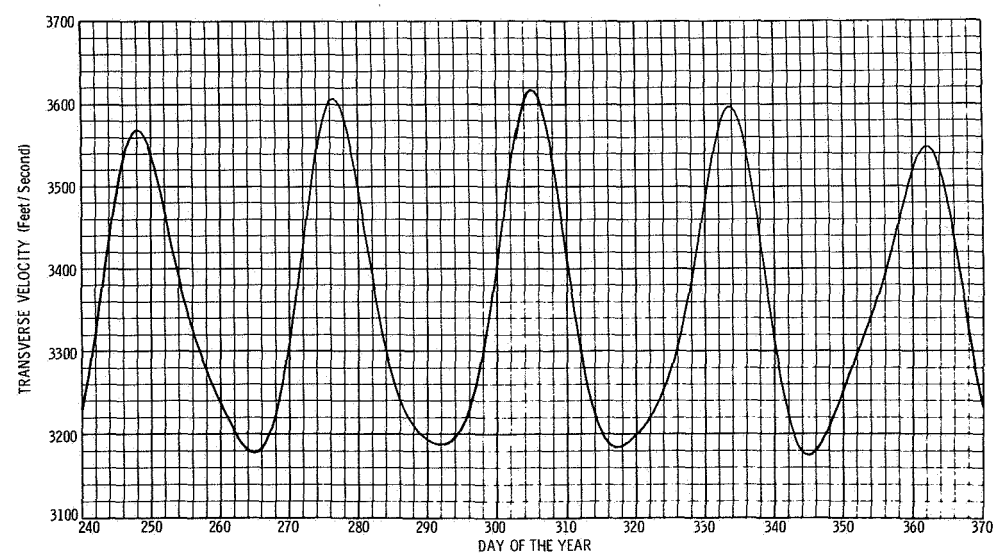
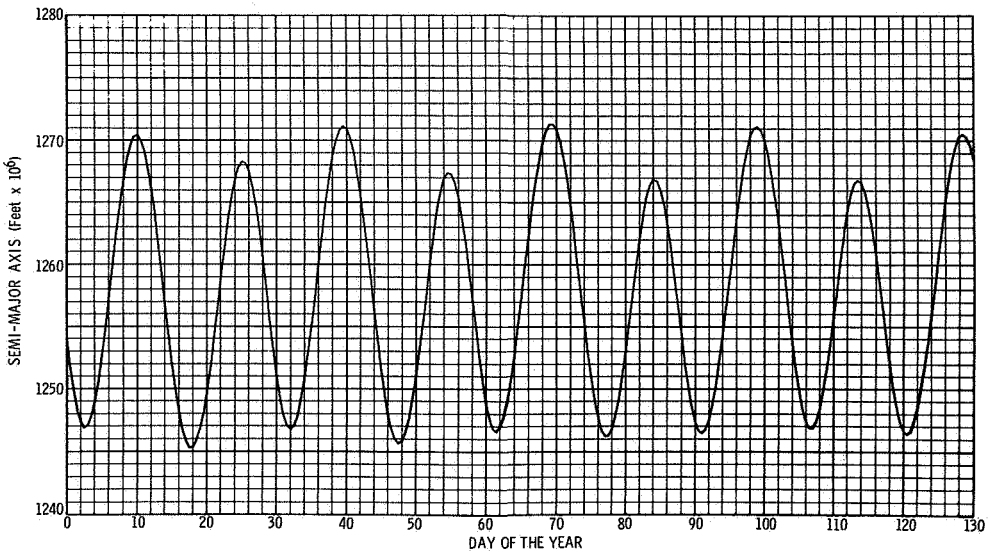
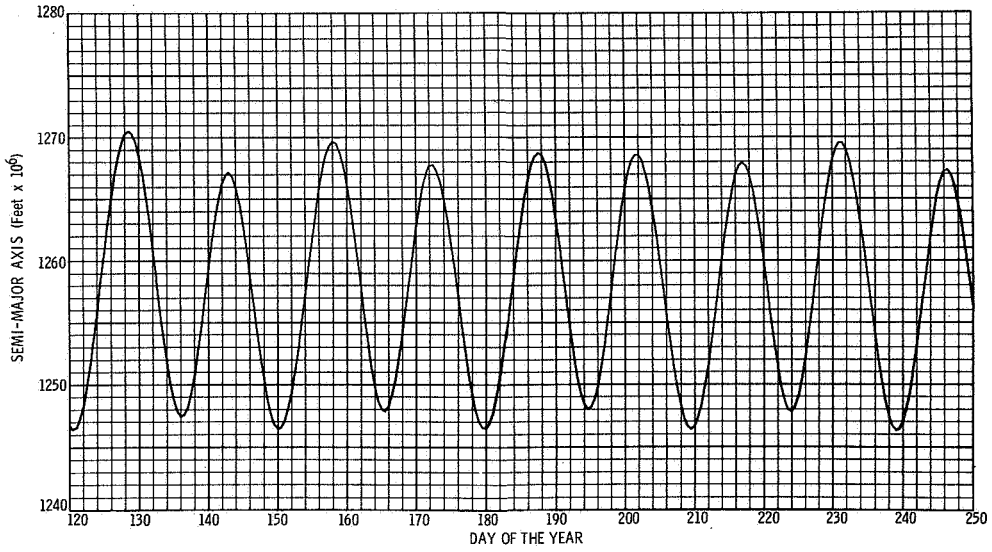


FIGURE B1967-5 TRANSVERSE VELOCITY OF THE MOON





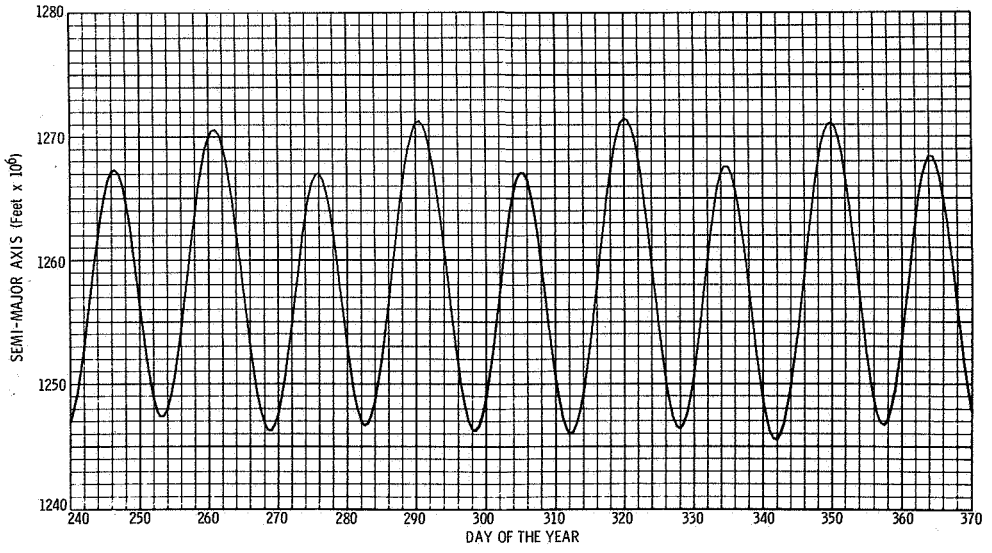


FIGURE B1967-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT

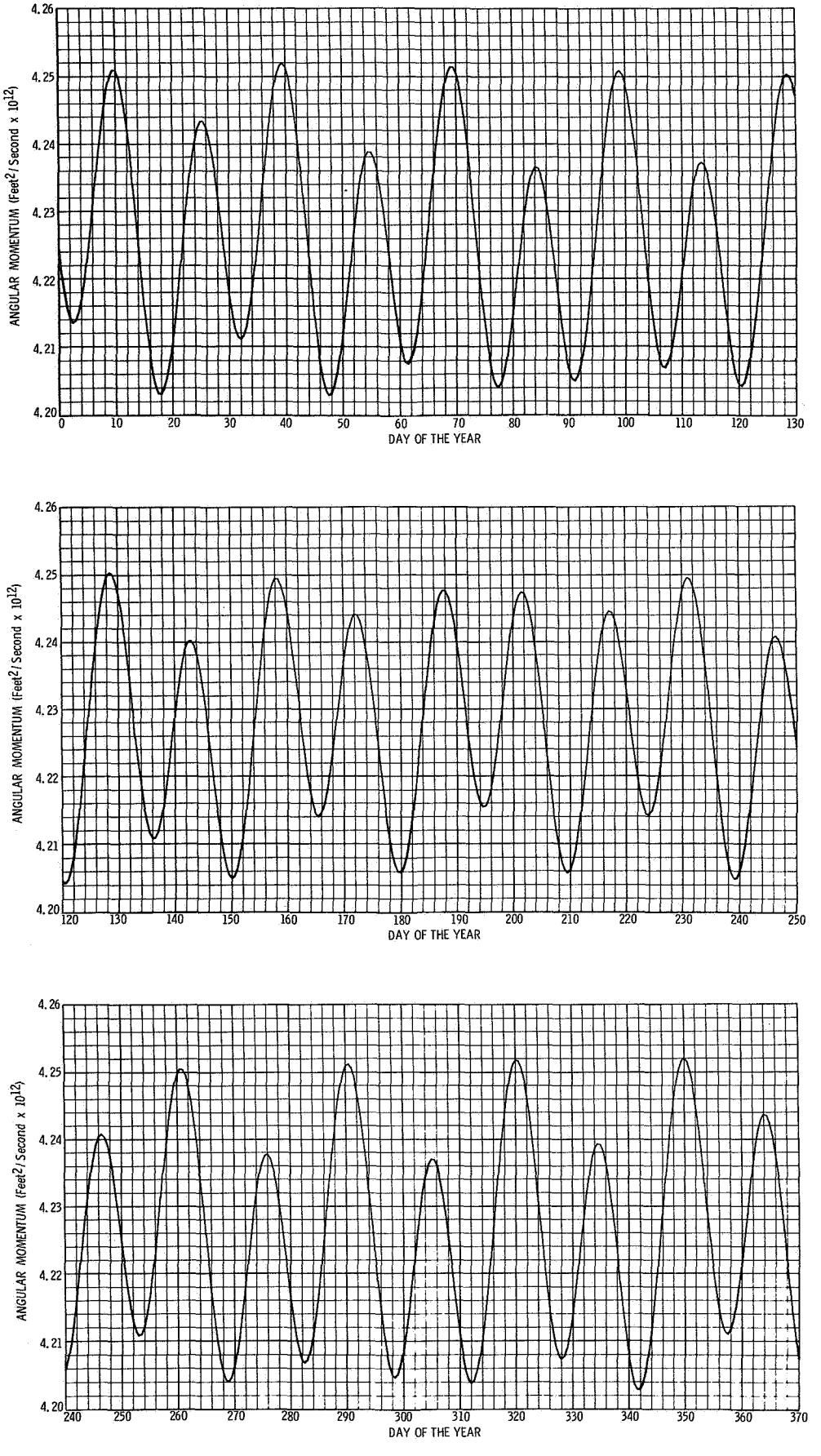
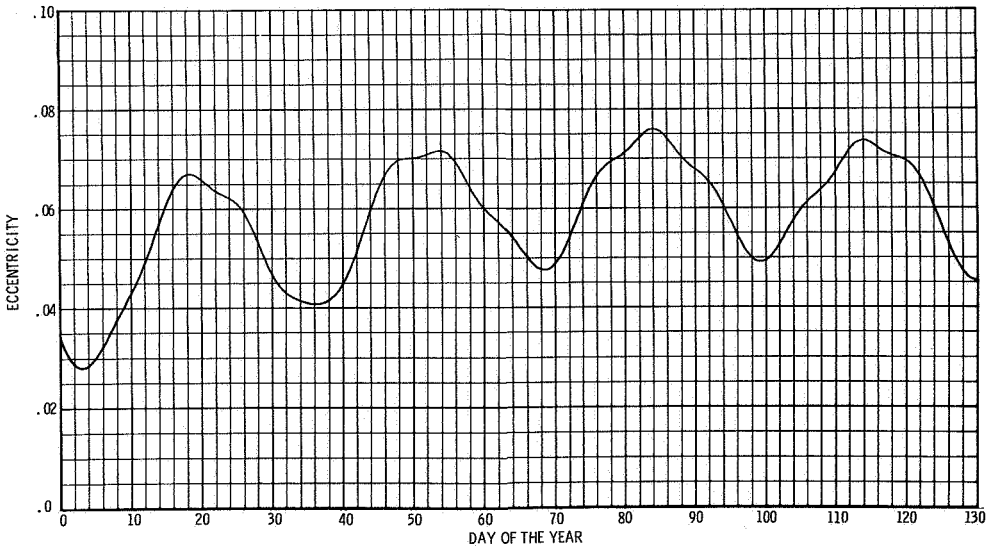
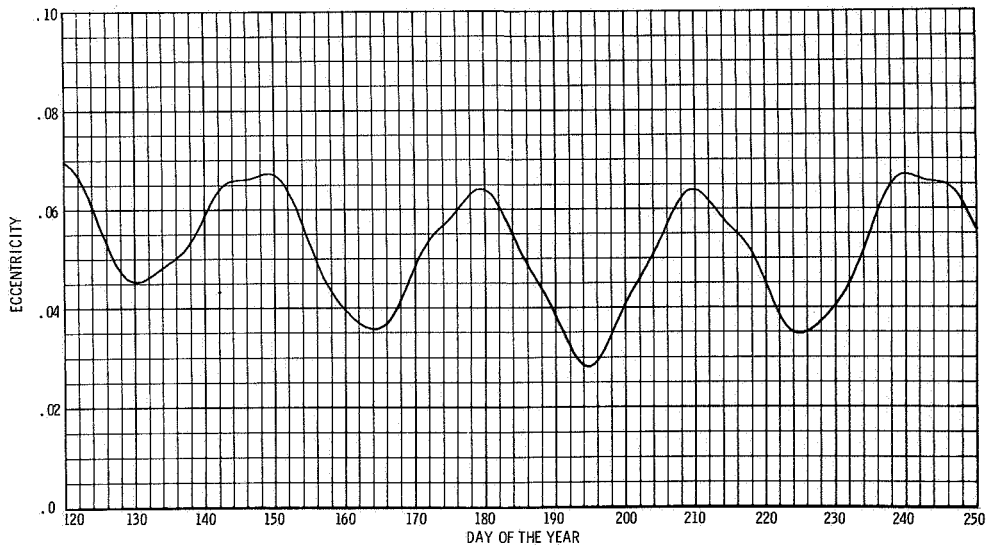


FIGURE B1967-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON





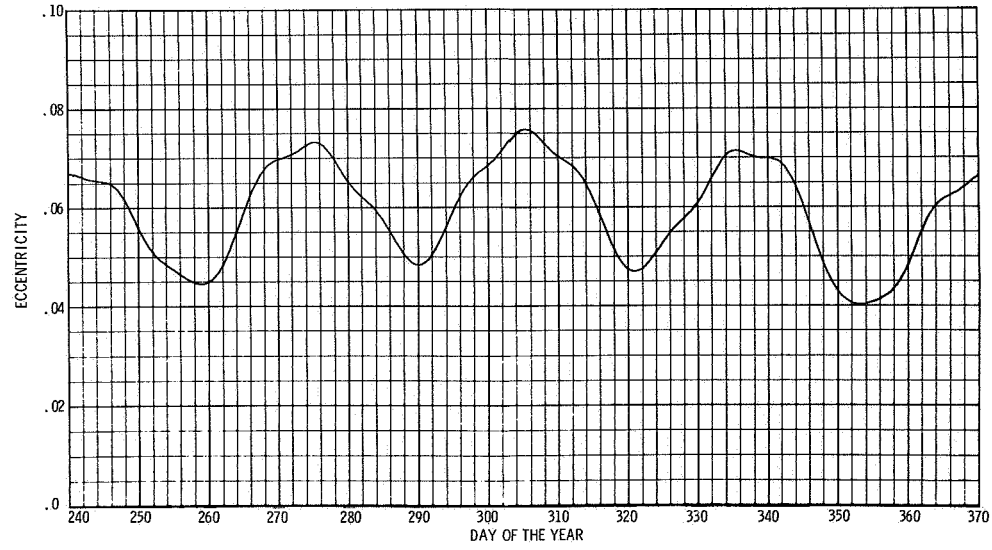
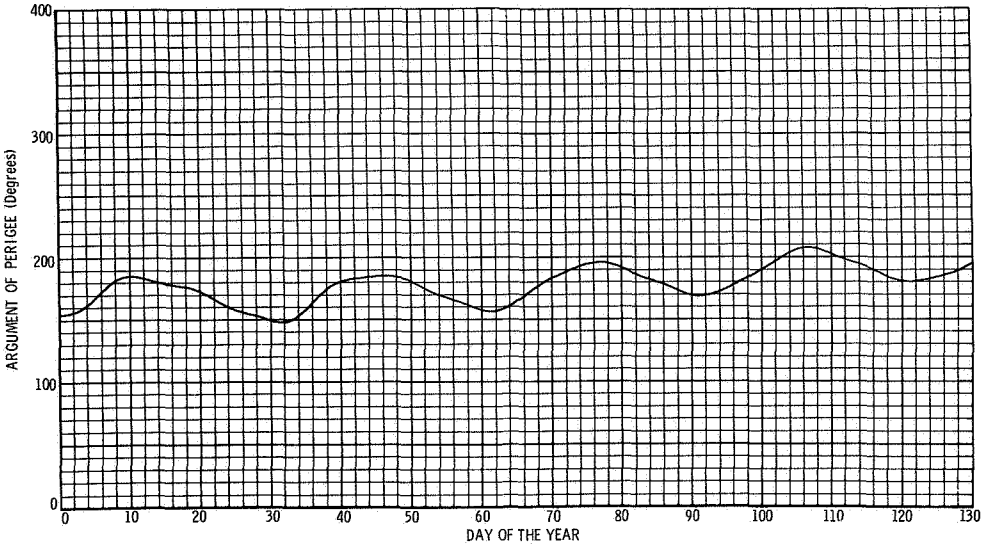
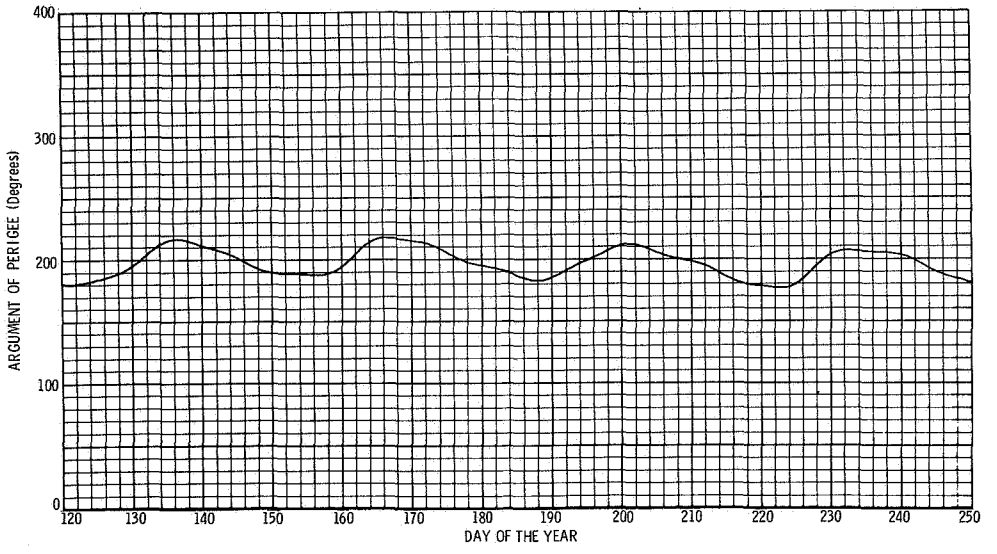


FIGURE B1967-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT





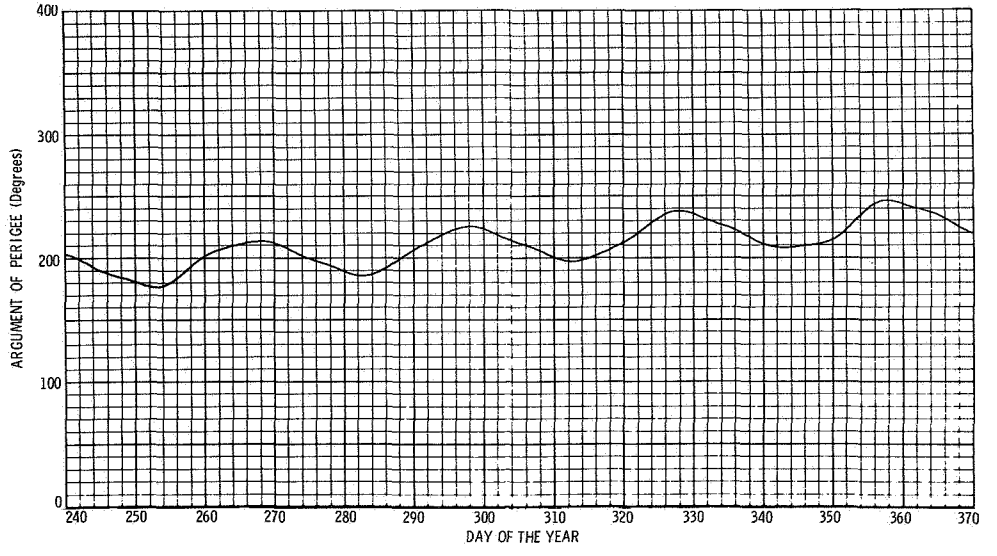
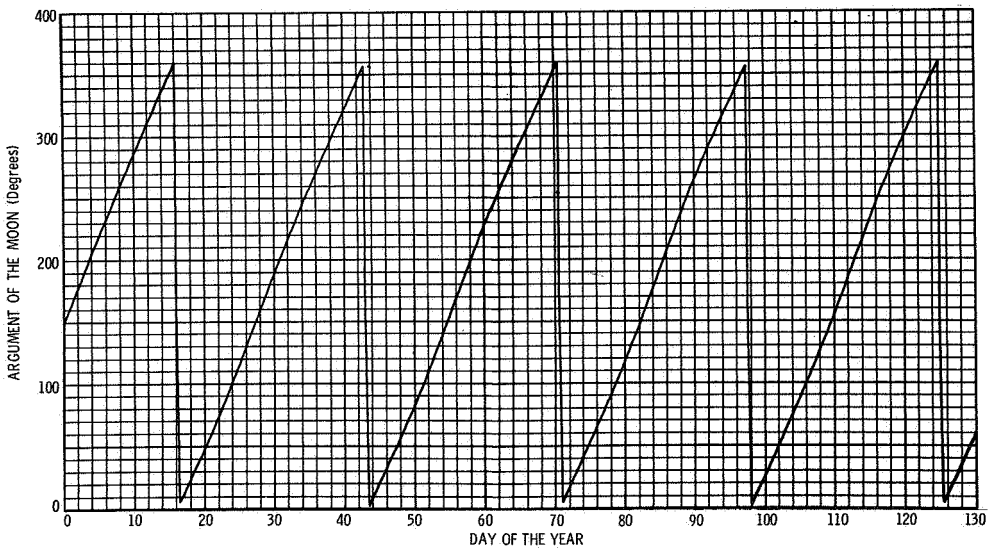
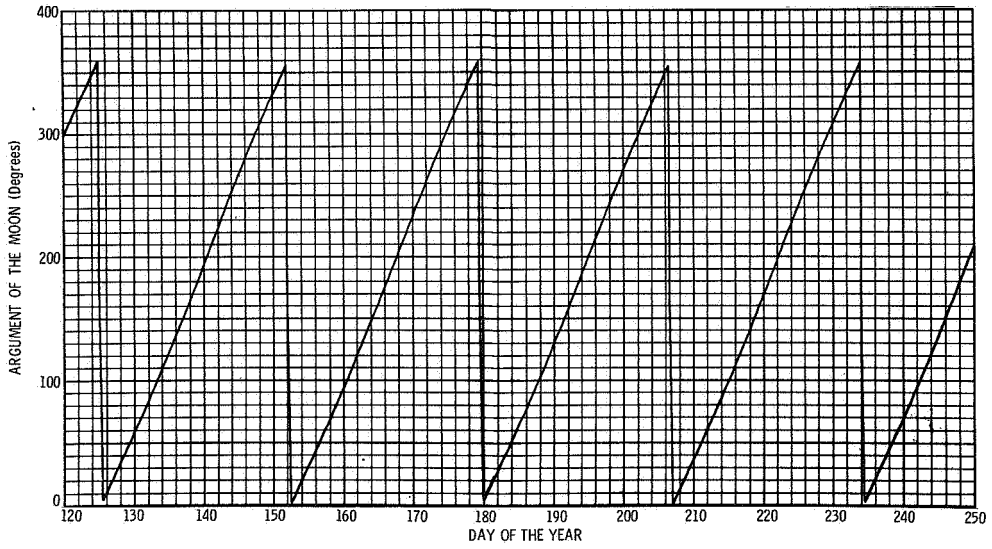


FIGURE B1967-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE





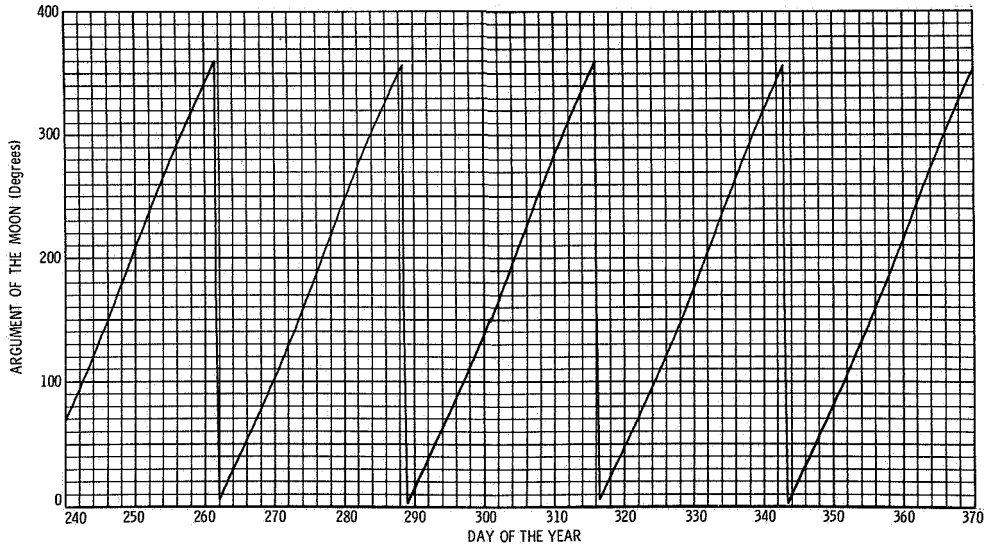


FIGURE B1967-10 ARGUMENT OF THE MOON'S POSITION

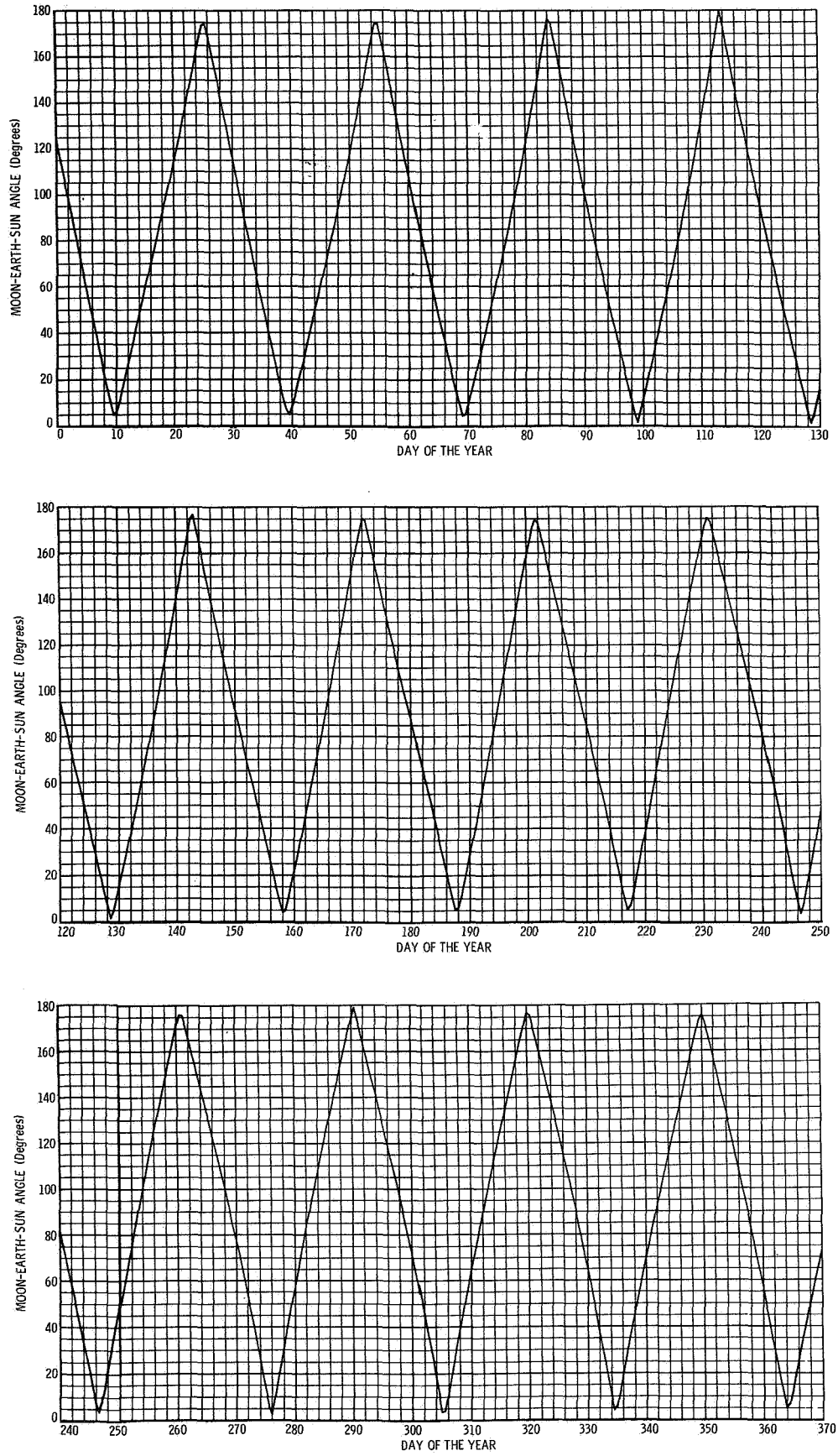


FIGURE B1967-11 MOON-EARTH-SUN ANGLE

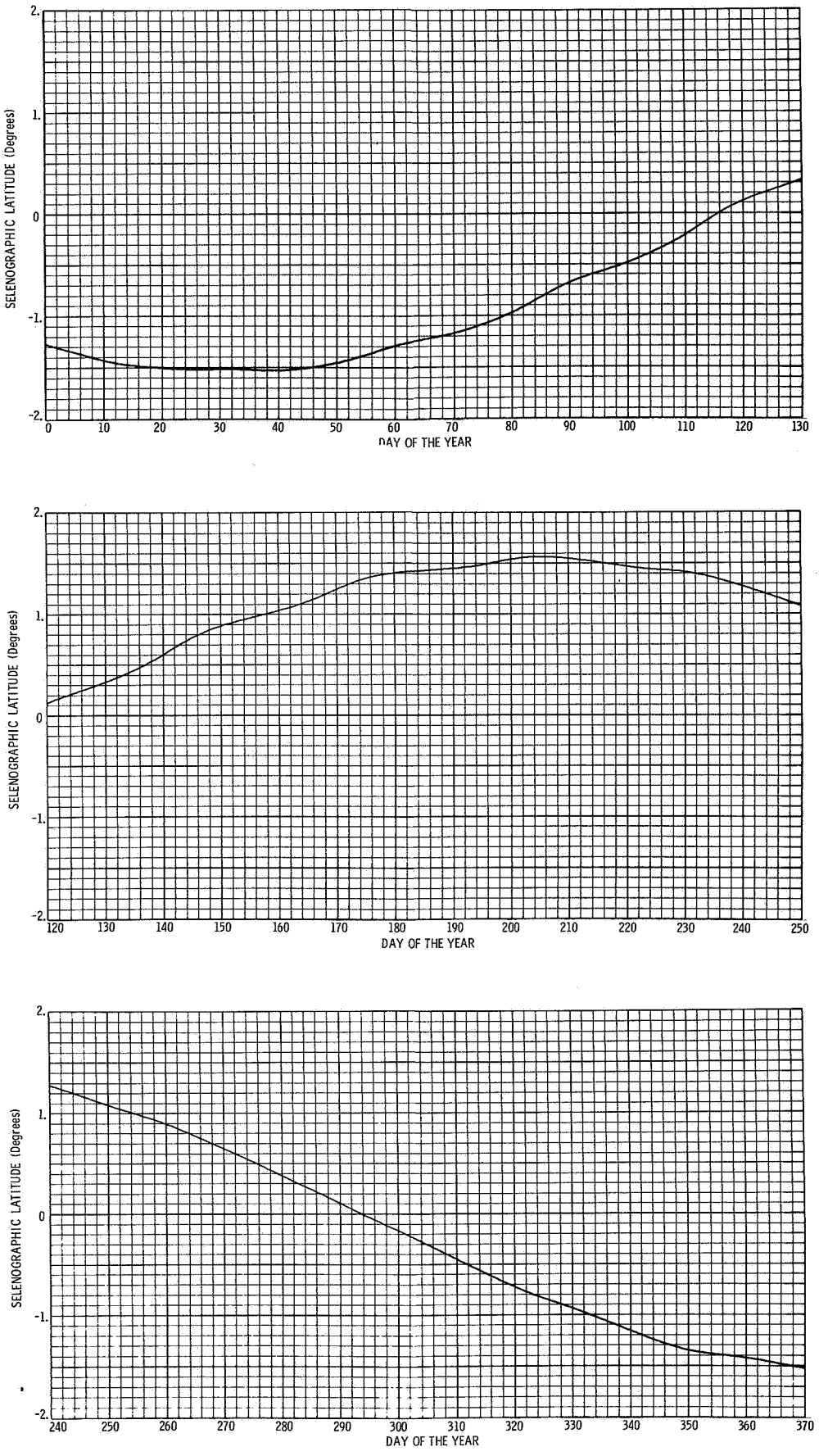
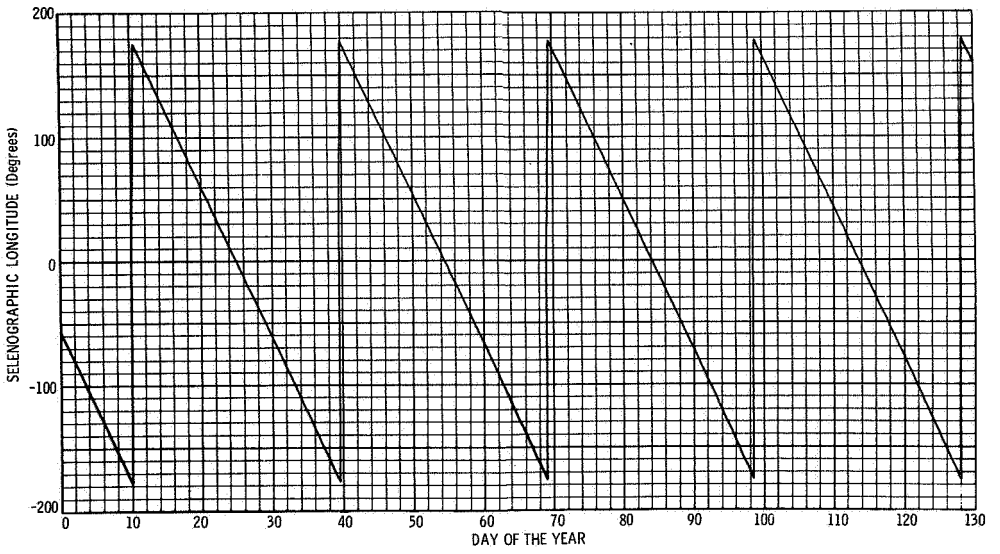
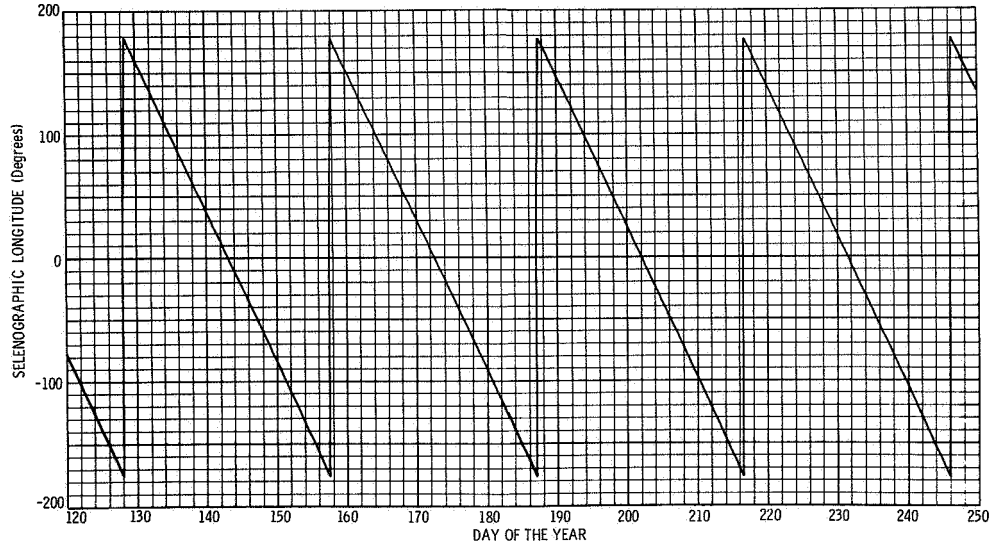


FIGURE B1967-12 SELENOGRAPHIC LATITUDE OF THE SUN





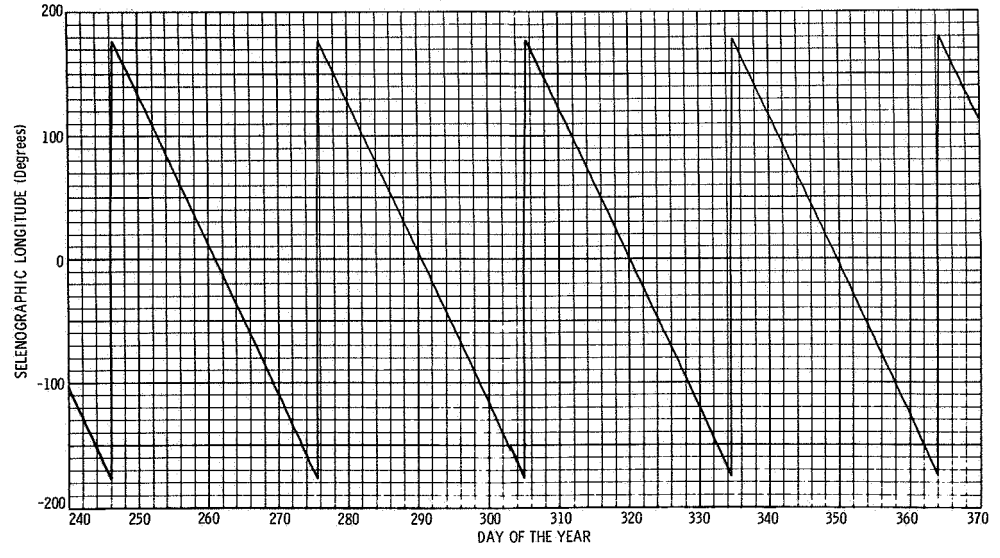
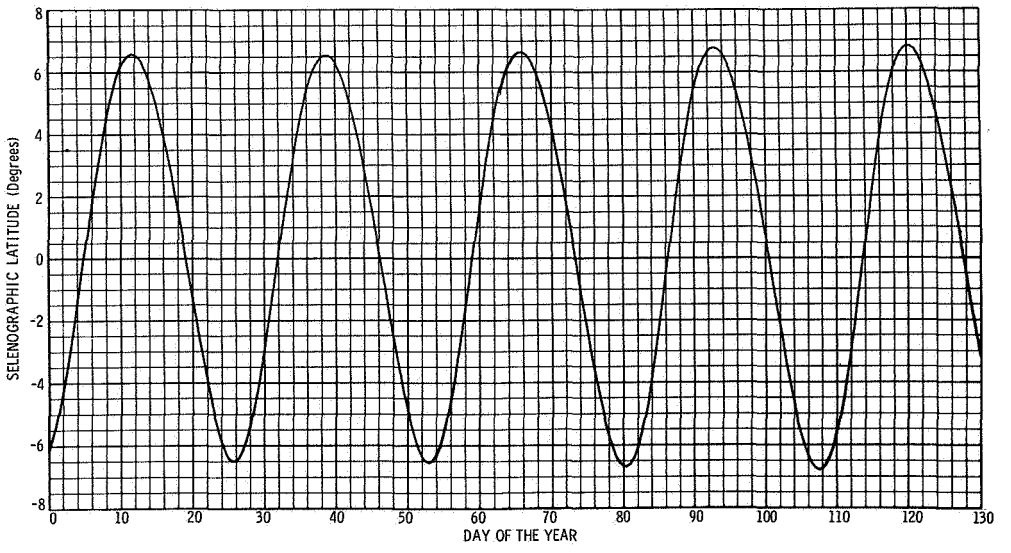
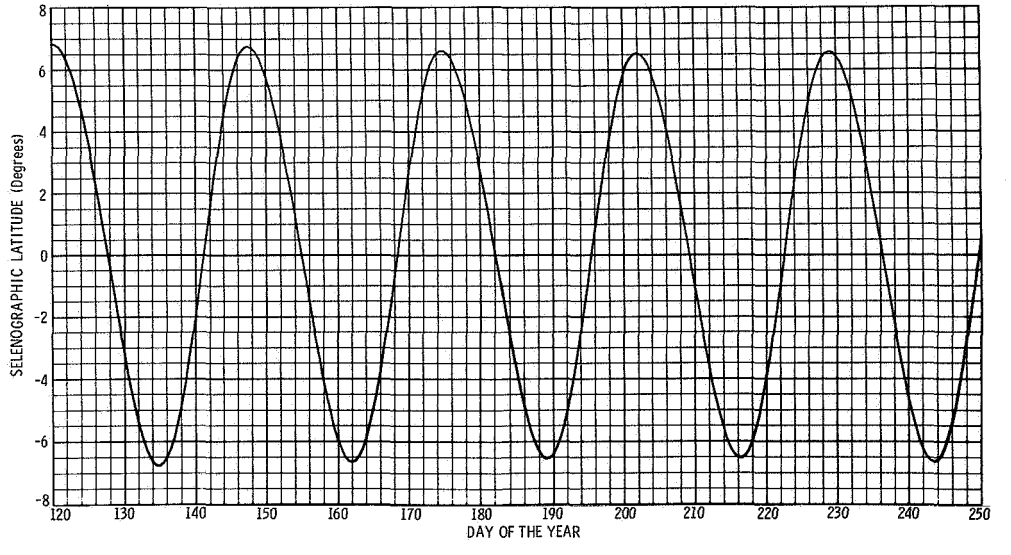


FIGURE B1967-13 SELENOGRAPHIC LONGITUDE OF THE SUN





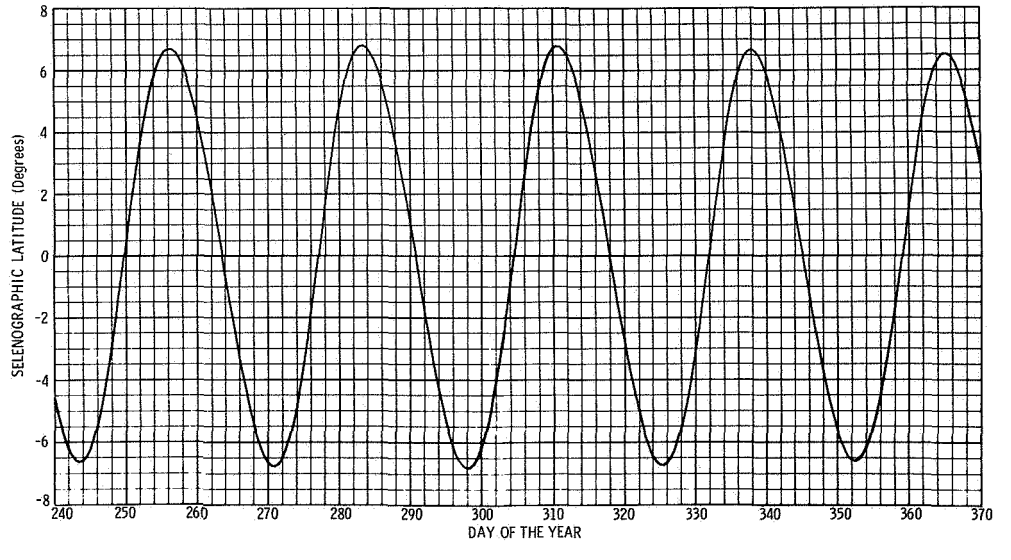


FIGURE B1967-14 SELENOGRAPHIC LATITUDE OF THE EARTH

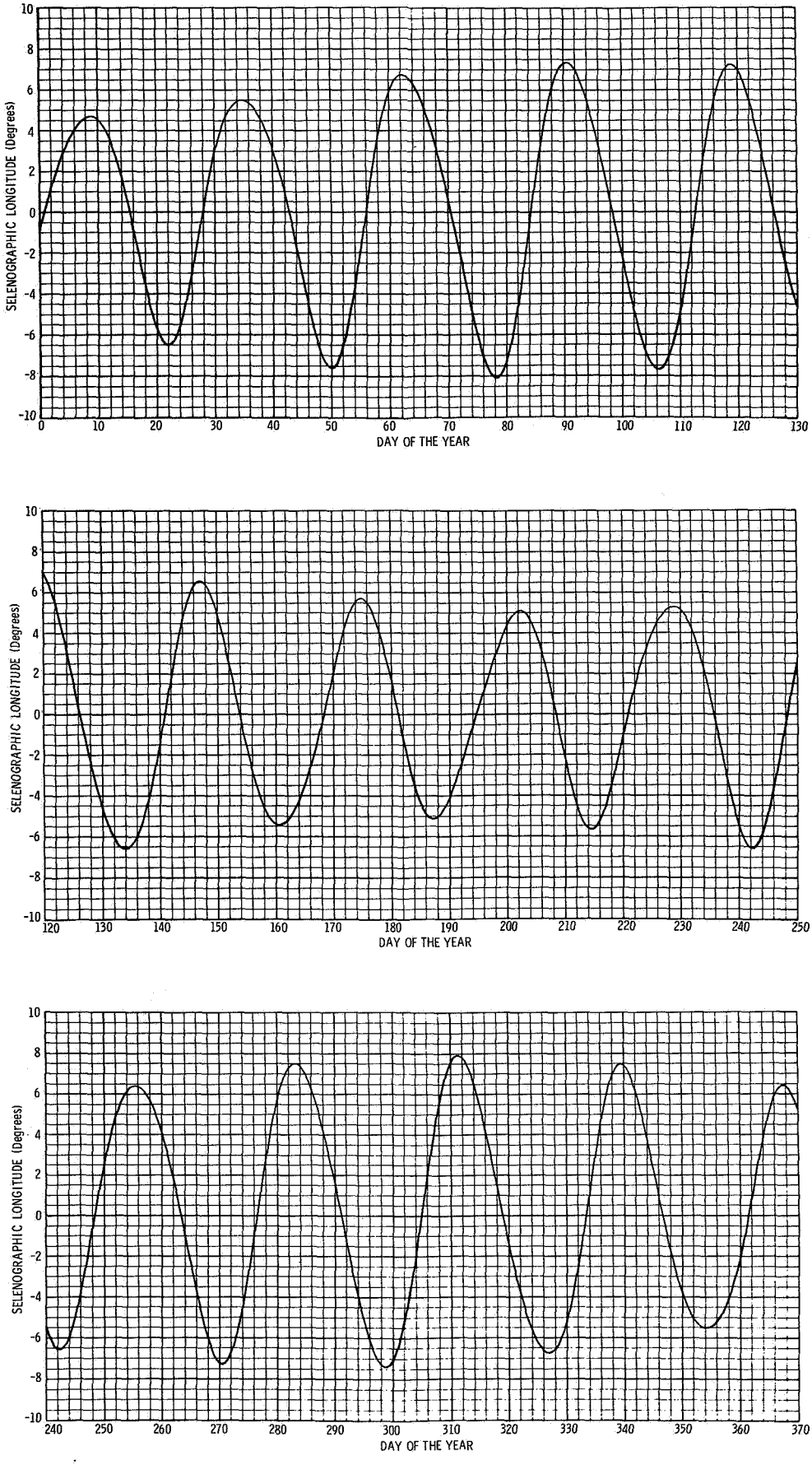


FIGURE B1967-15 SELENOGRAPHIC LONGITUDE OF THE EARTH

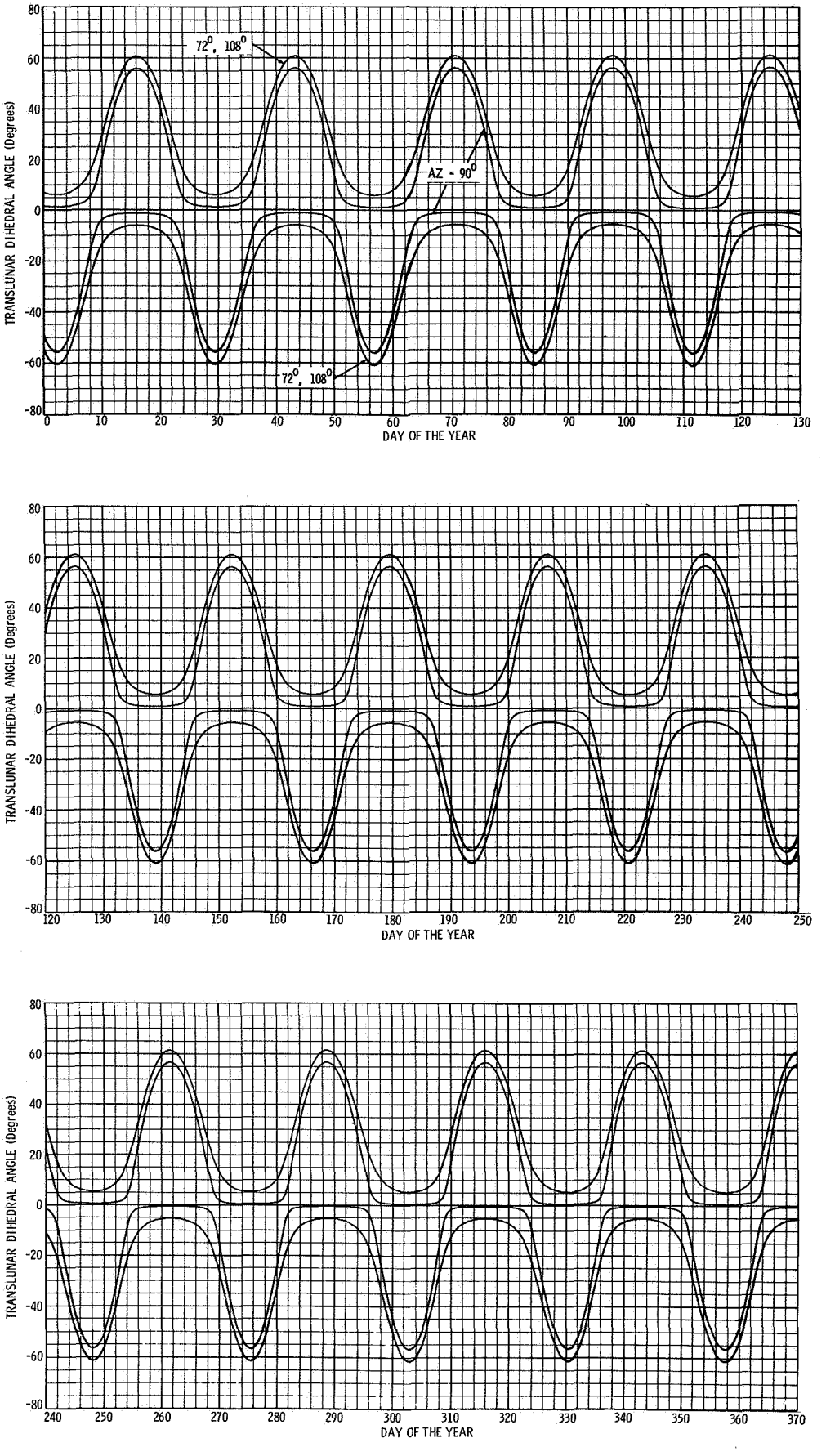


FIGURE B1967-16 TRANSLUNAR DIHEDRAL ANGLES

1968

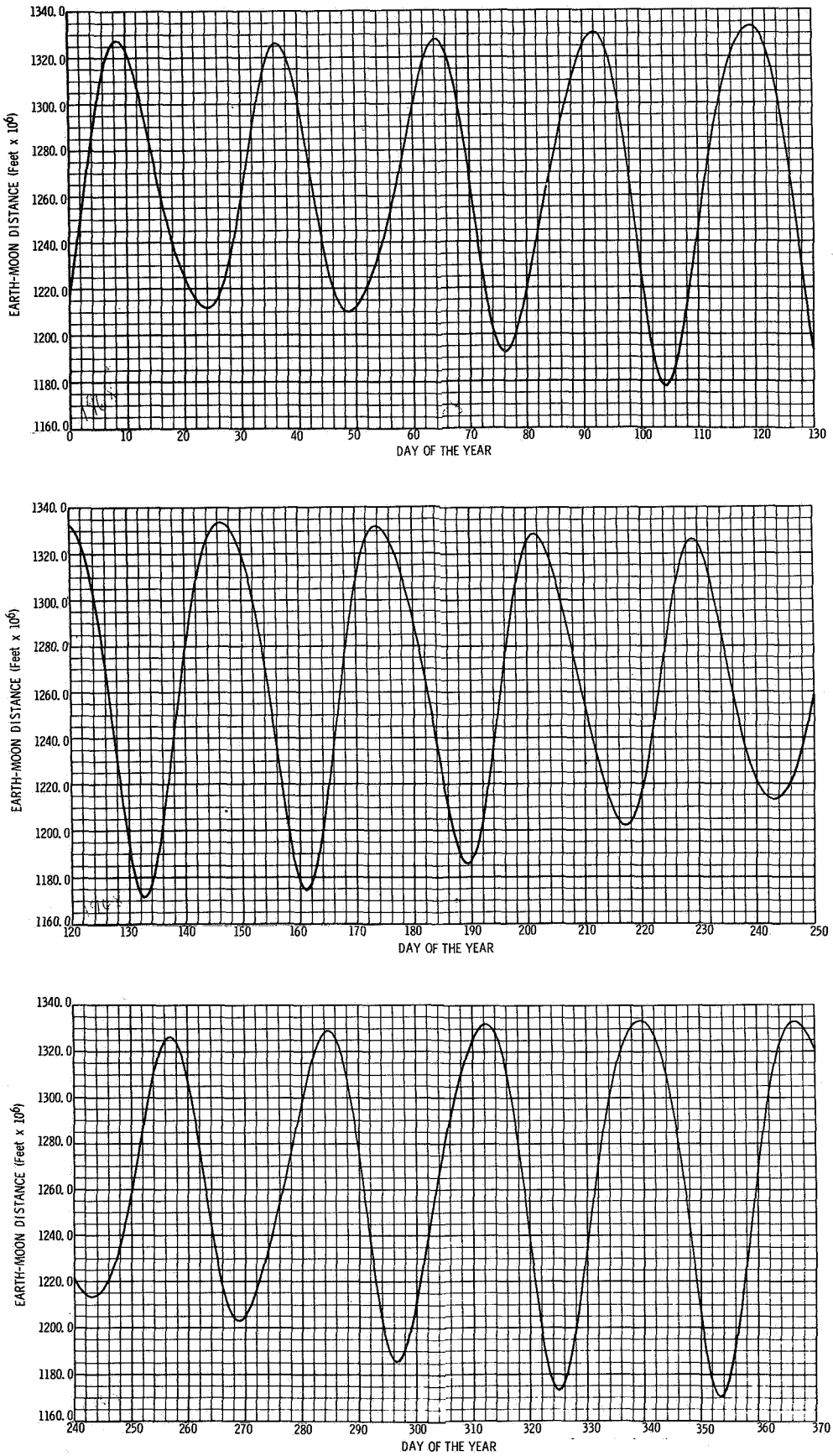
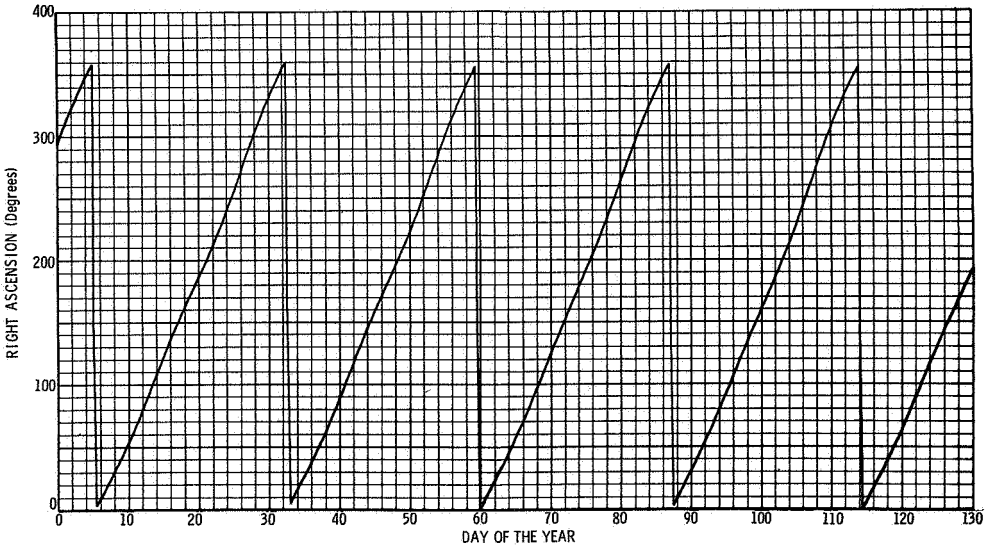
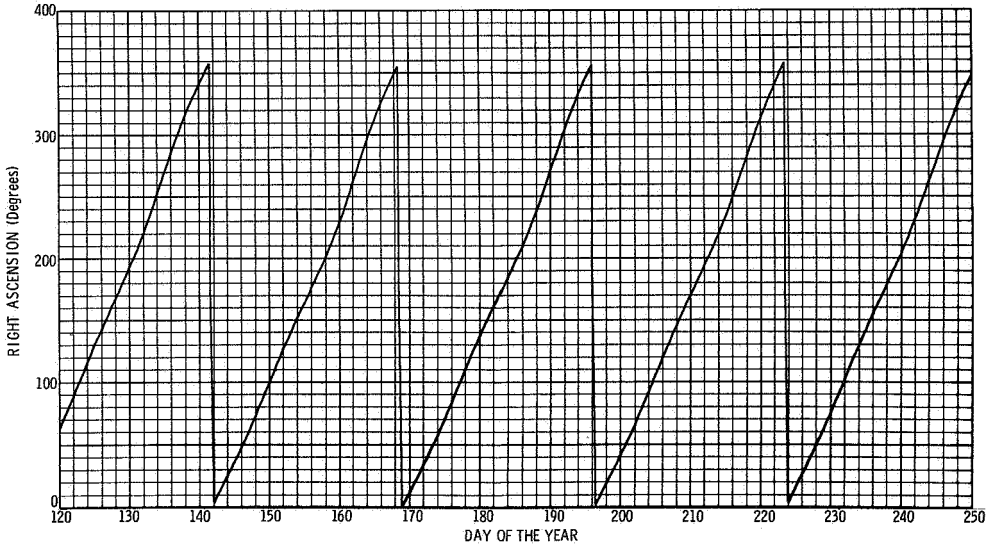


FIGURE B1968-1 EARTH-MOON DISTANCE





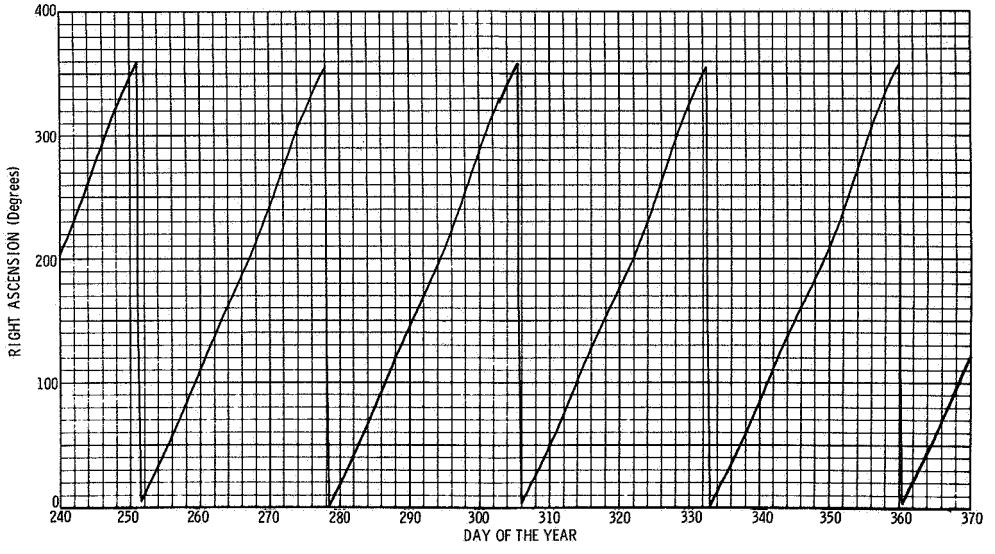


FIGURE B1968-2 RIGHT ASCENSION OF THE MOON

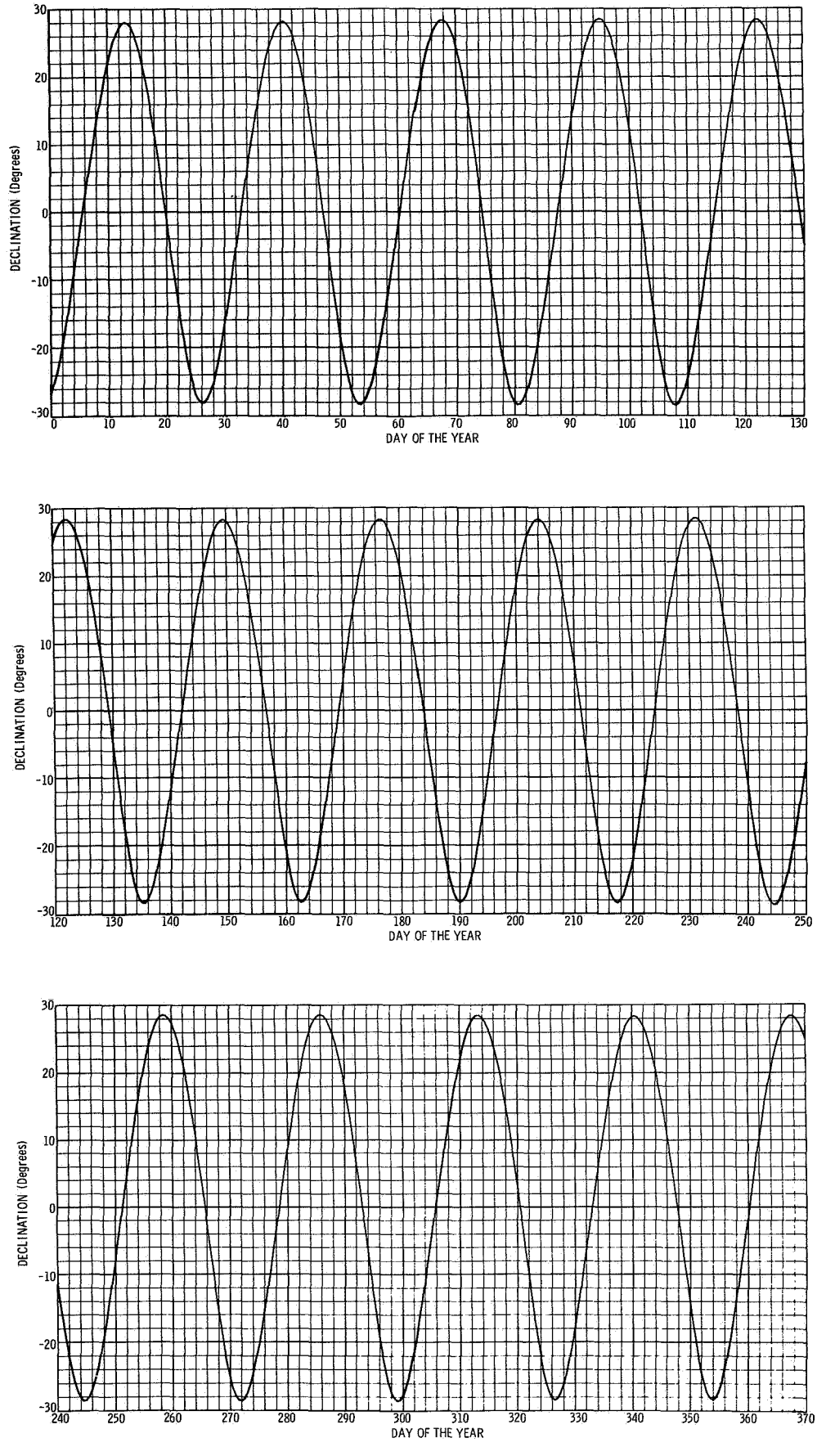
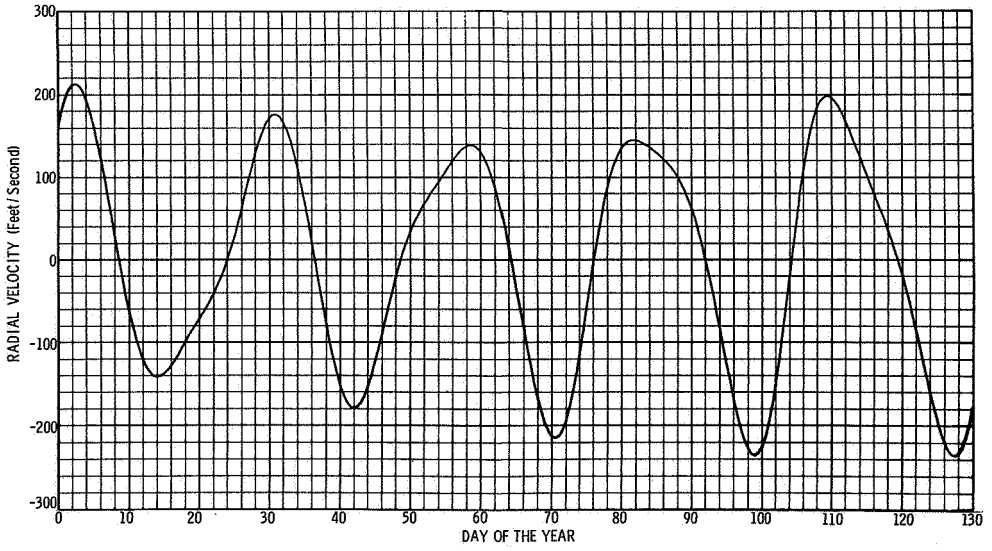
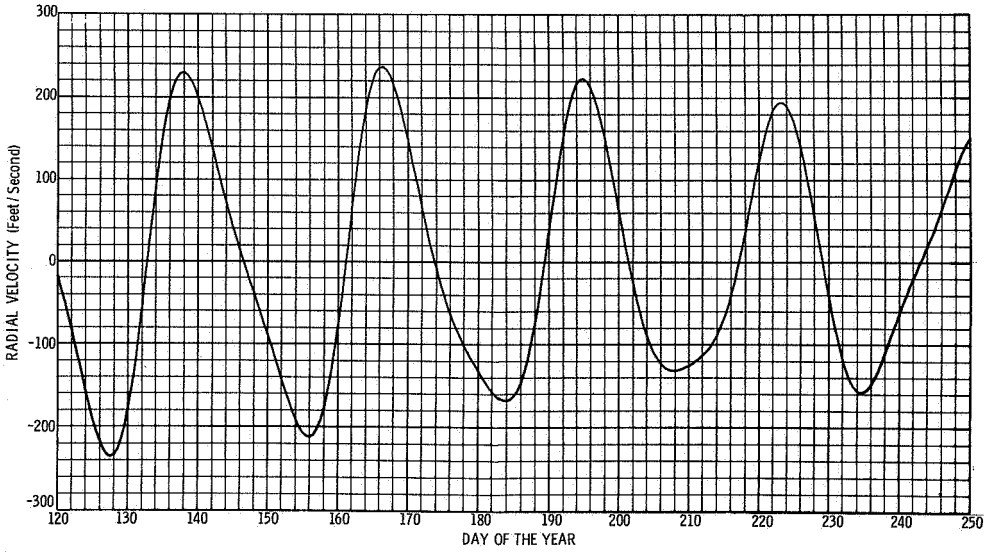


FIGURE B1968-3 DECLINATION OF THE MOON





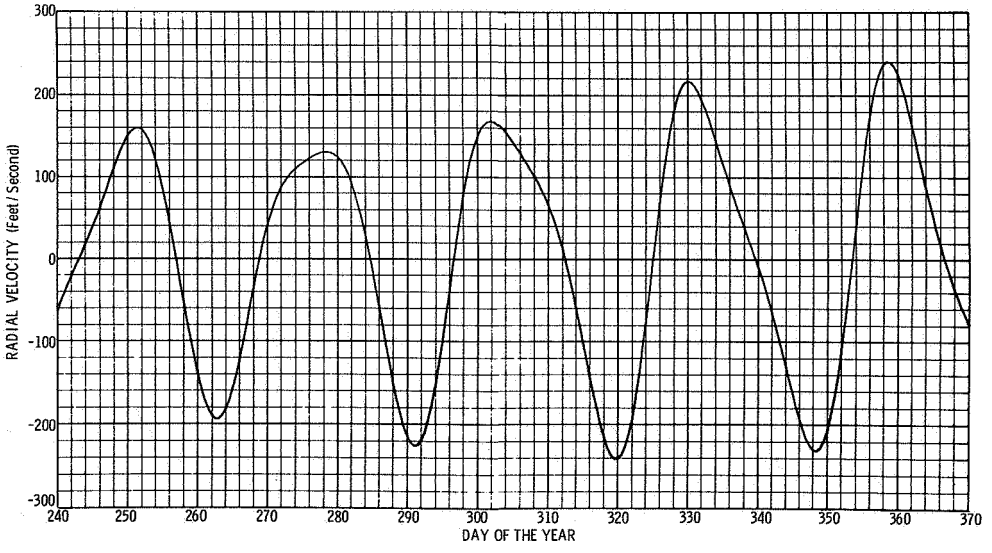
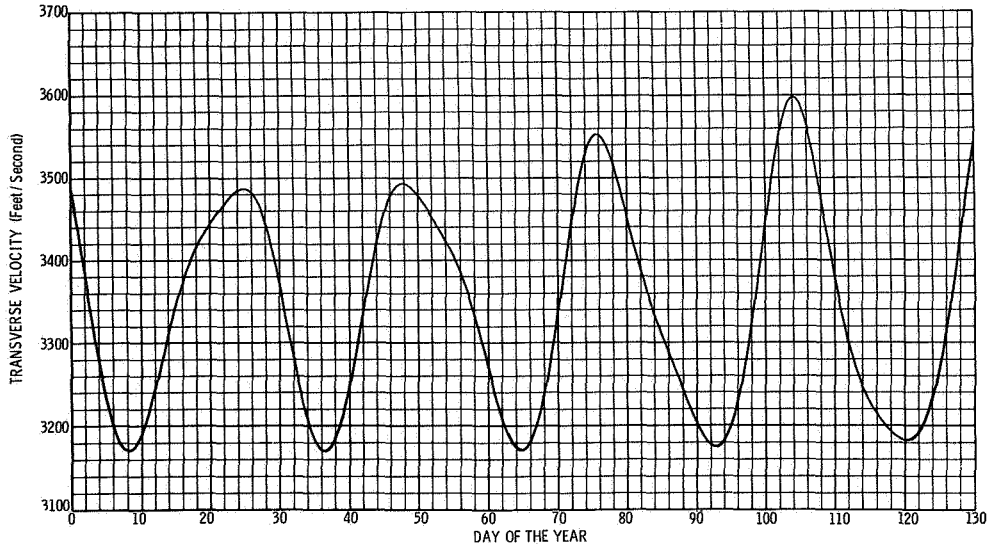
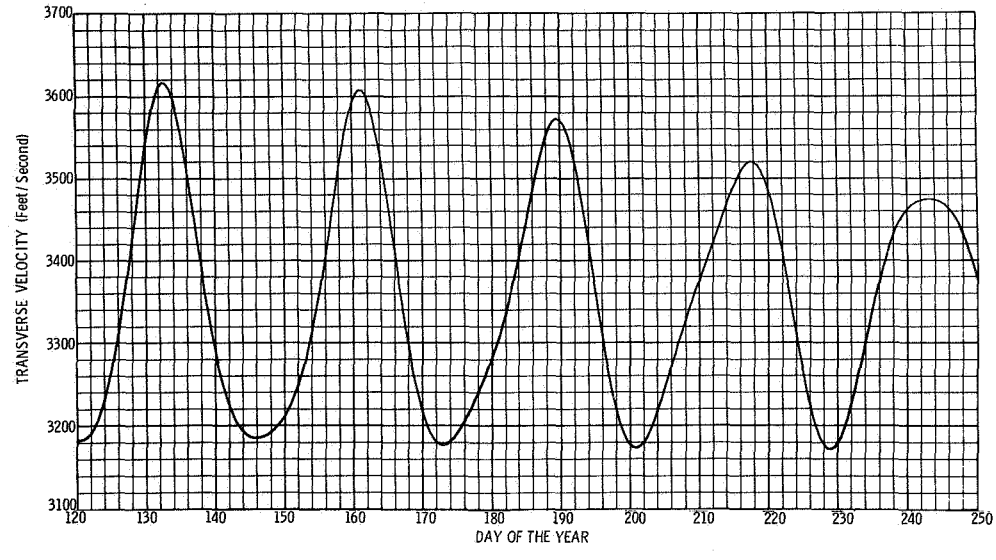


FIGURE B 1968-4 RADIAL VELOCITY OF THE MOON





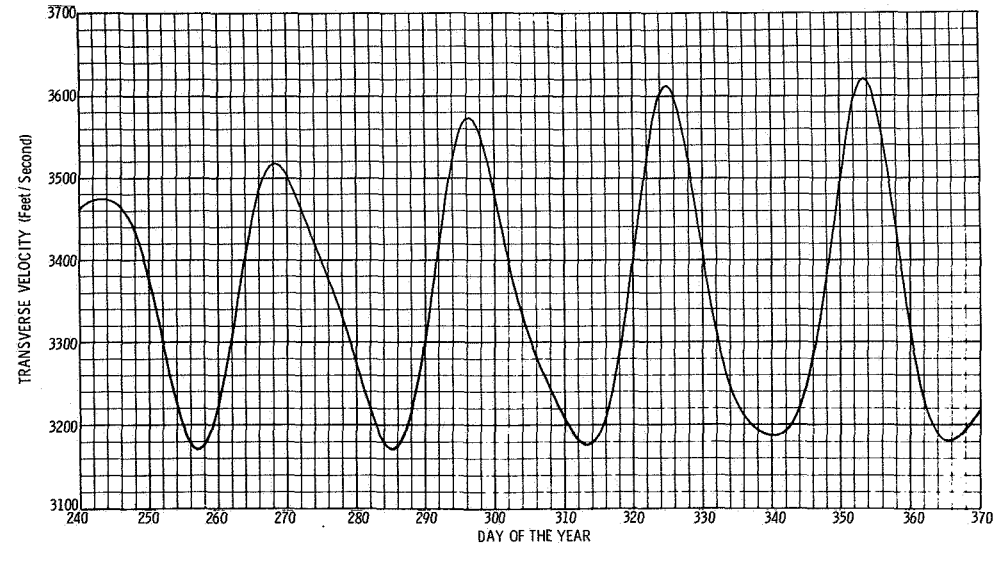


FIGURE B 1968-5 TRANSVERSE VELOCITY OF THE MOON

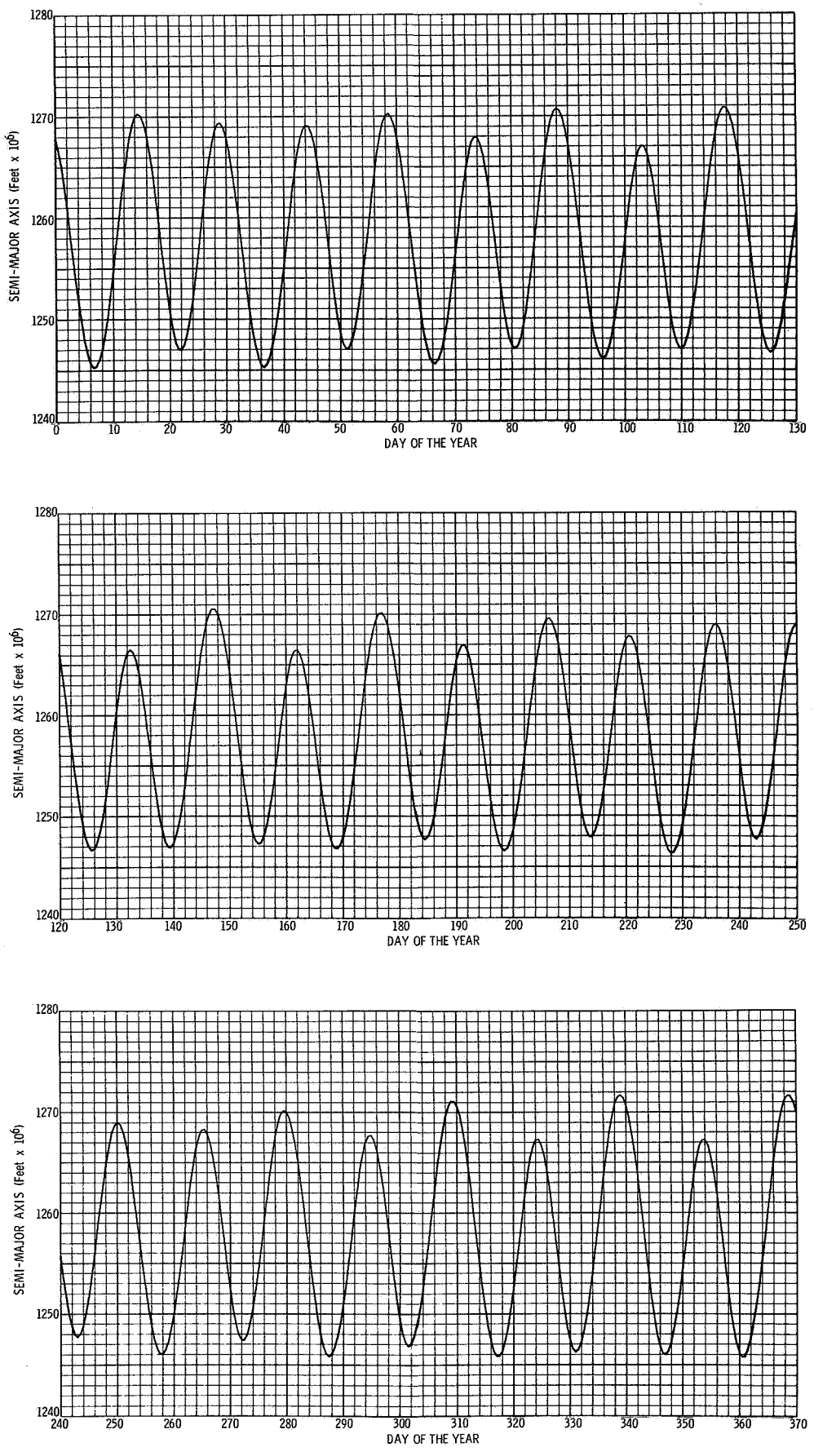


FIGURE B 1968-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT

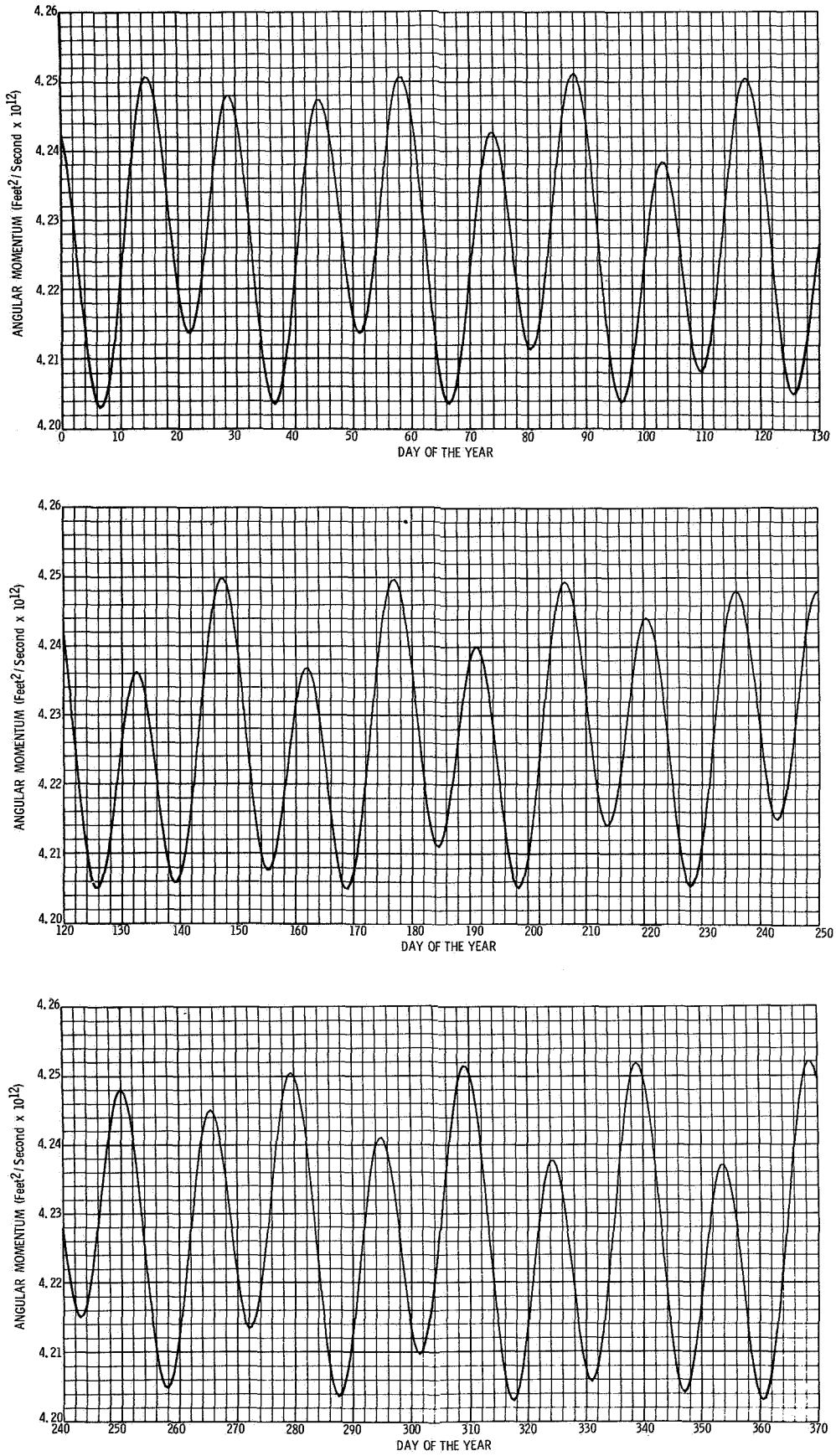
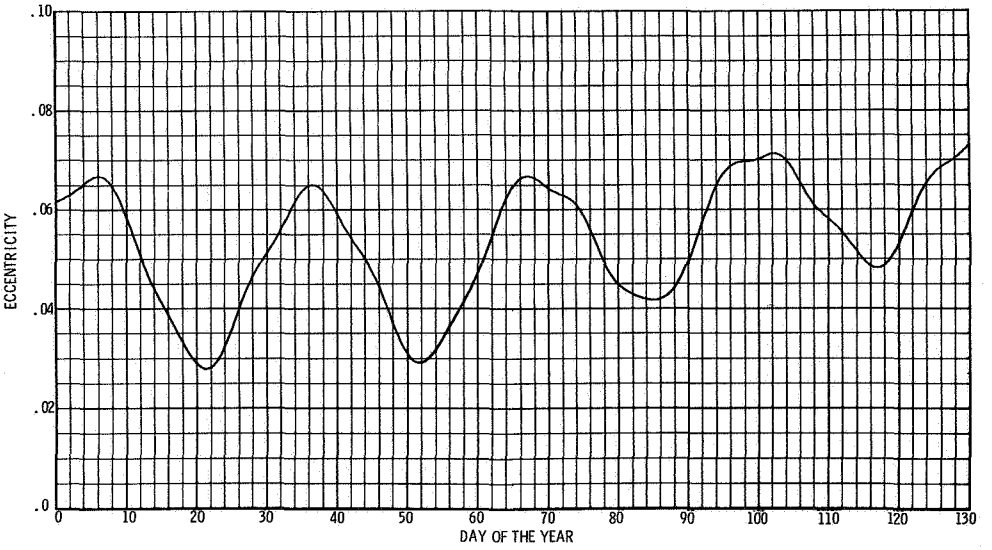
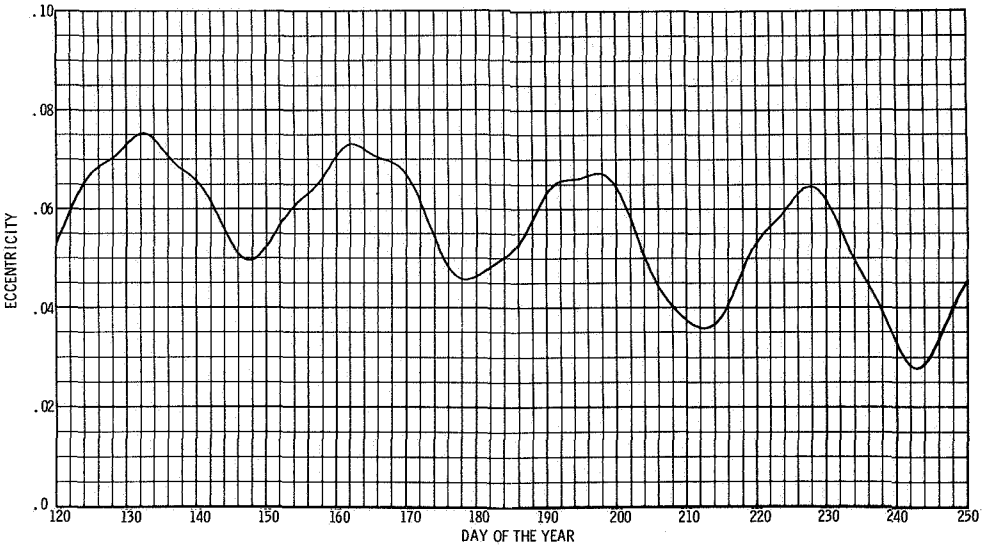


FIGURE B 1968-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON





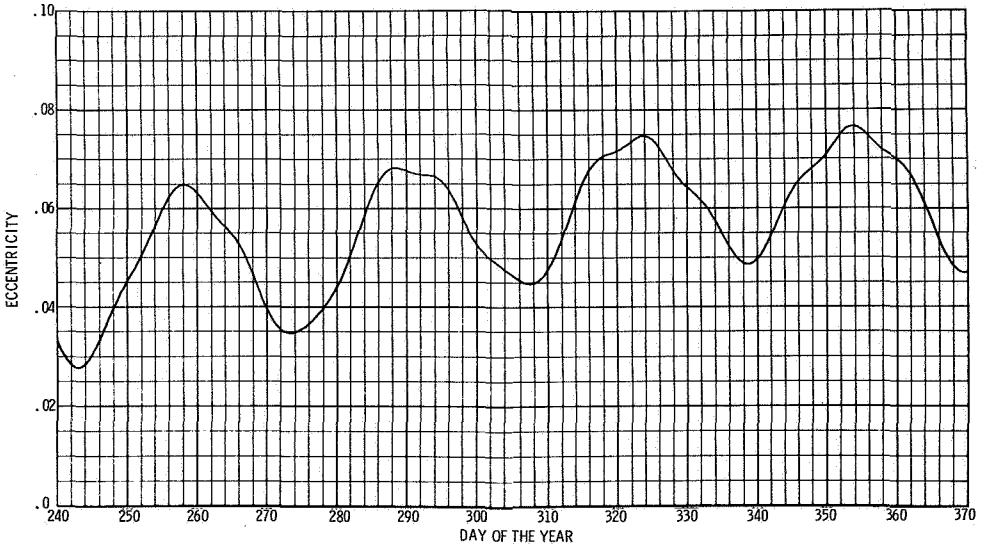


FIGURE B 1968-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT

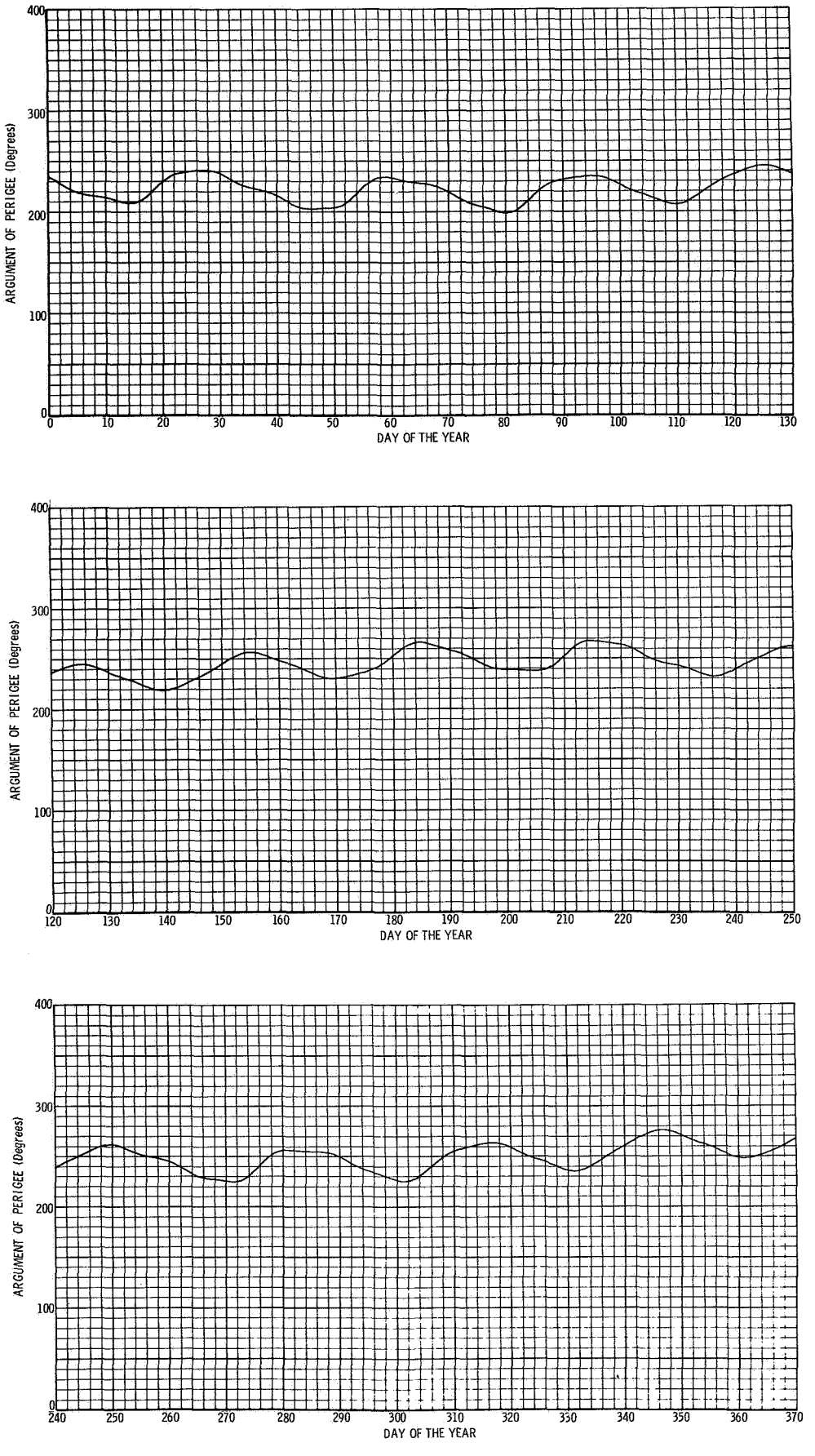
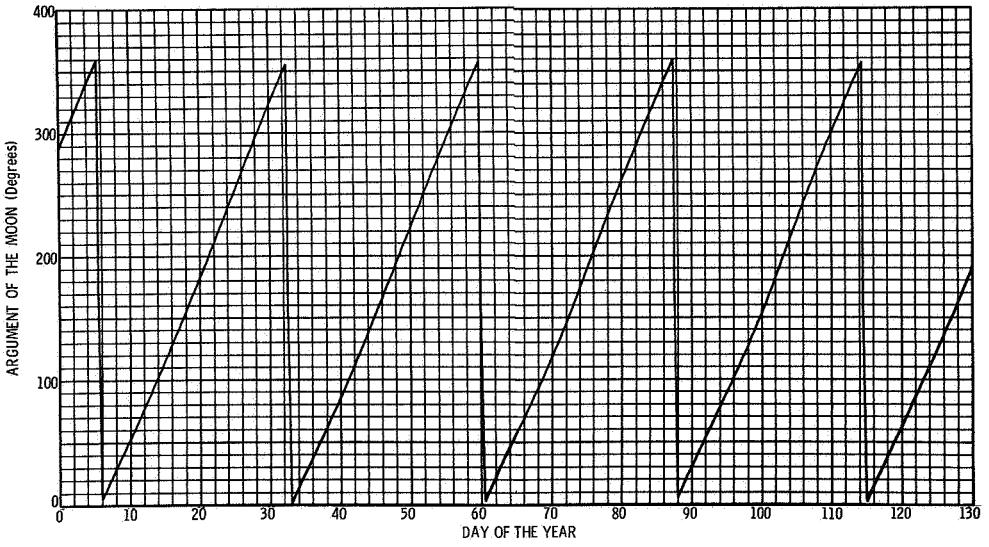
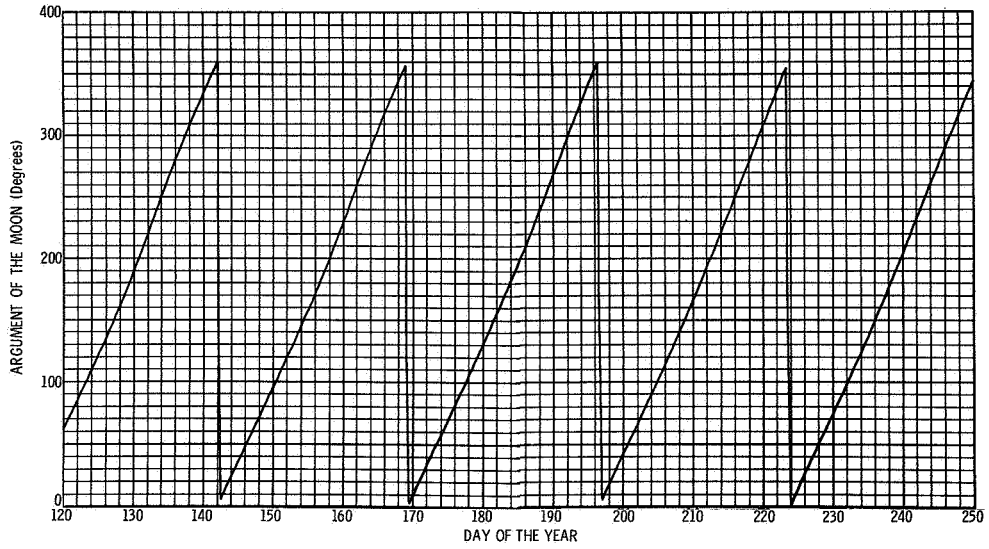


FIGURE B 1968-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE





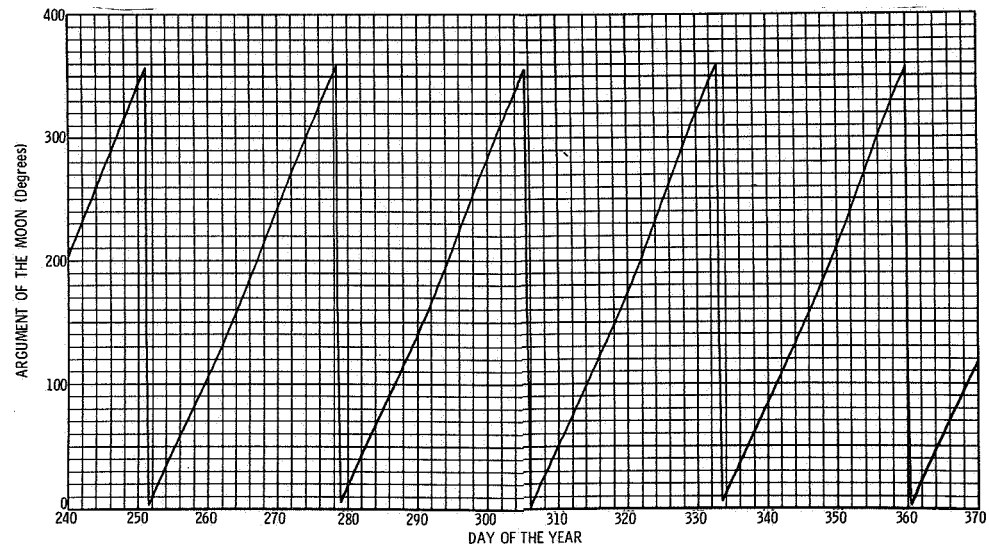
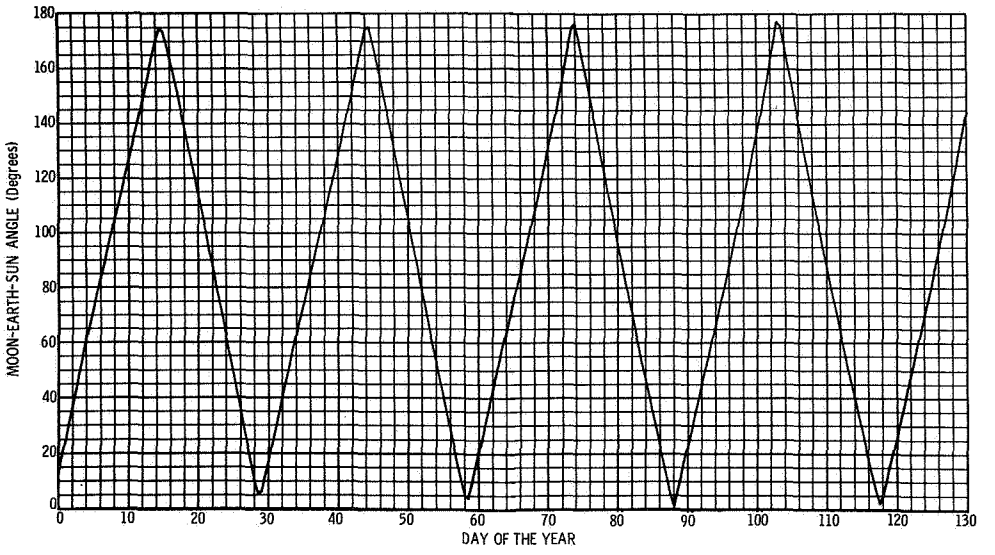
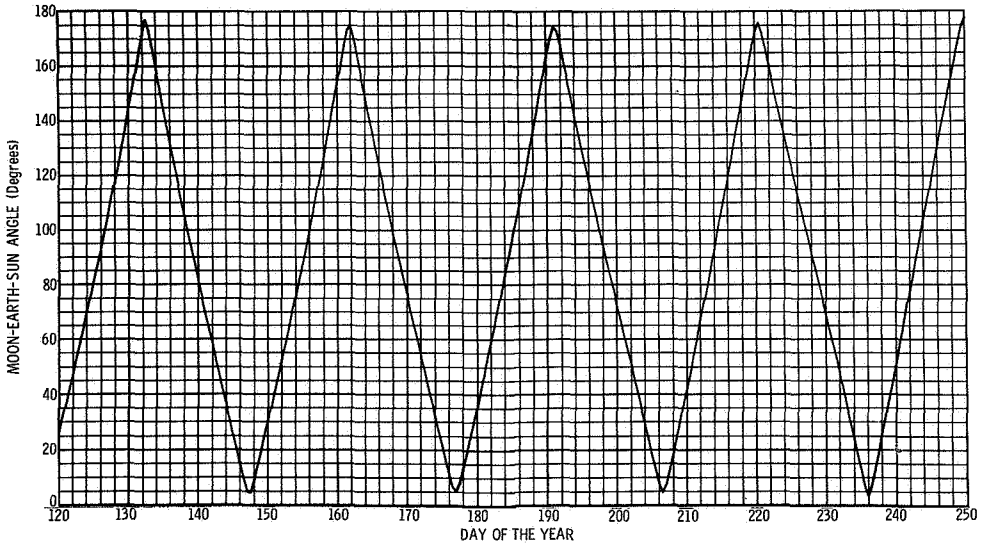


FIGURE B 1968-10 ARGUMENT OF THE MOON'S POSITION





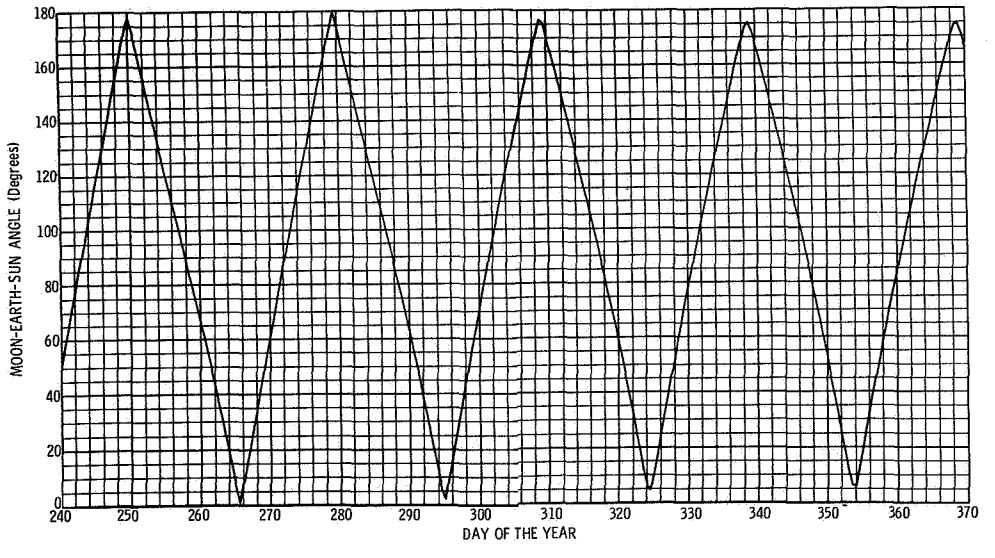
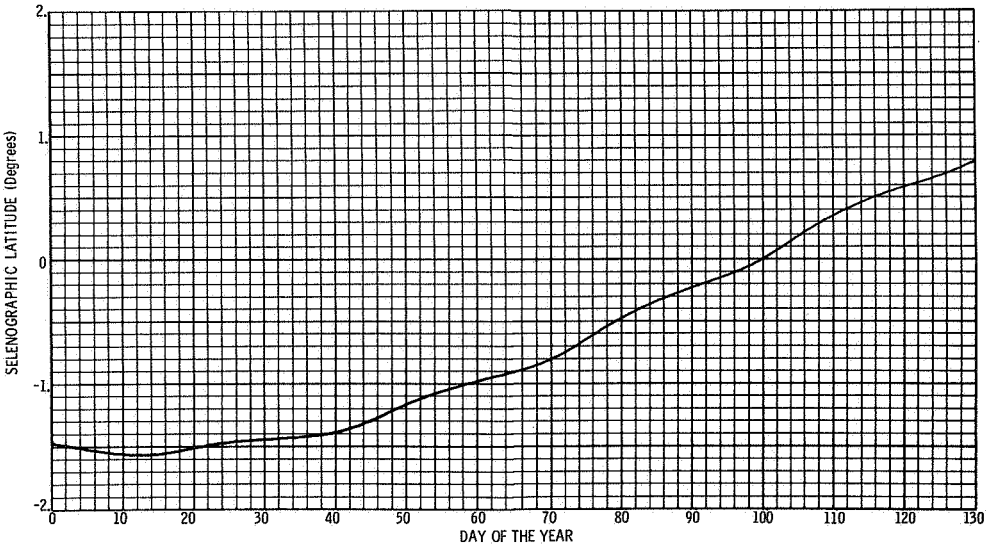
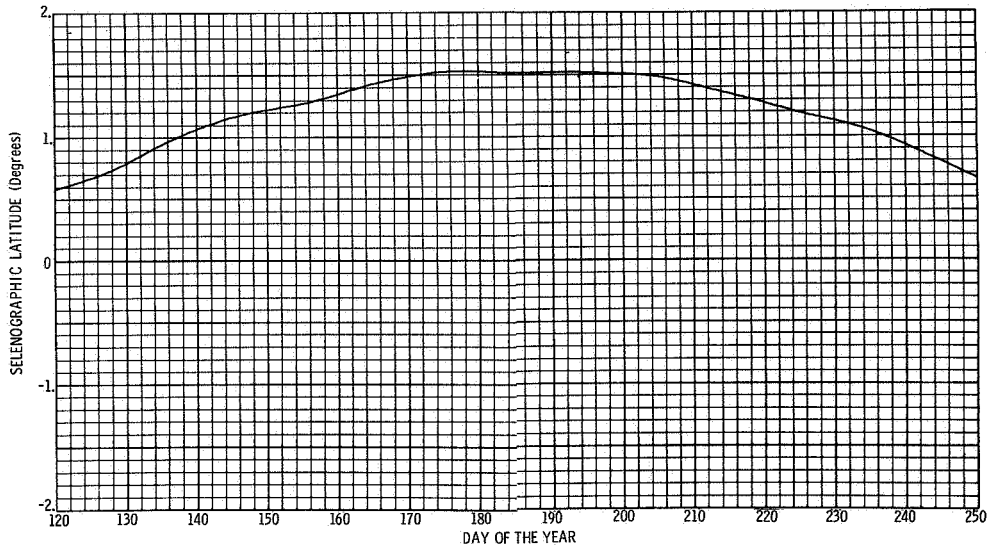


FIGURE B 1968-11 MOON-EARTH-SUN ANGLE





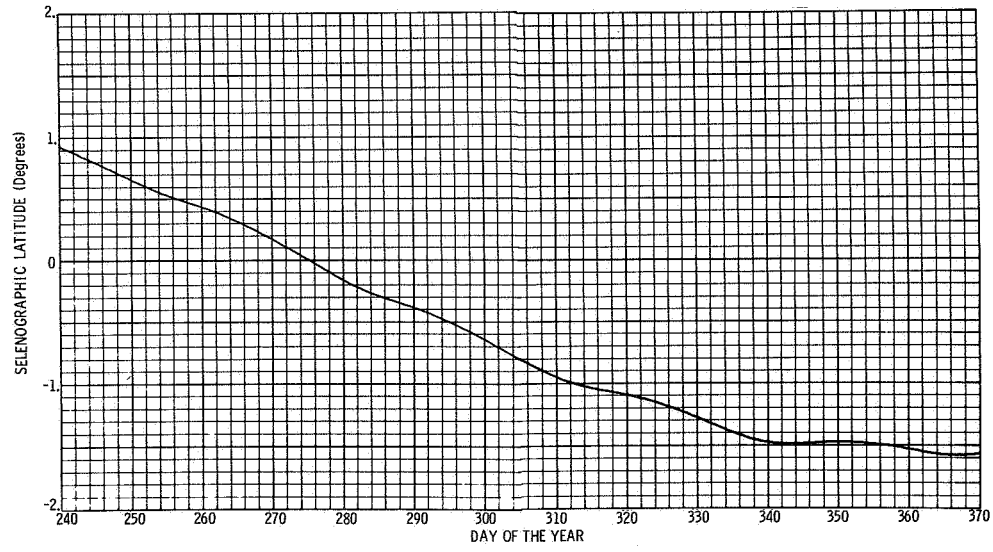


FIGURE B 1968-12 SELENOGRAPHIC LATITUDE OF THE SUN

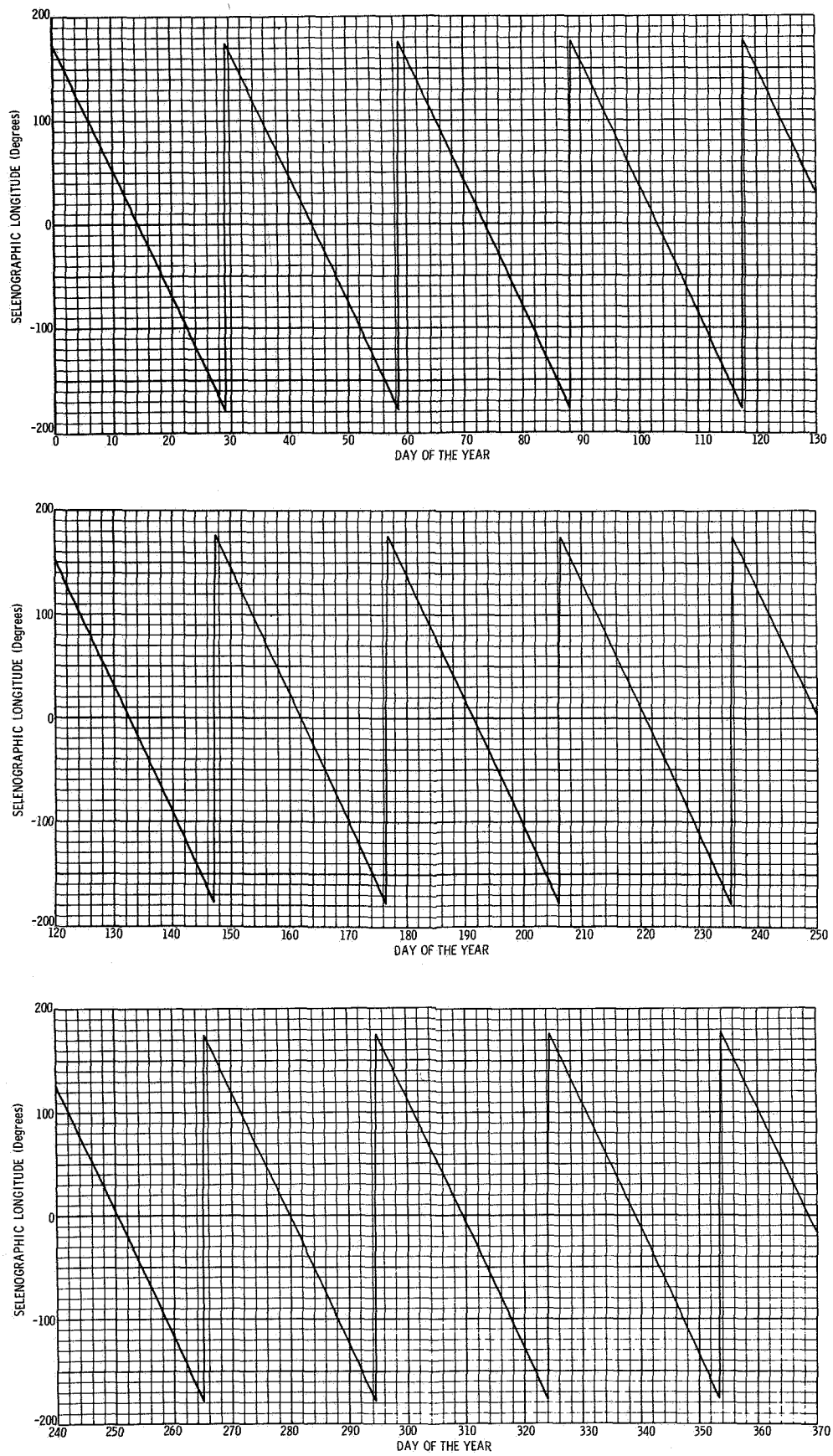


FIGURE B 1968-13 SELENOGRAPHIC LONGITUDE OF THE SUN

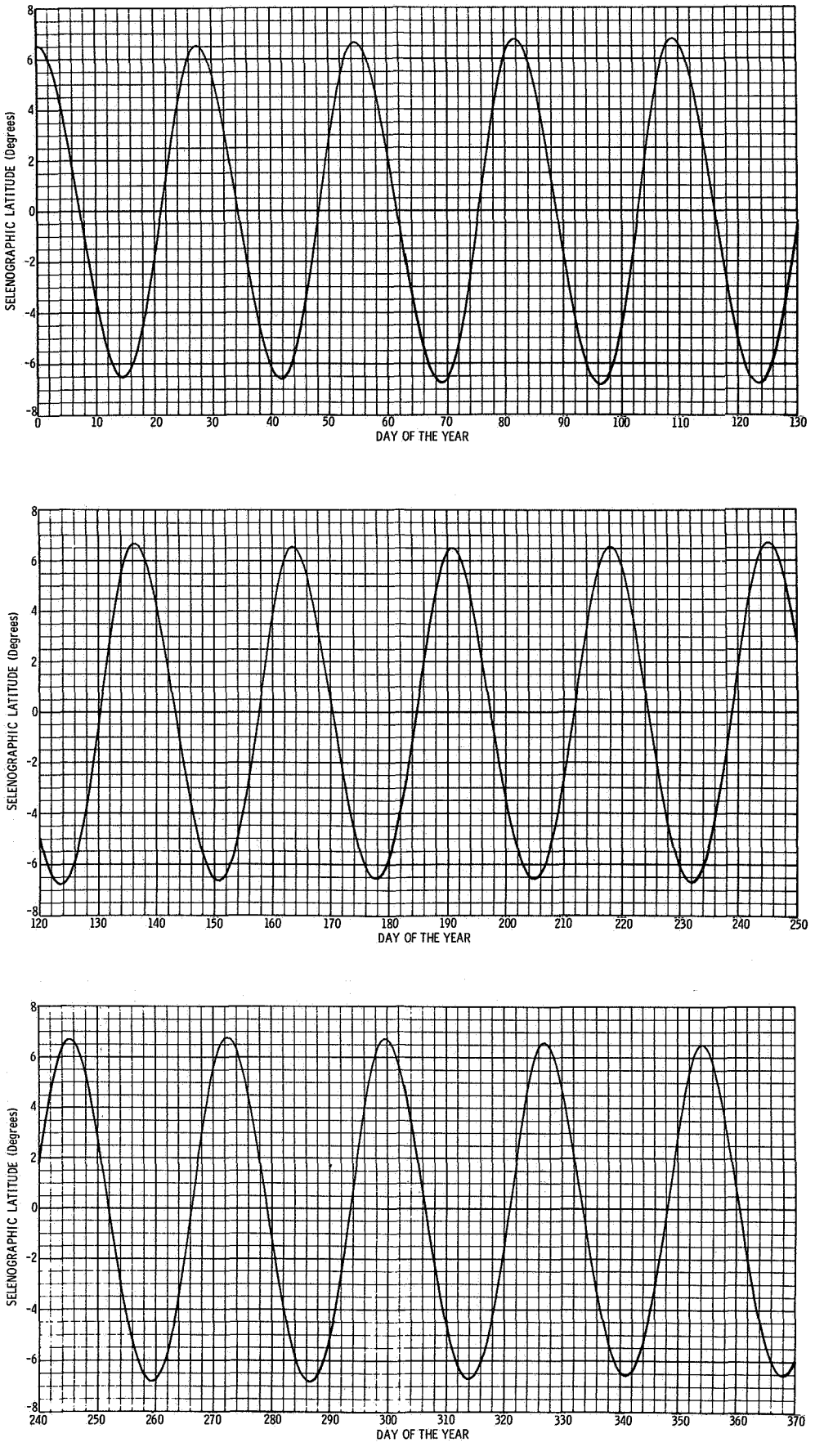


FIGURE B1968-14 SELENOGRAPHIC LATITUDE OF THE EARTH

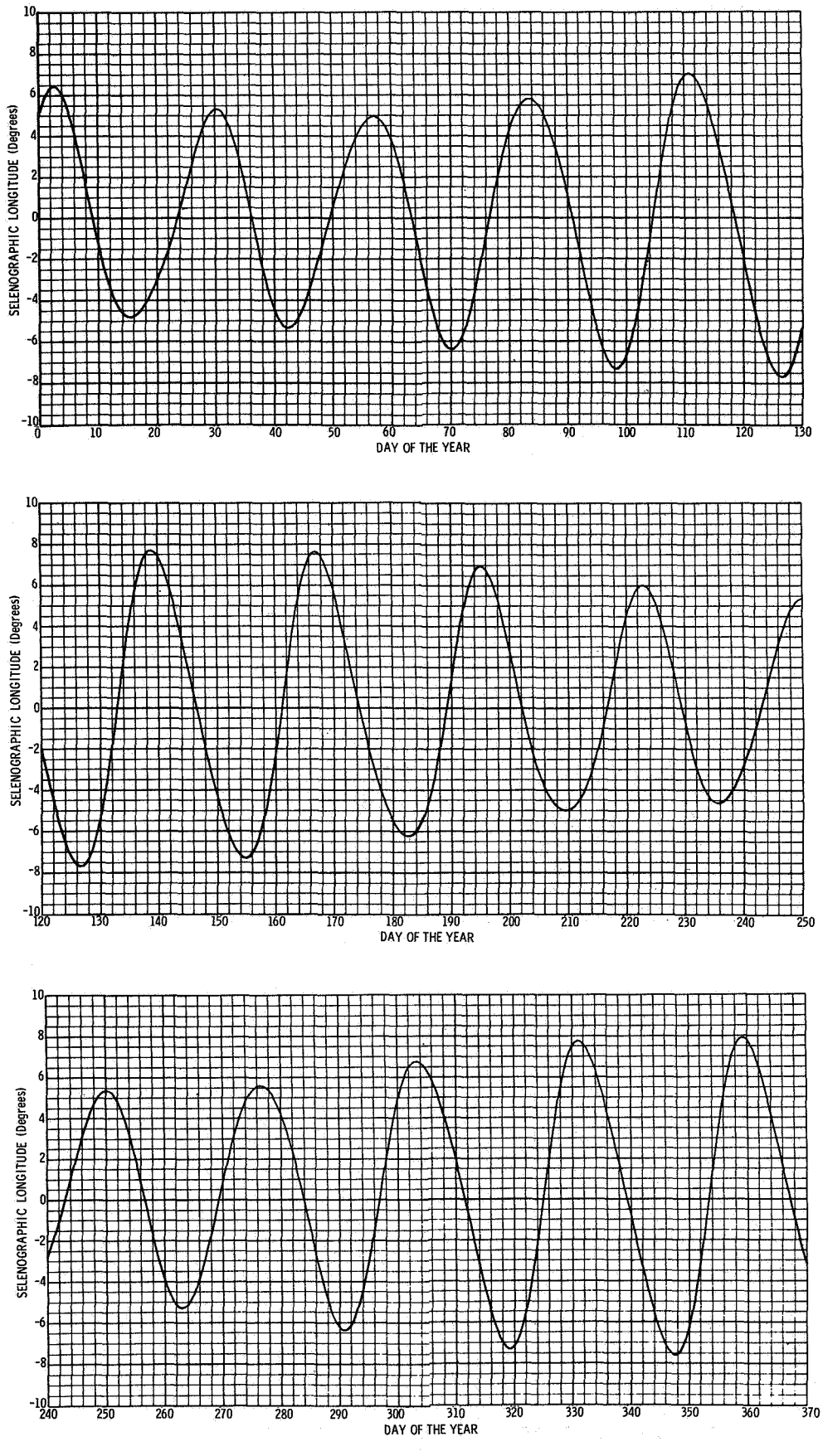
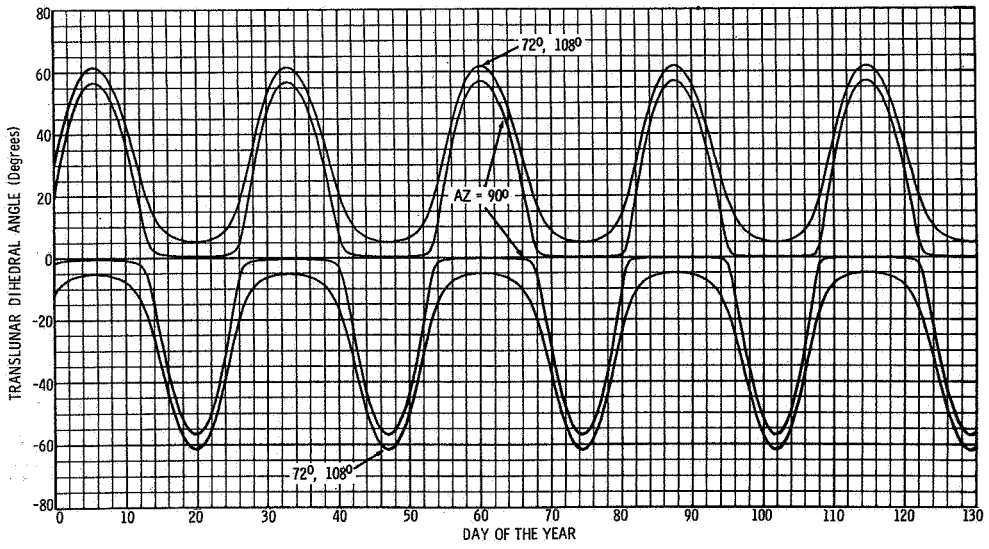
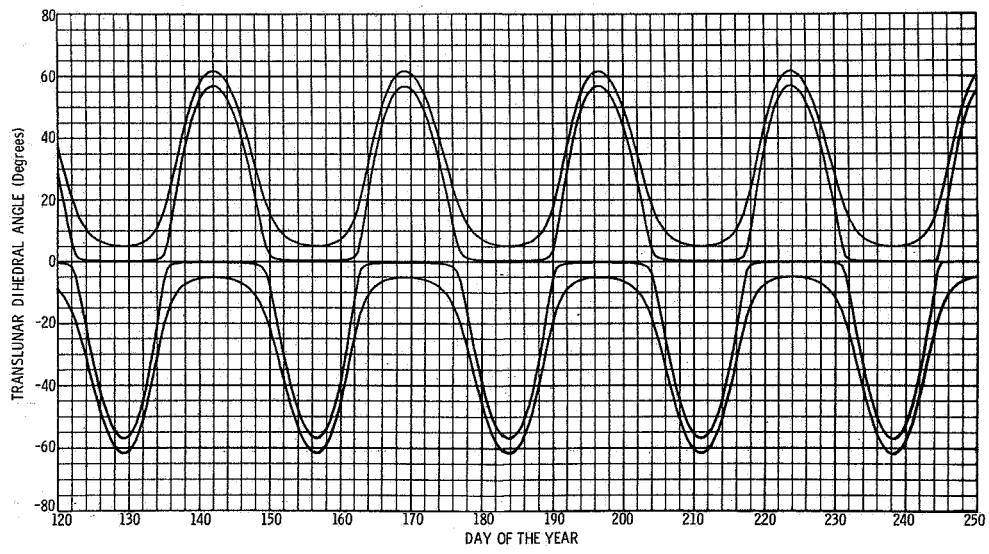


FIGURE B 1968-15 SELENOGRAPHIC LONGITUDE OF THE EARTH





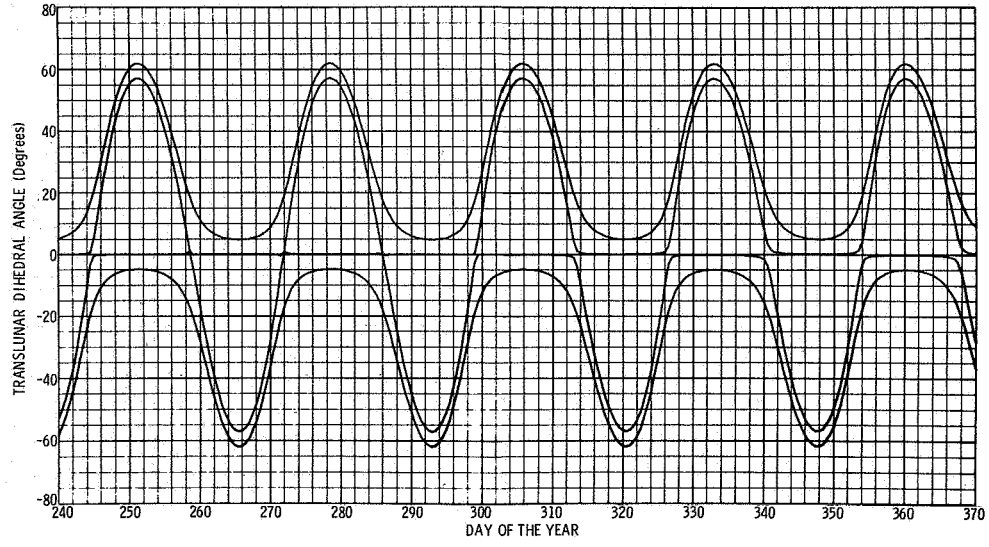


FIGURE B1968-16 TRANSLUNAR DIHEDRAL ANGLES

1969

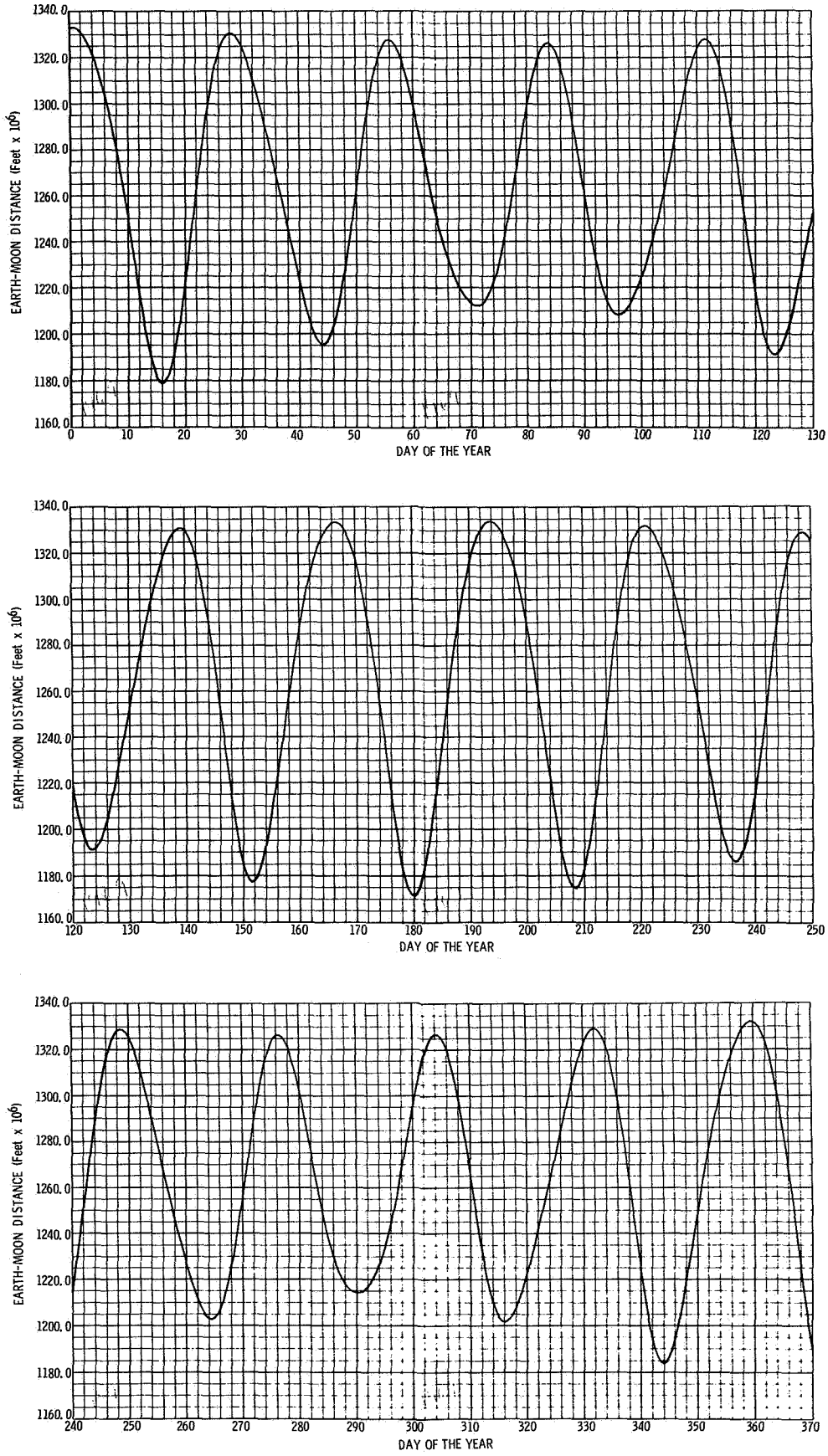
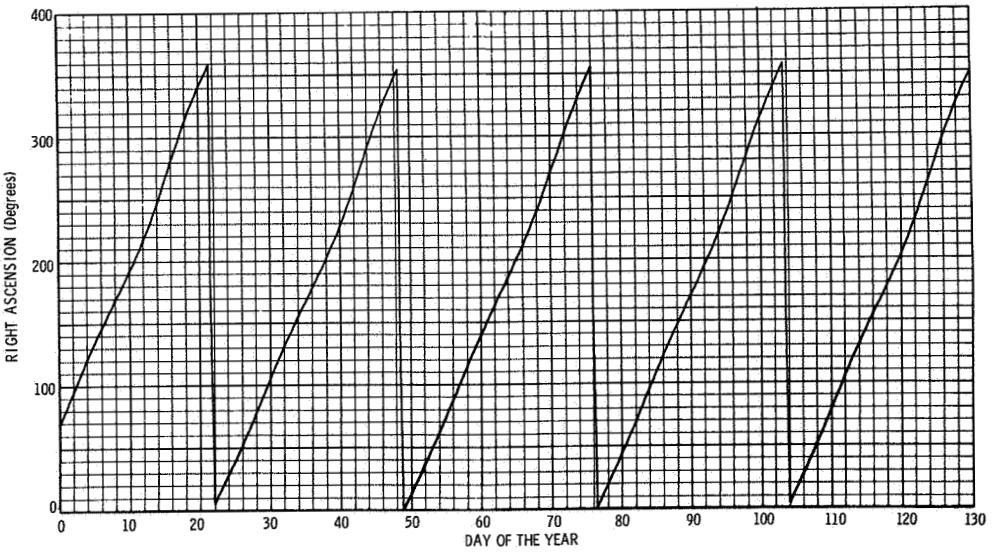
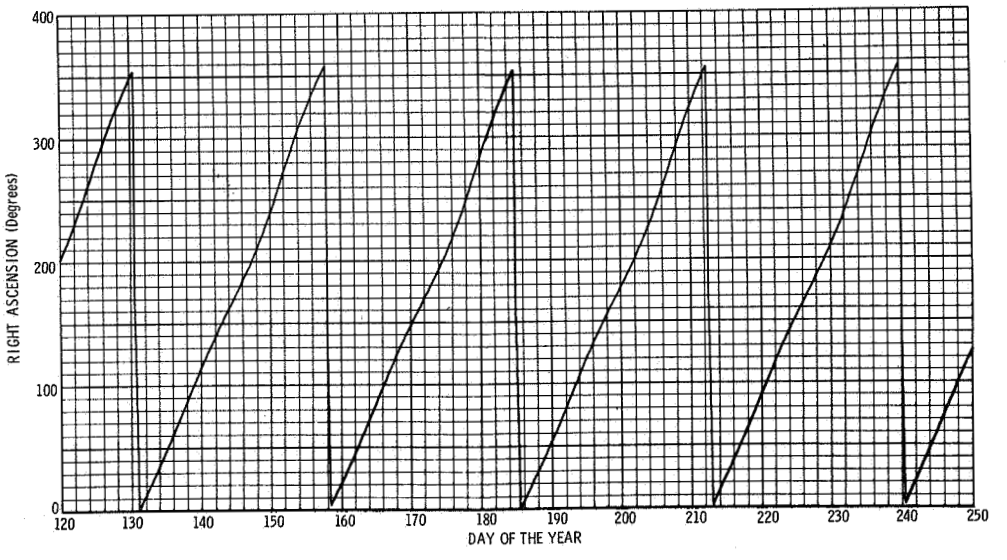


FIGURE B1969-1 EARTH-MOON DISTANCE





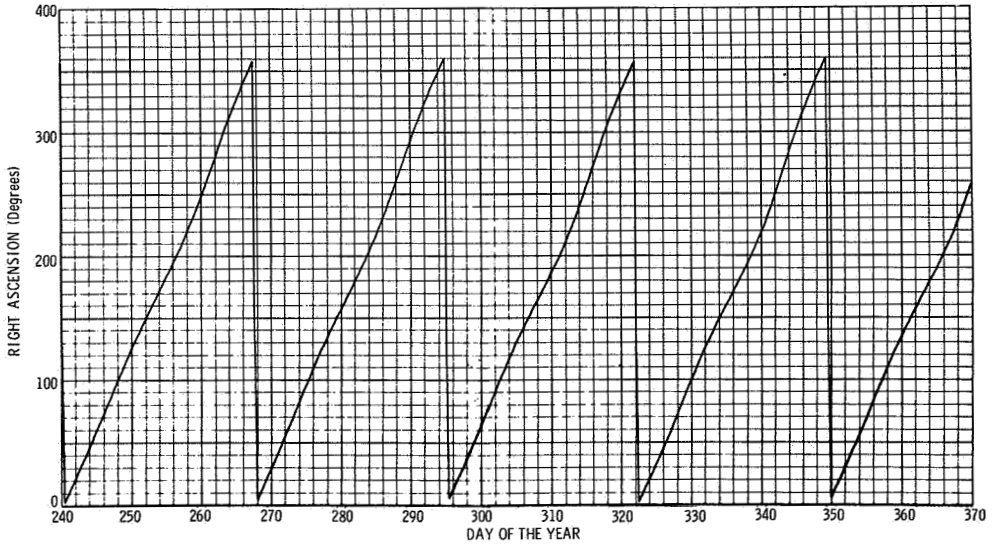


FIGURE B1969-2 RIGHT ASCENSION OF THE MOON

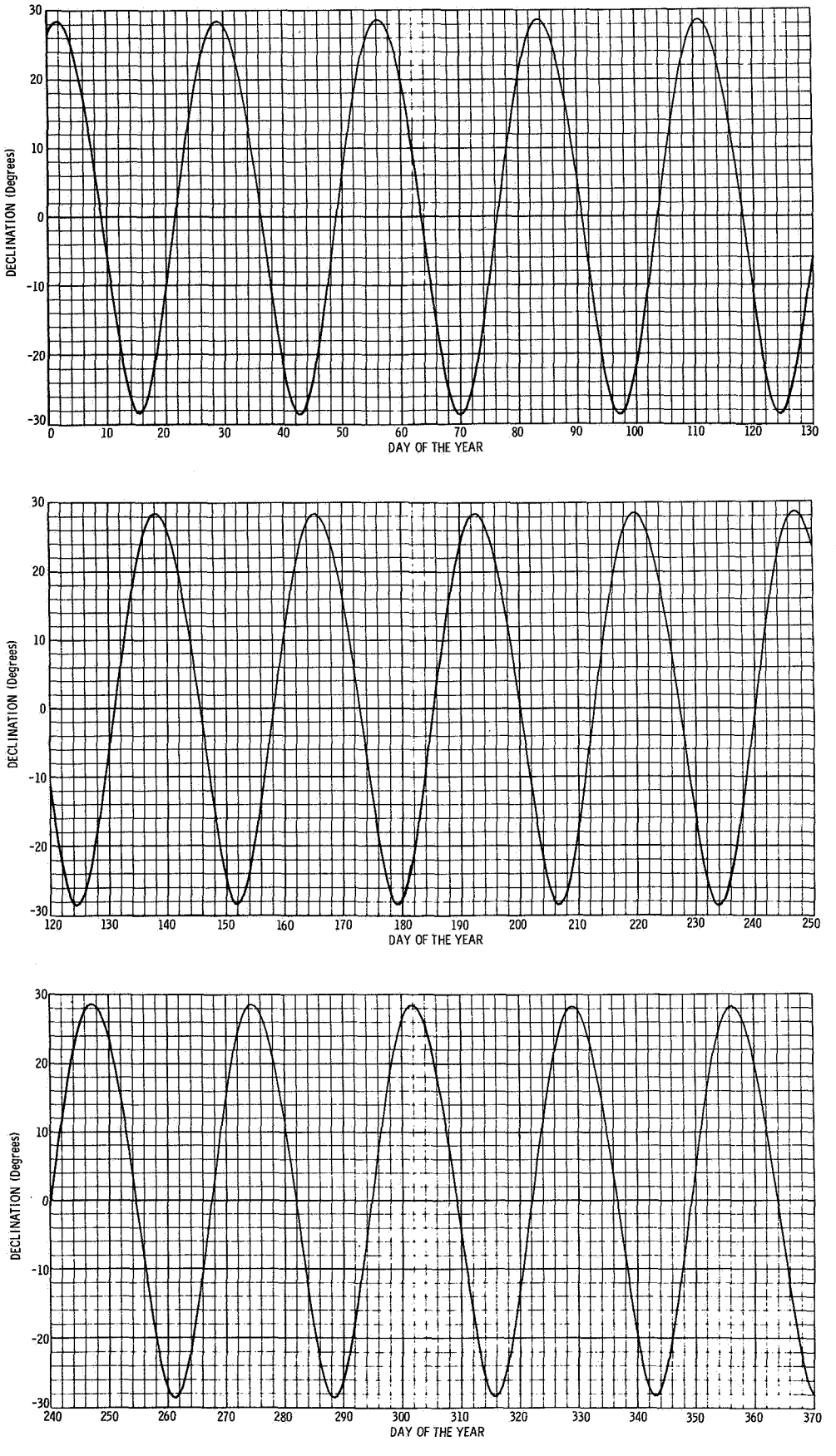
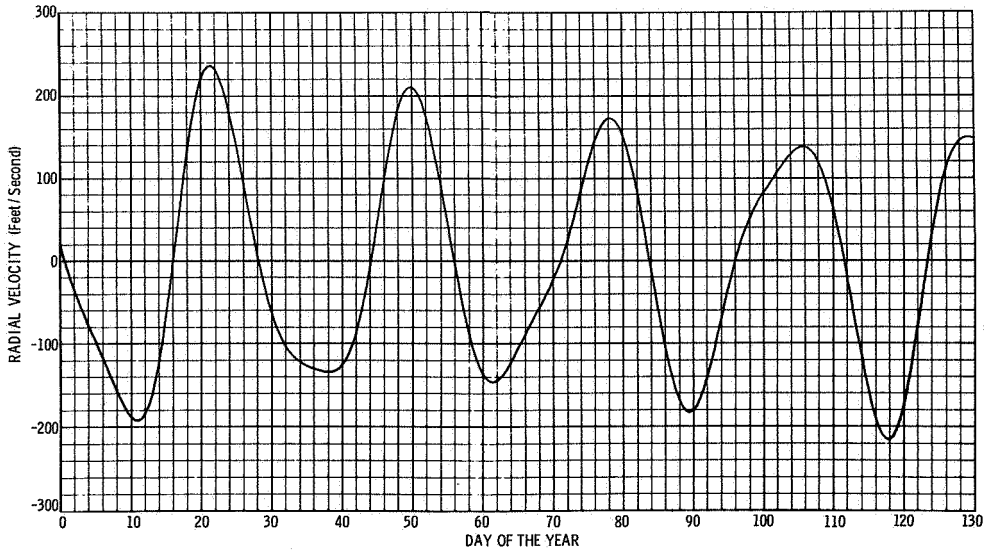
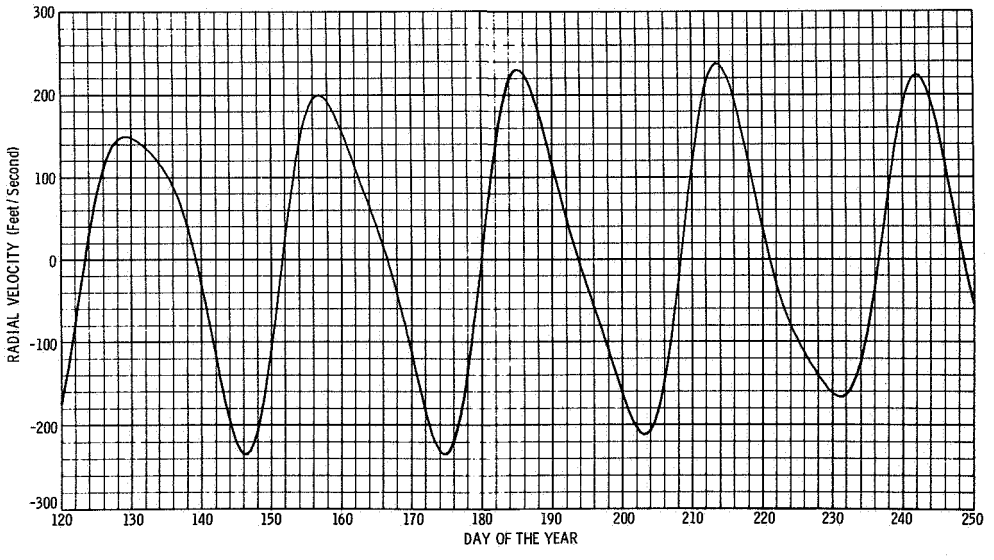


FIGURE B1969-3 DECLINATION OF THE MOON





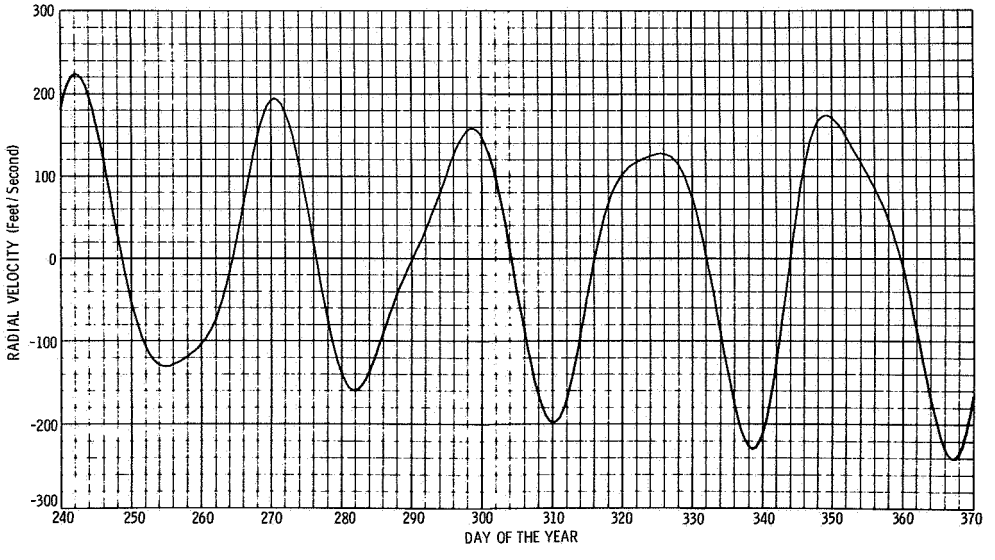
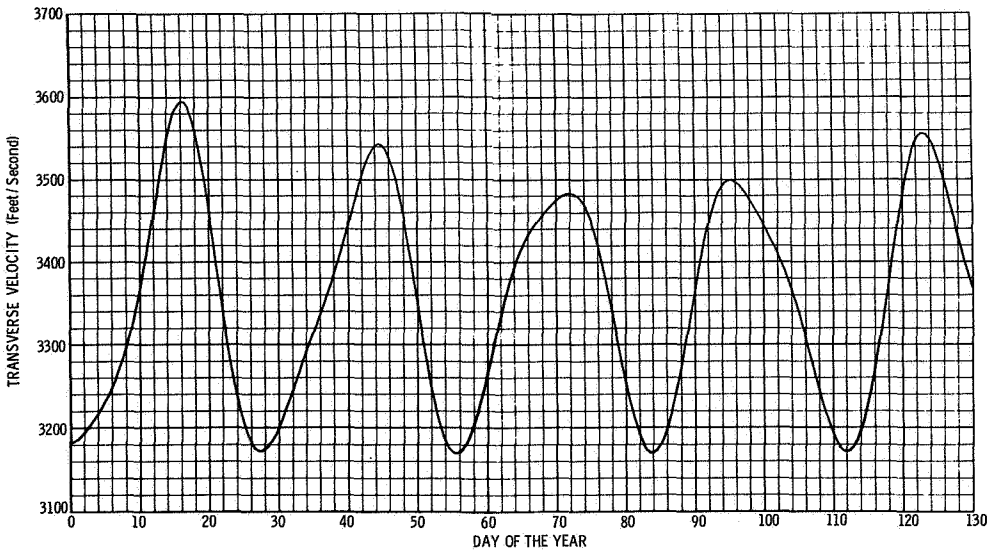
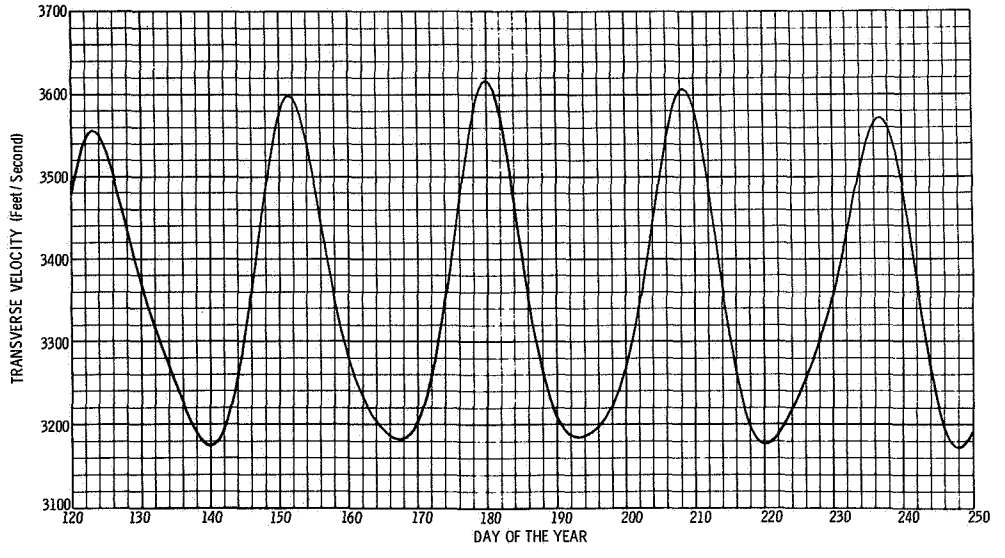


FIGURE B1969-4 RADIAL VELOCITY OF THE MOON





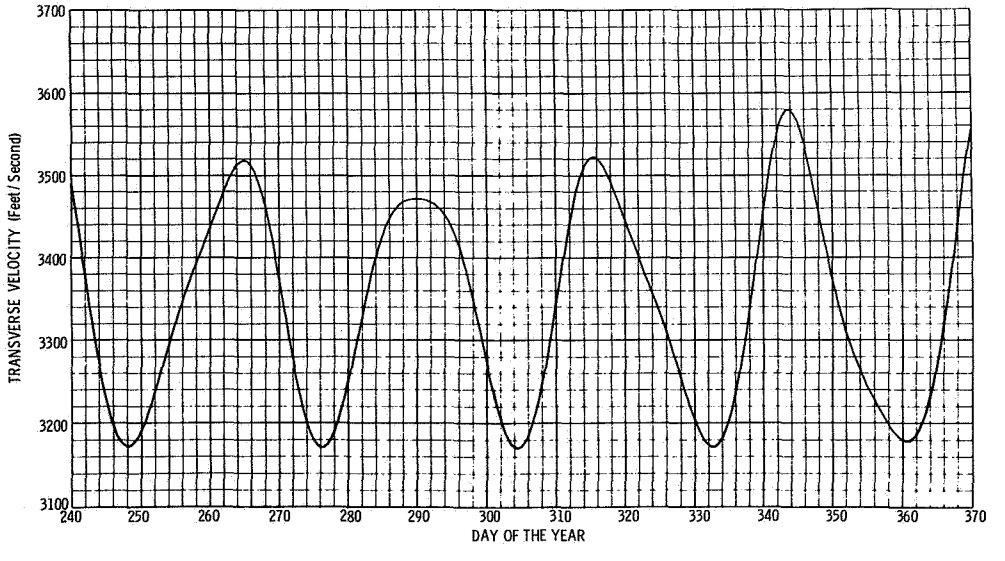
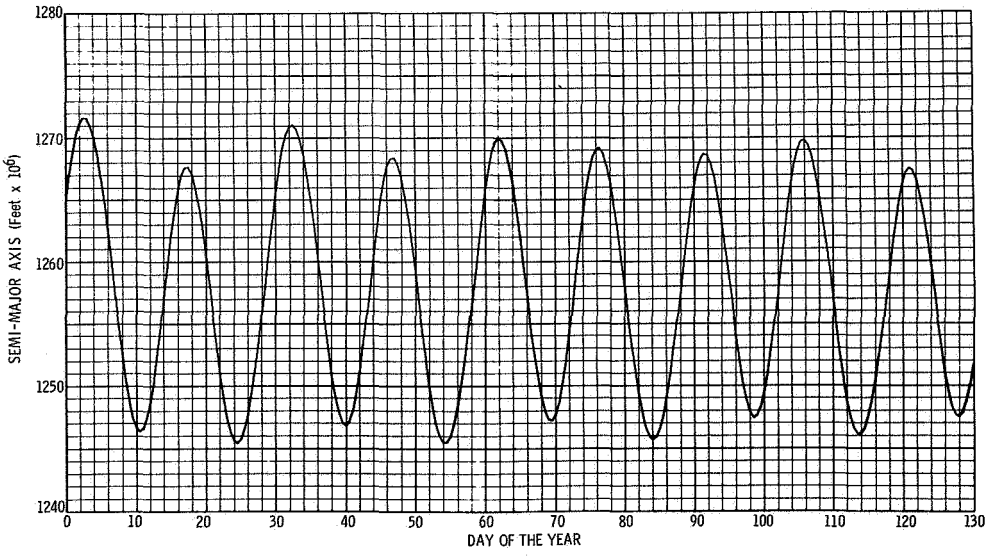
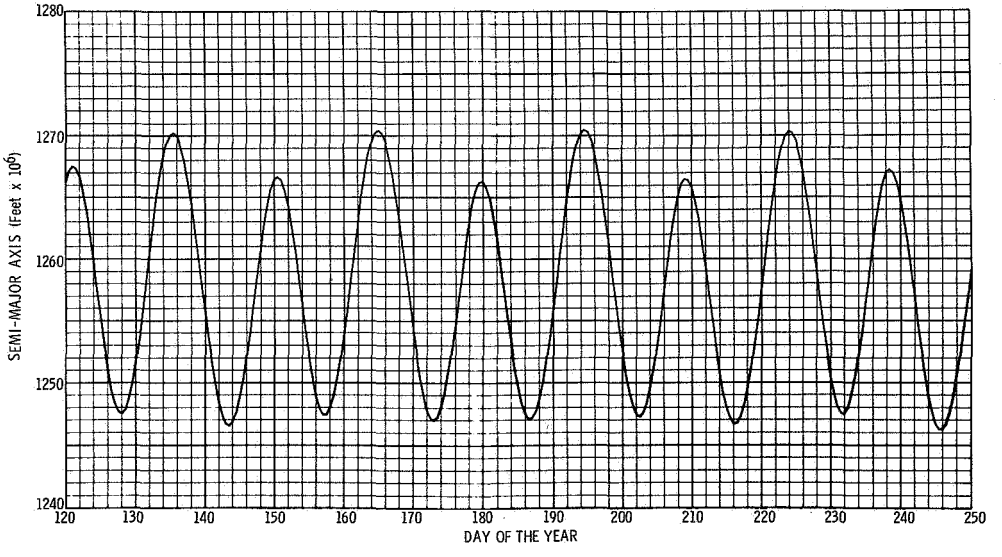


FIGURE B1969-5 TRANSVERSE VELOCITY OF THE MOON





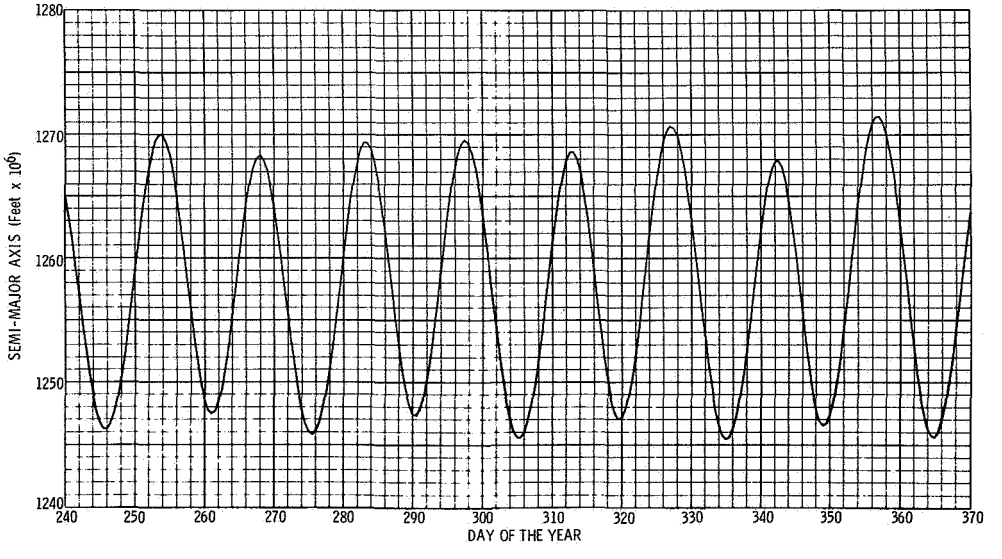
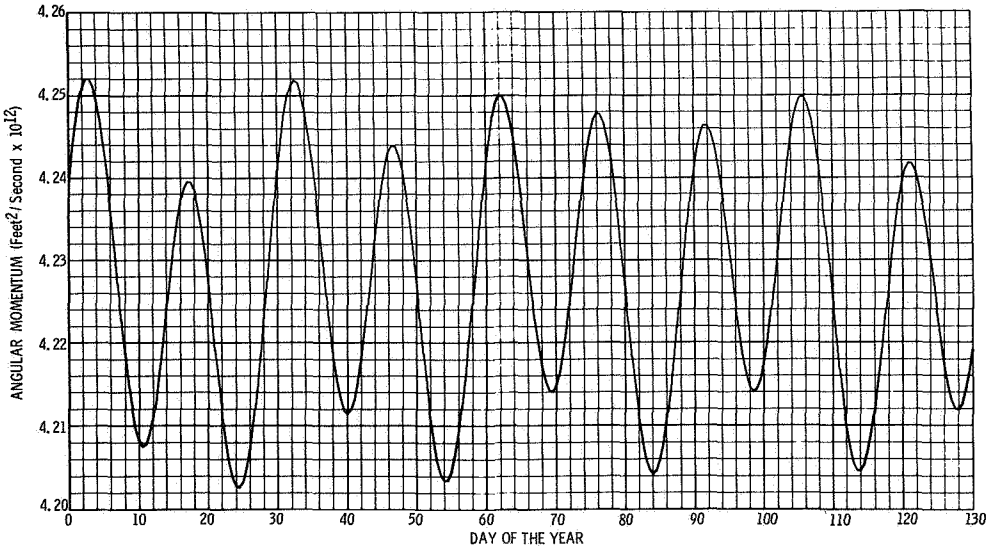
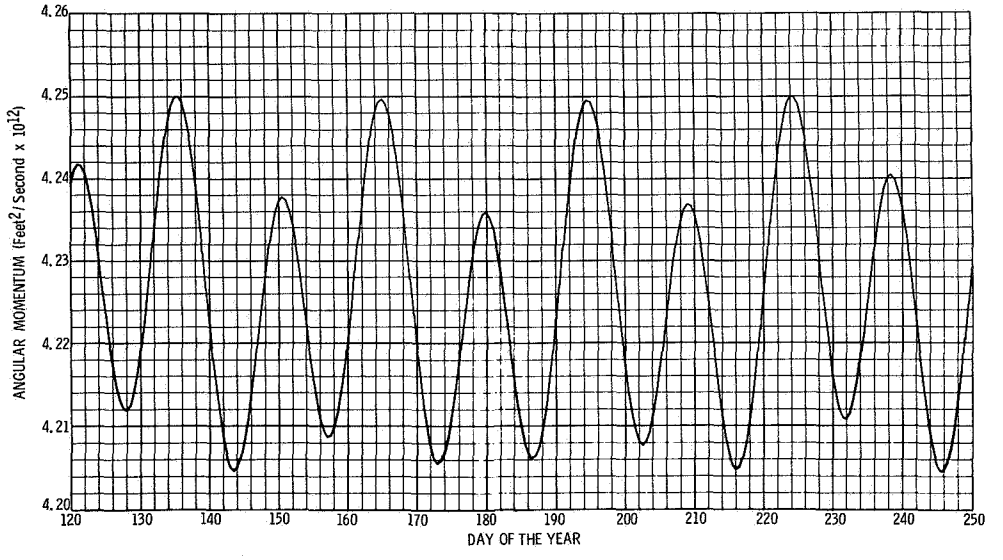


FIGURE B1969-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT





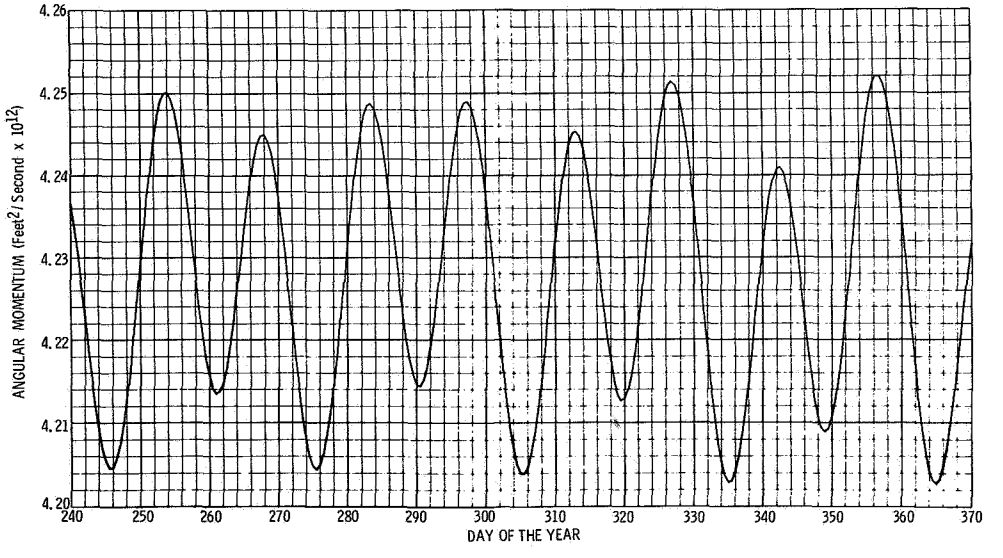


FIGURE B1969-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON

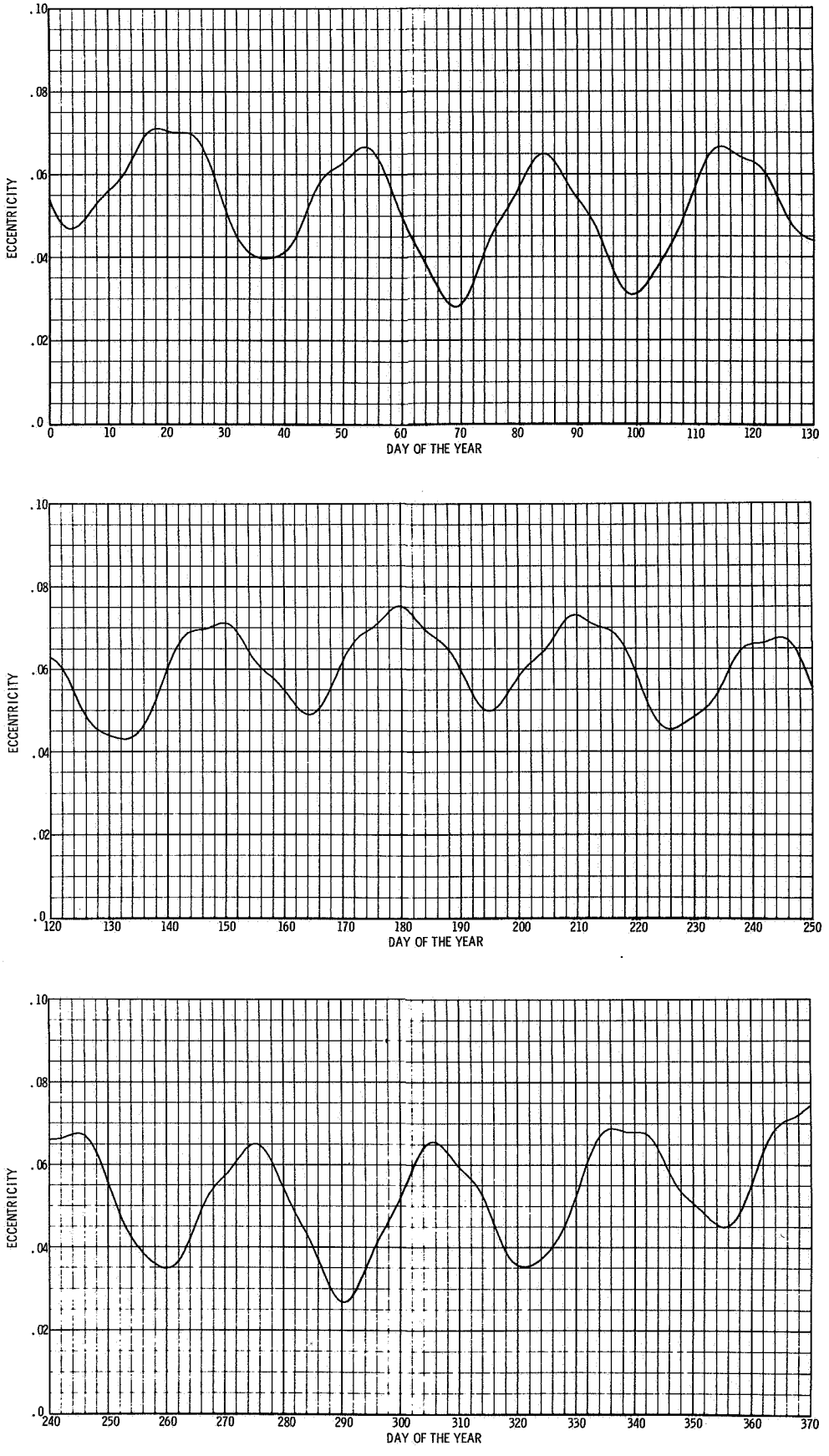
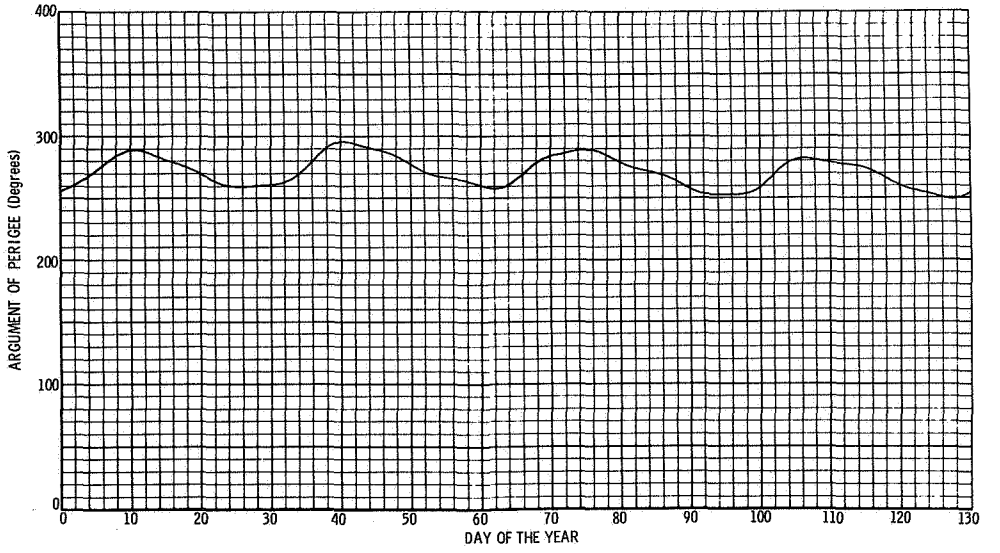
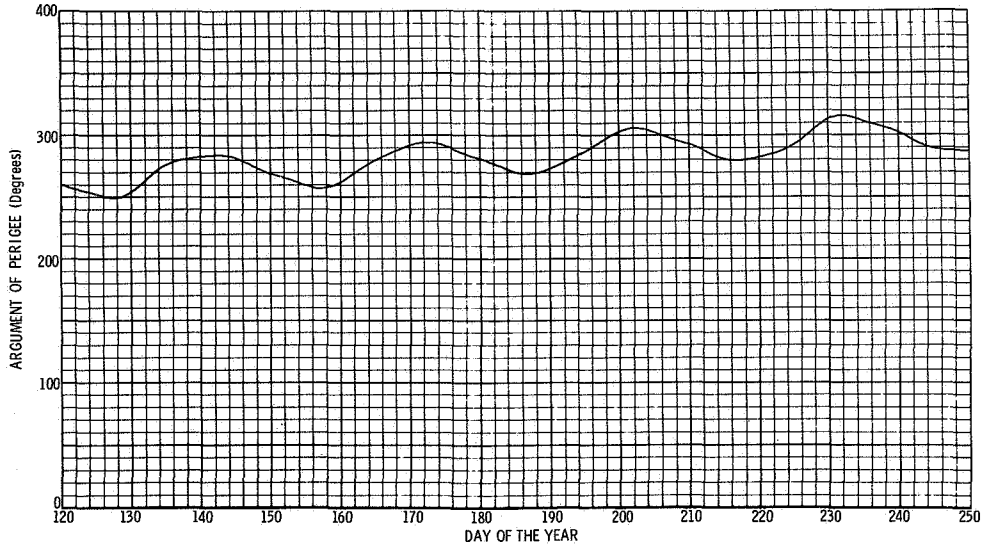


FIGURE B1969-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT





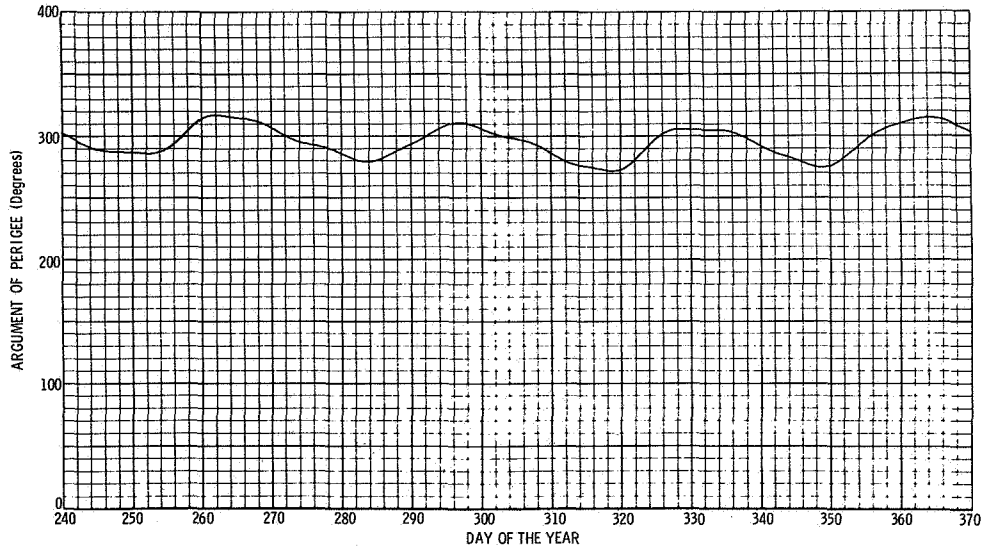
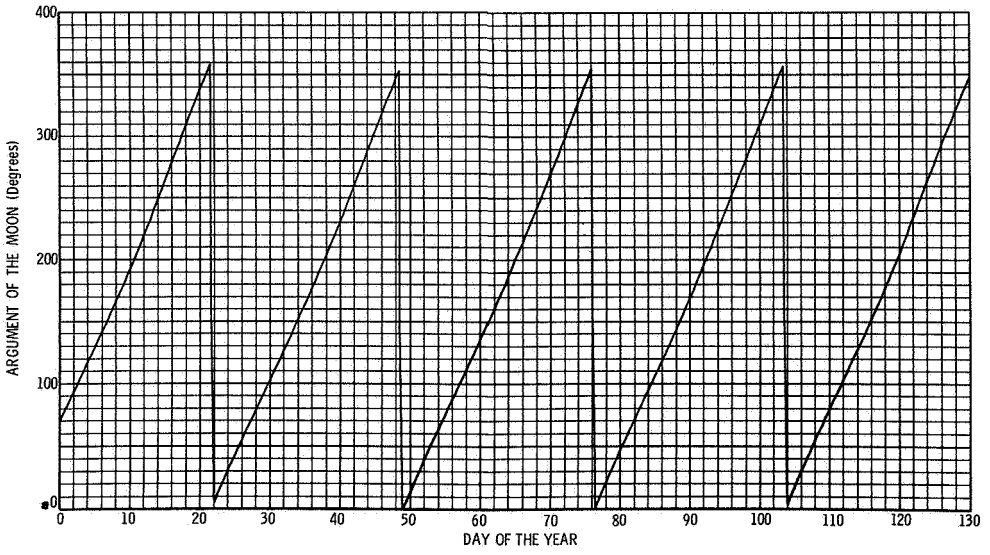
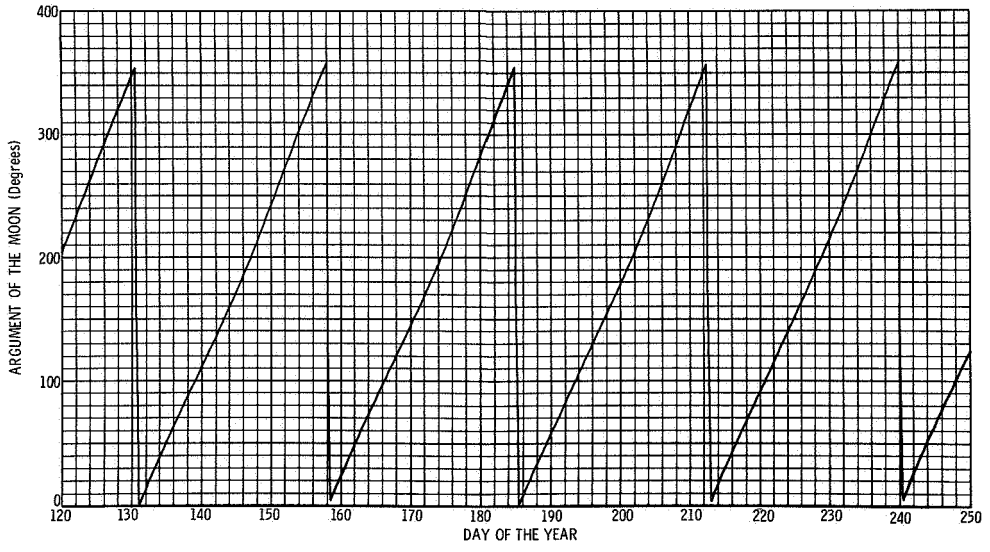


FIGURE B1969-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE





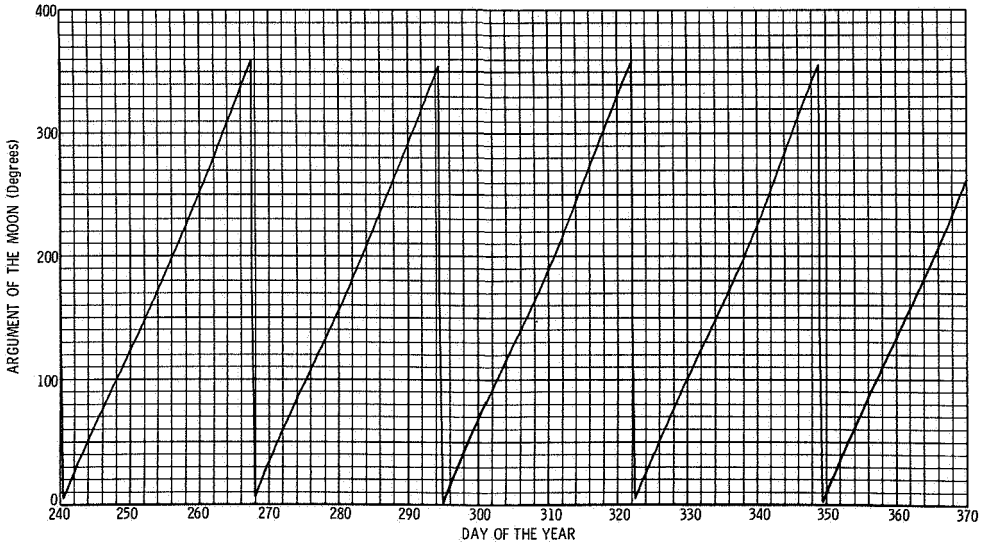


FIGURE B1969-10 ARGUMENT OF THE MOON'S POSITION

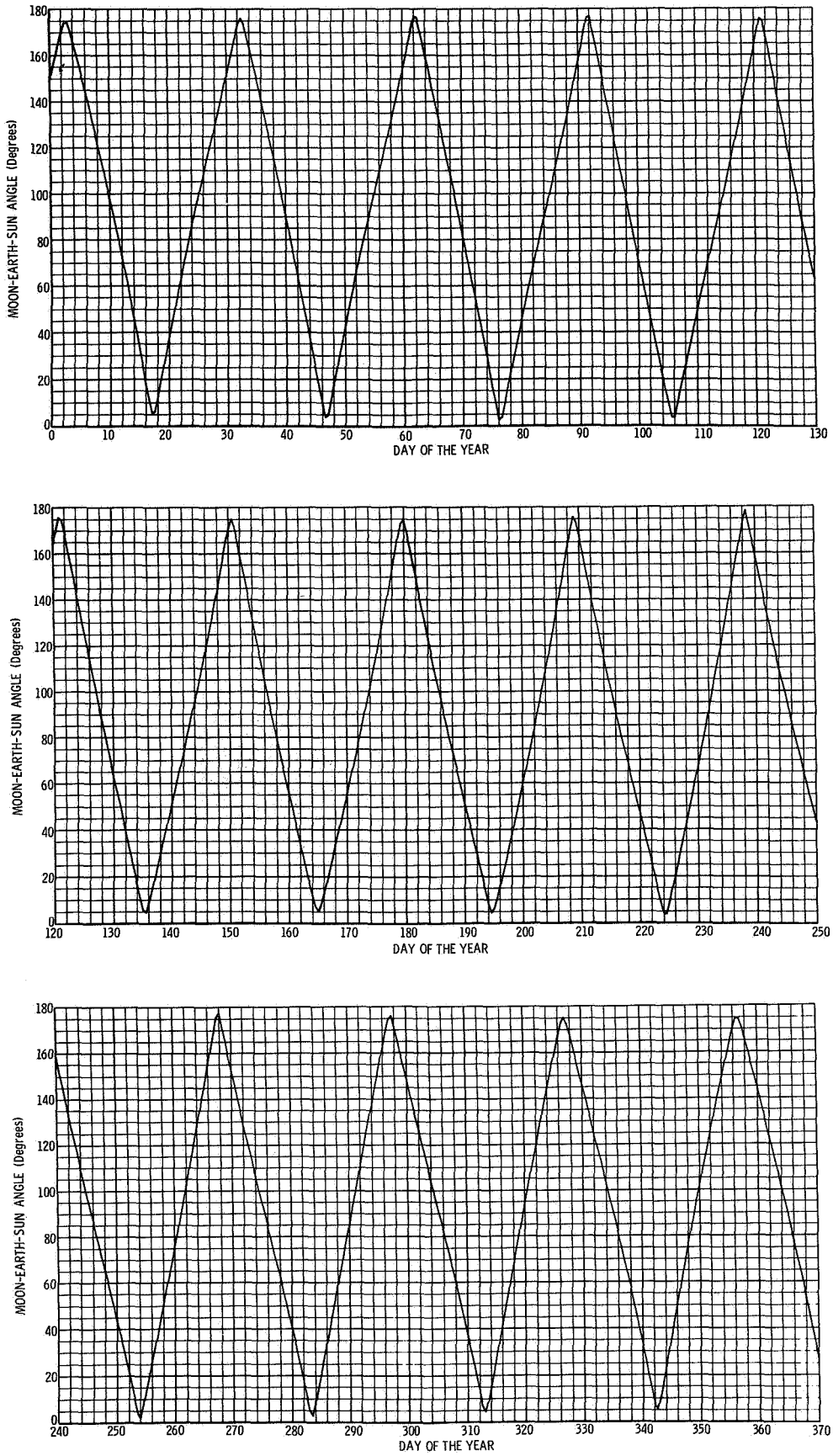
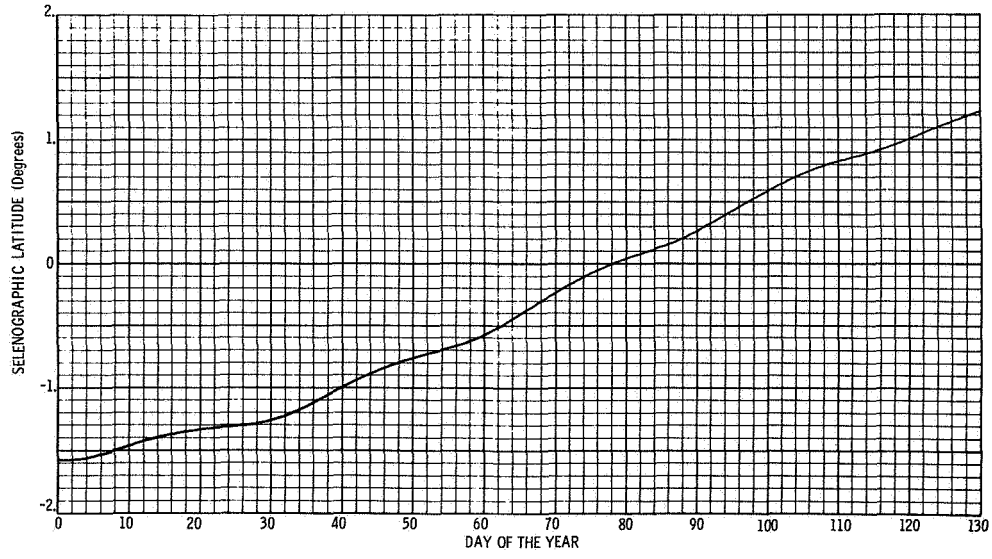
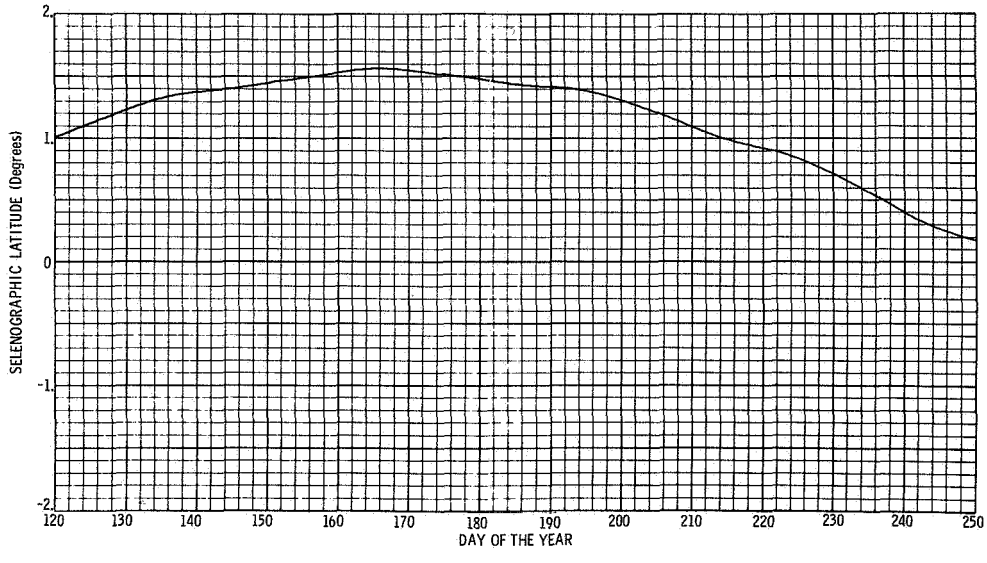


FIGURE B1969-11 MOON-EARTH-SUN ANGLE





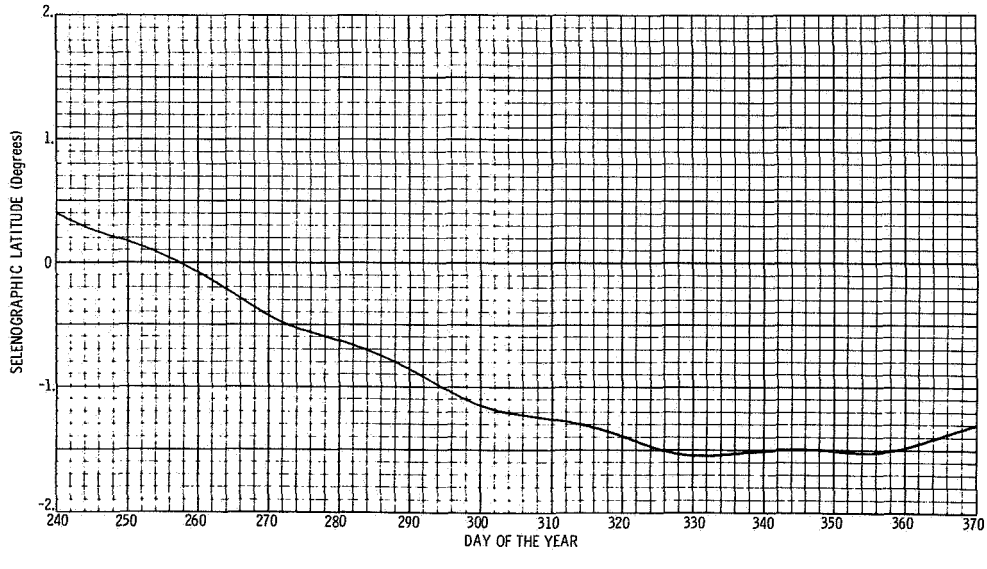


FIGURE B1969-12 SELENOGRAPHIC LATITUDE OF THE SUN

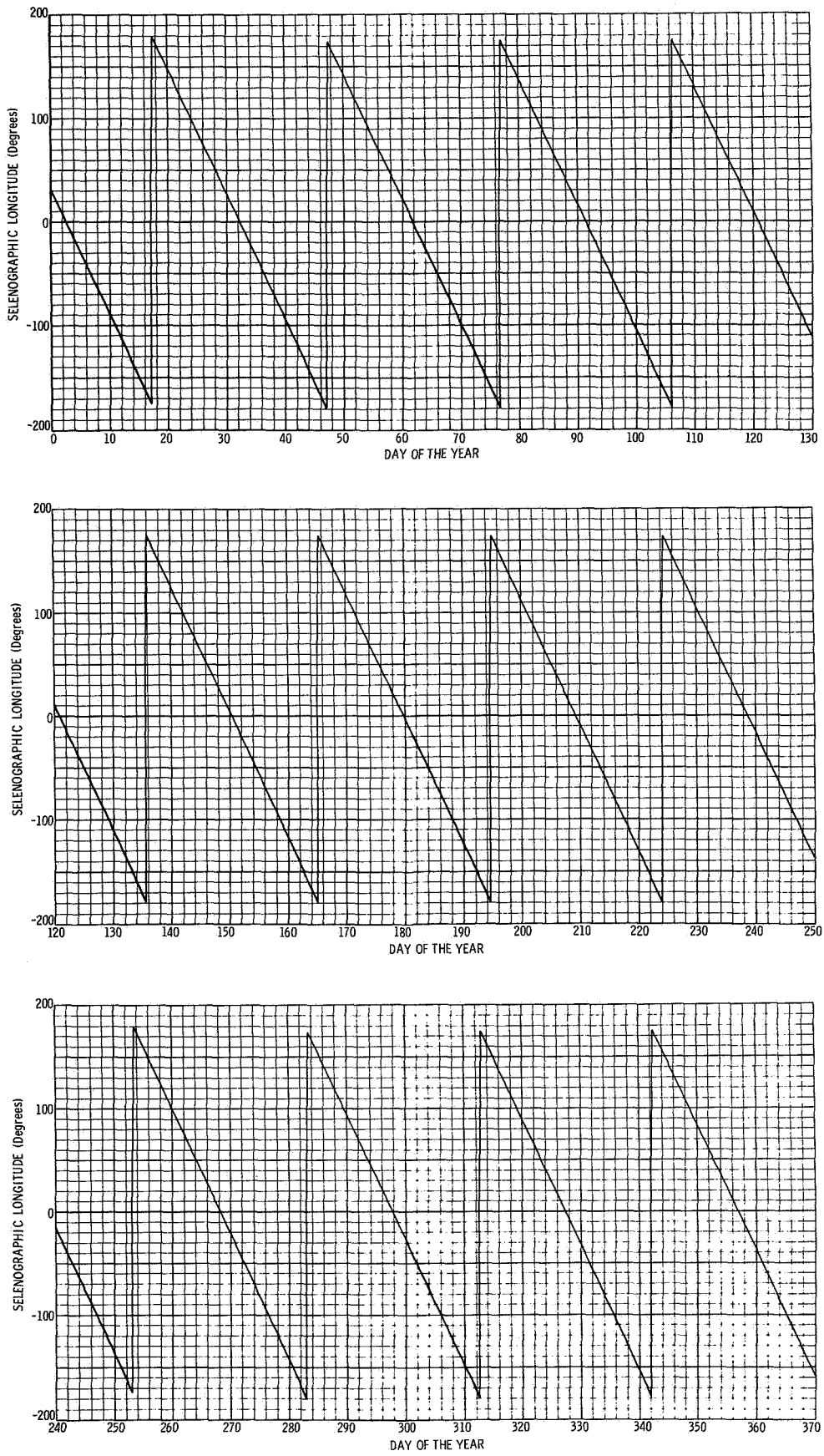
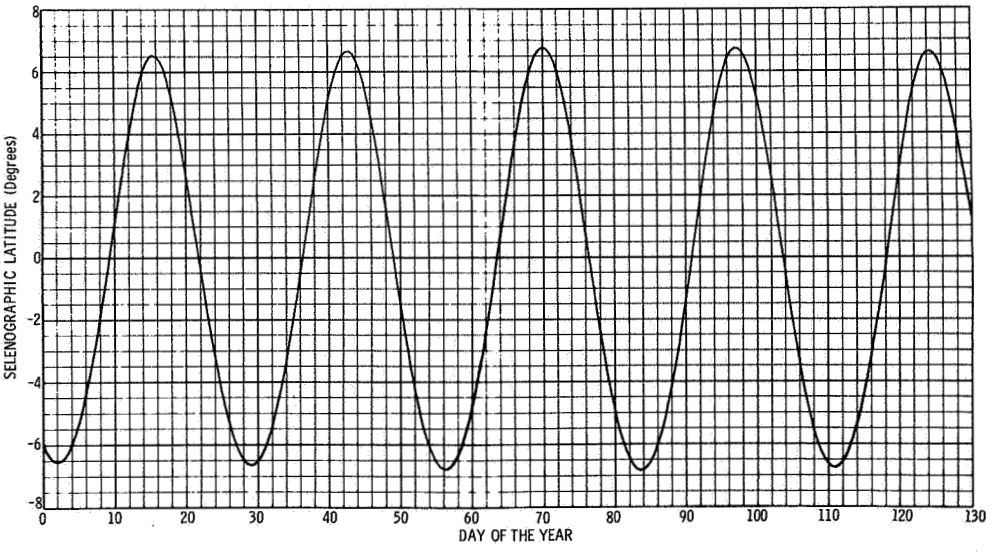
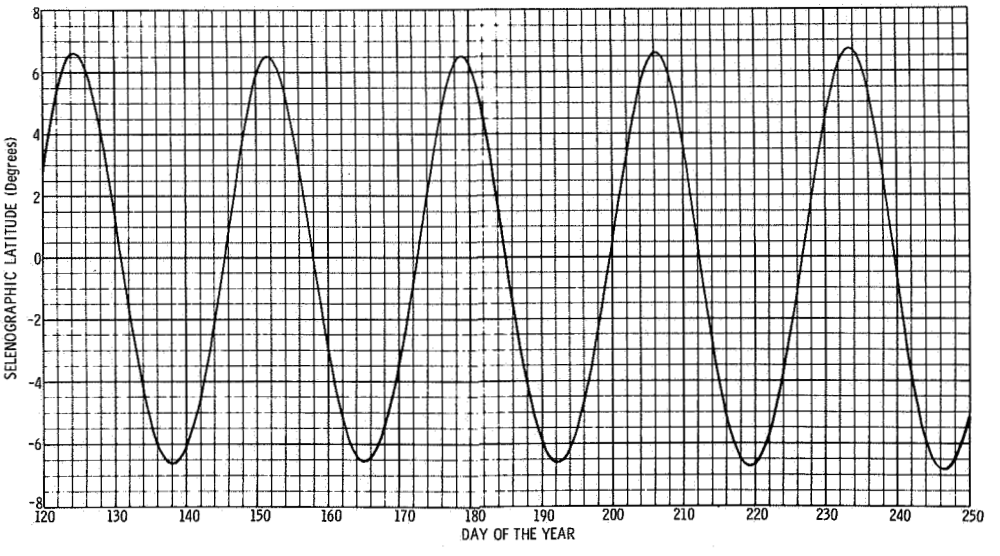


FIGURE B1969-13 SELENOGRAPHIC LONGITUDE OF THE SUN





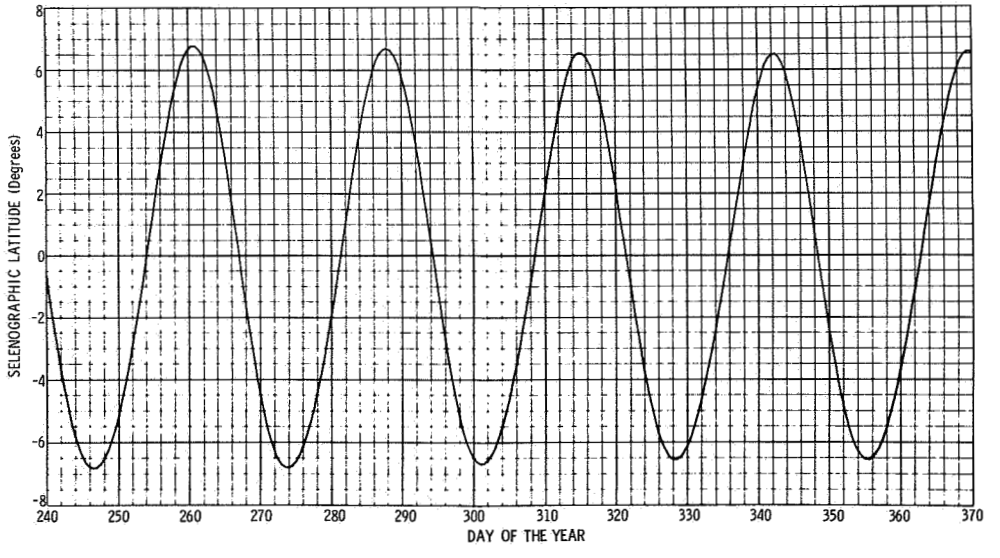


FIGURE B1969-14 SELENOGRAPHIC LATITUDE OF THE EARTH

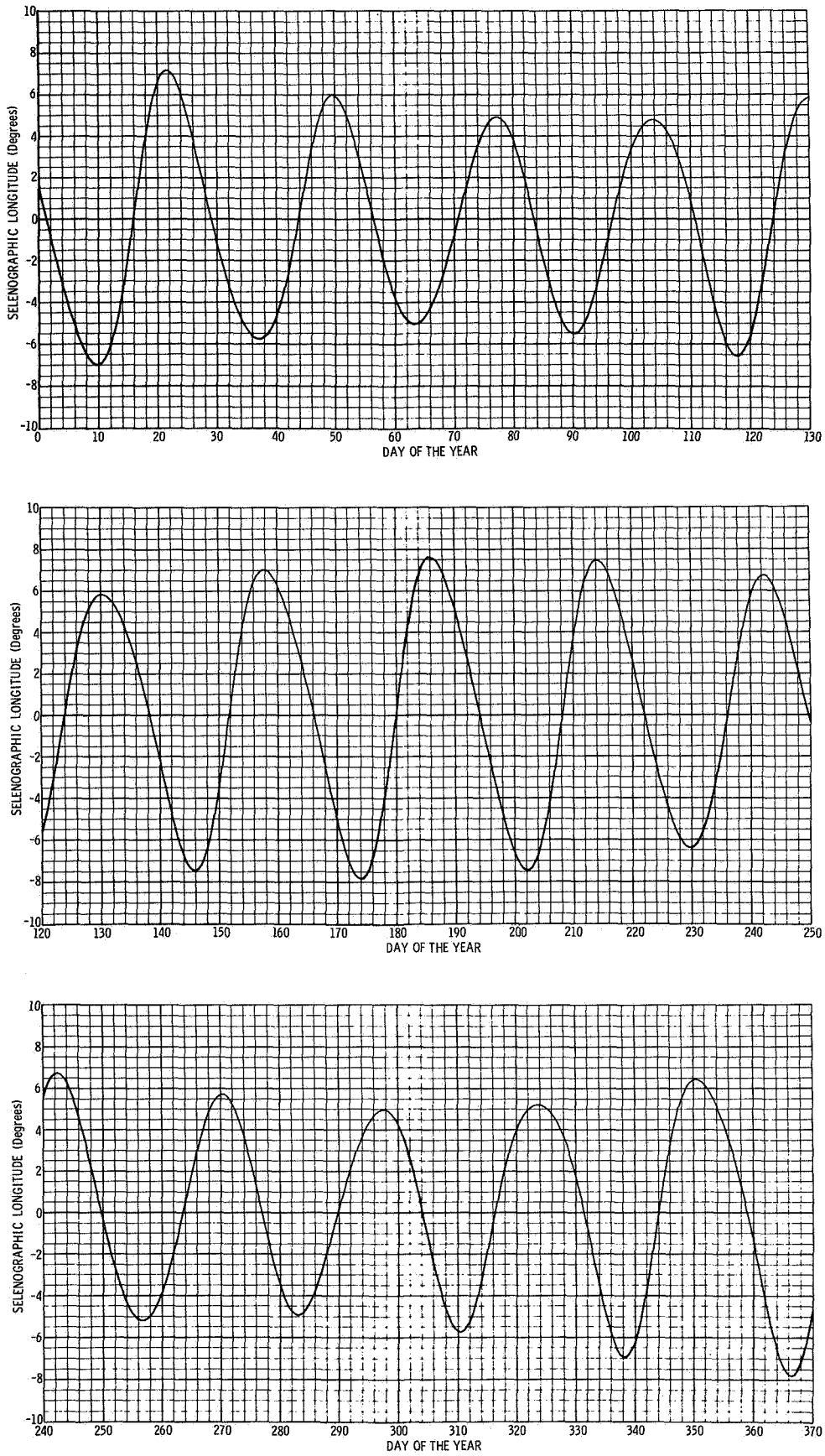


FIGURE B1969-15 SELENOGRAPHIC LONGITUDE OF THE EARTH

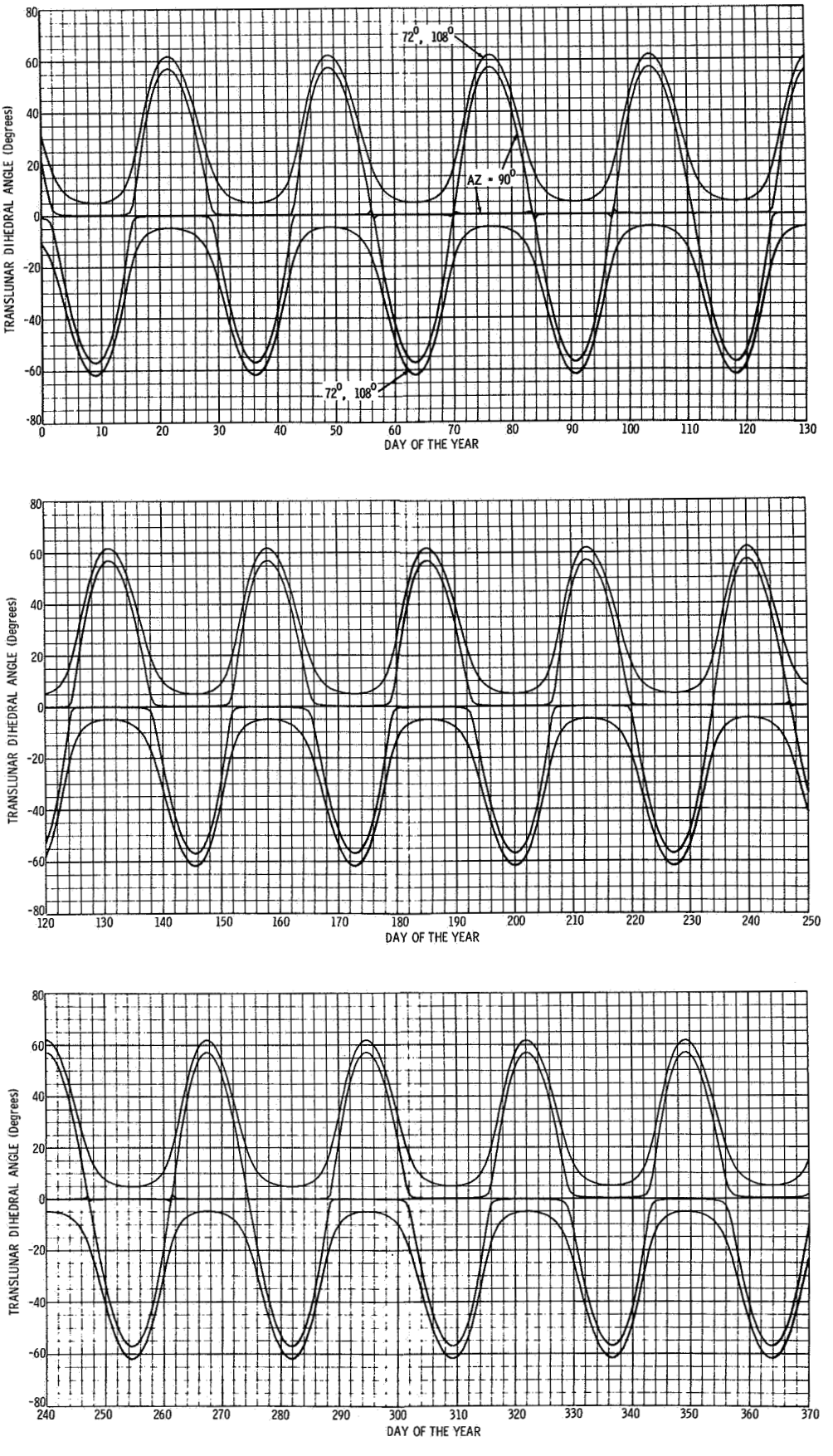
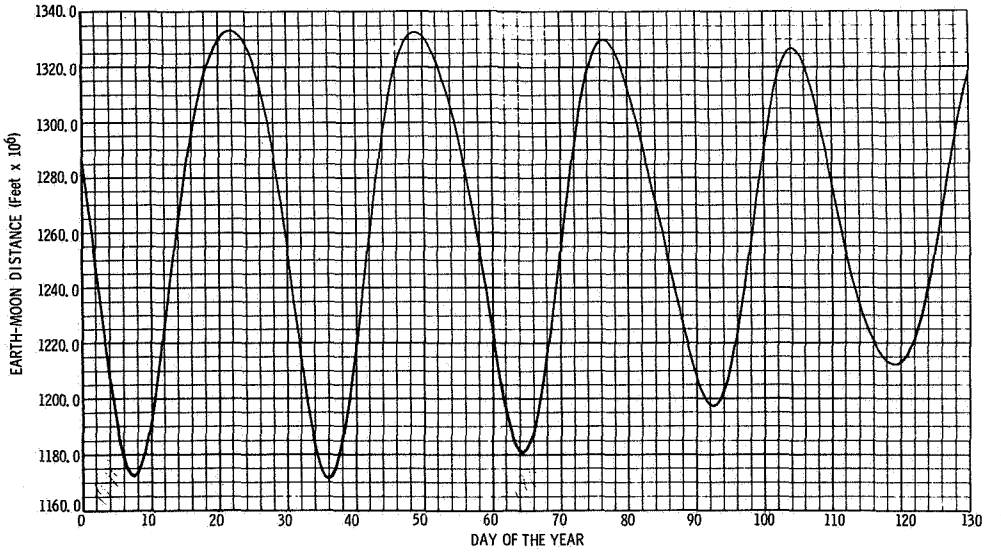
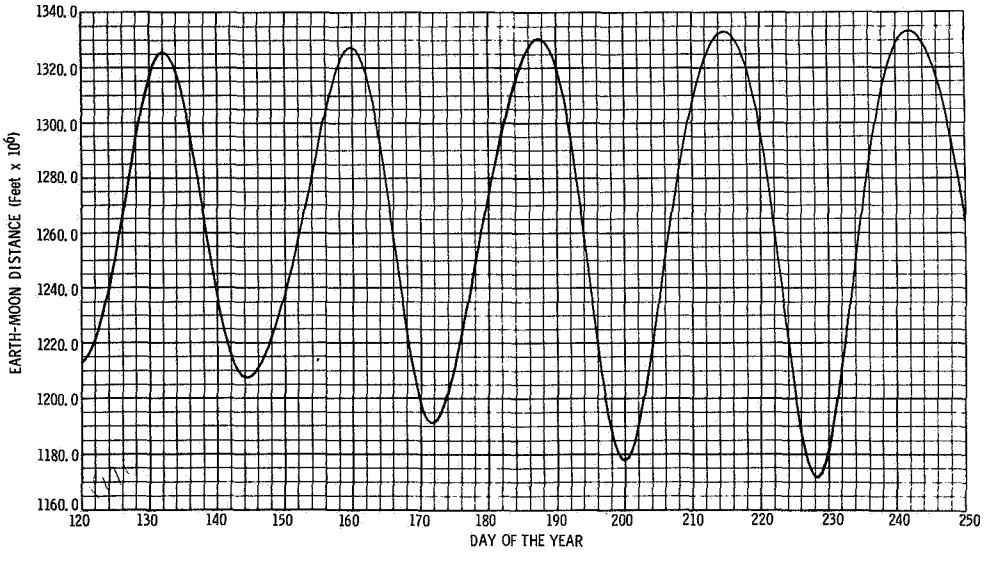


FIGURE B1969-16 TRANSLUNAR DIHEDRAL ANGLES

1970





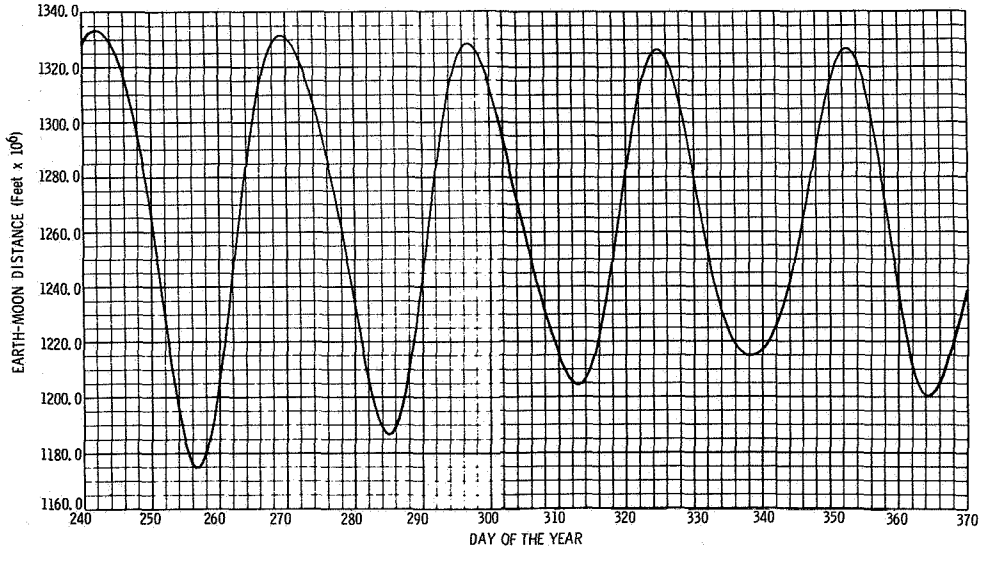
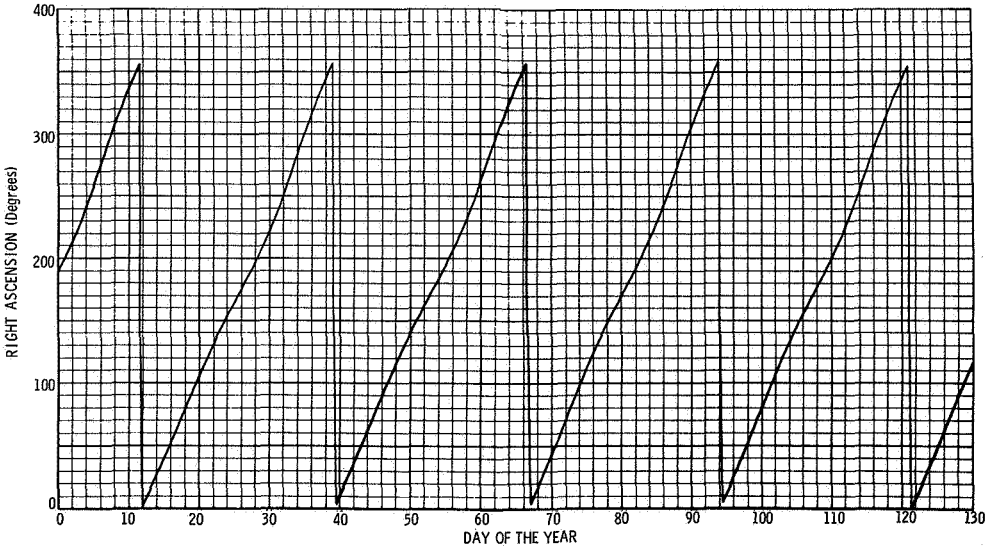
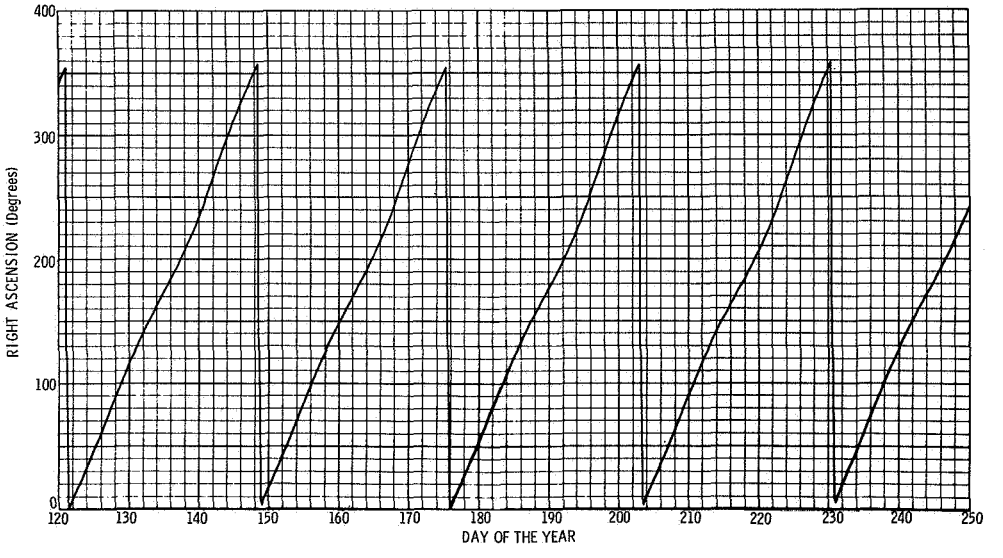


FIGURE B1970-1 EARTH-MOON DISTANCE





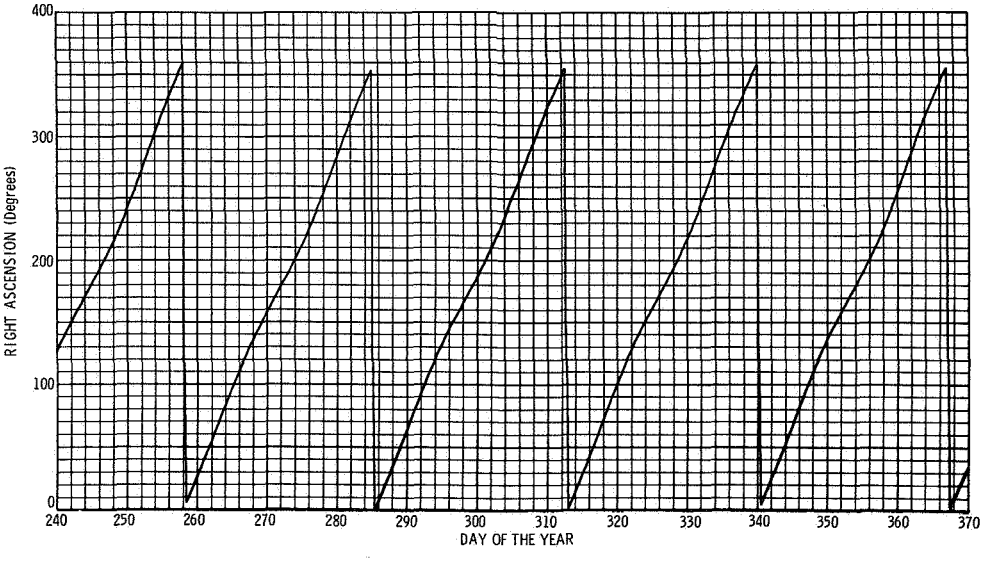
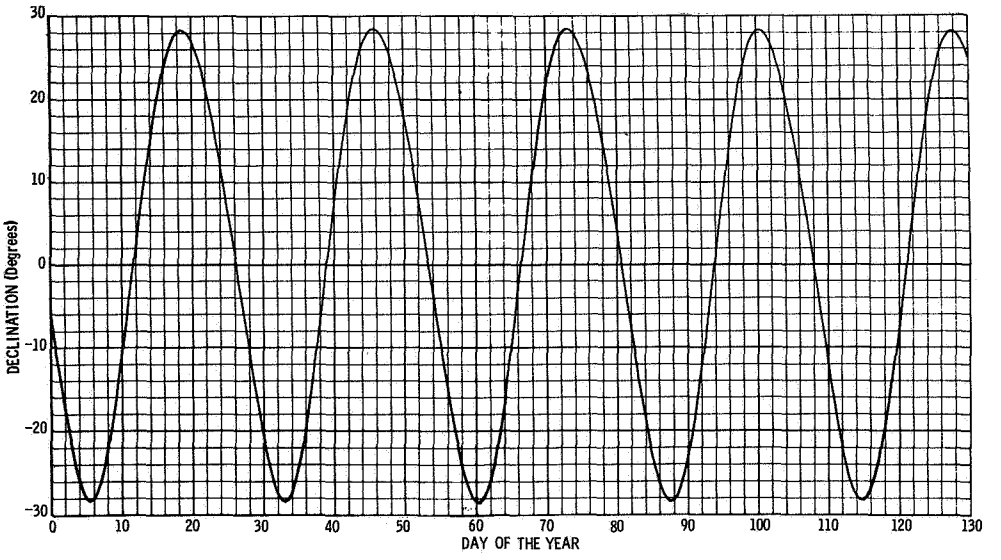
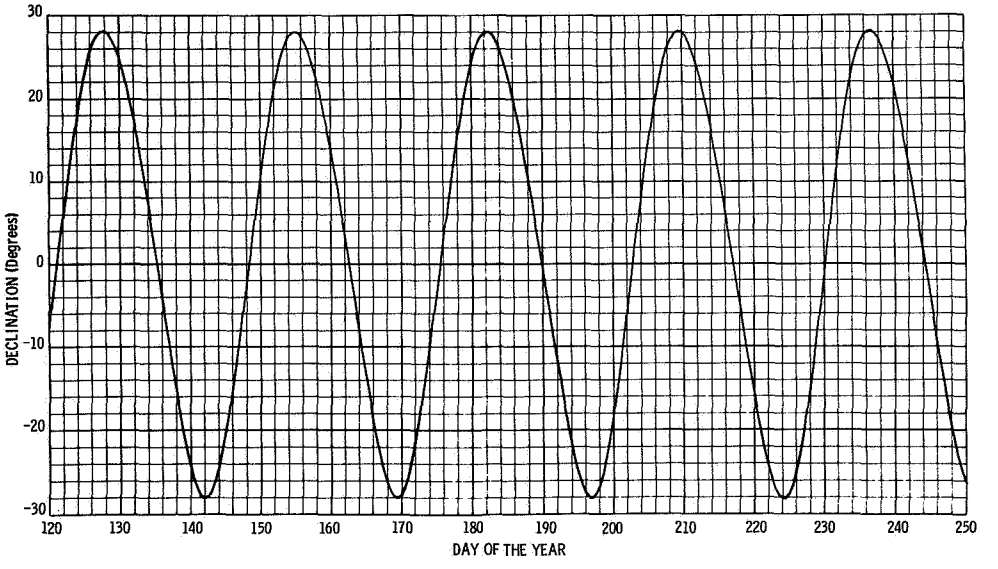


FIGURE B1970-2 RIGHT ASCENSION OF THE MOON





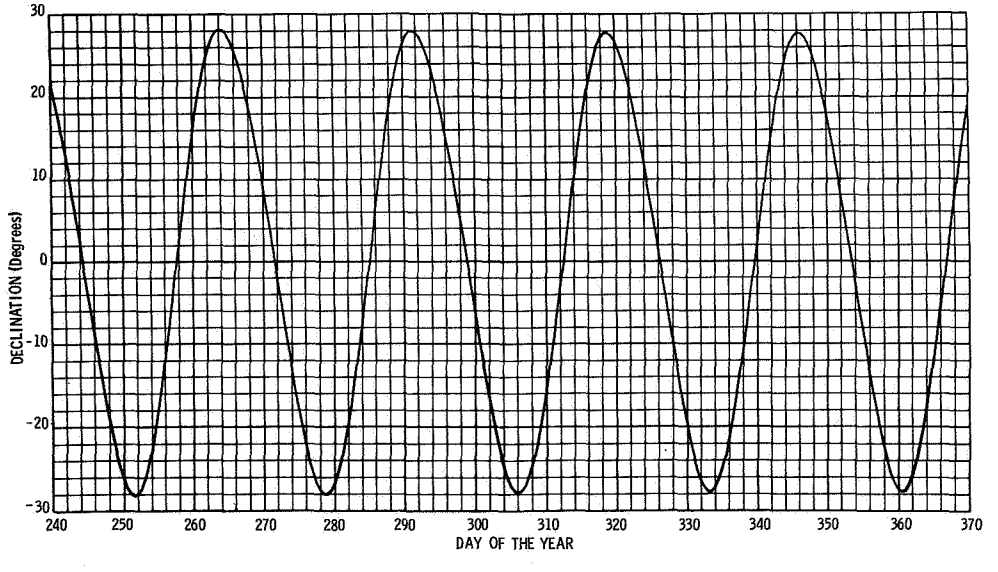
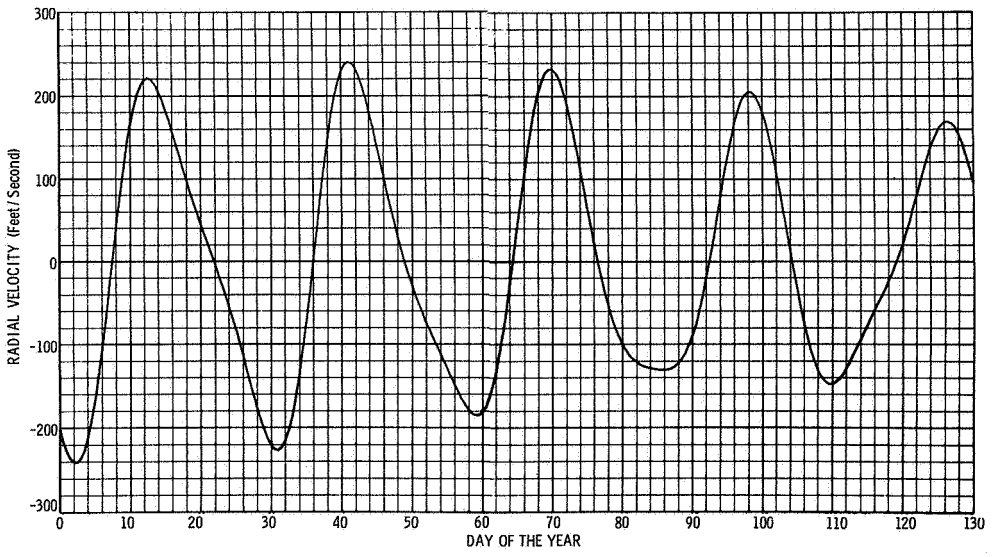
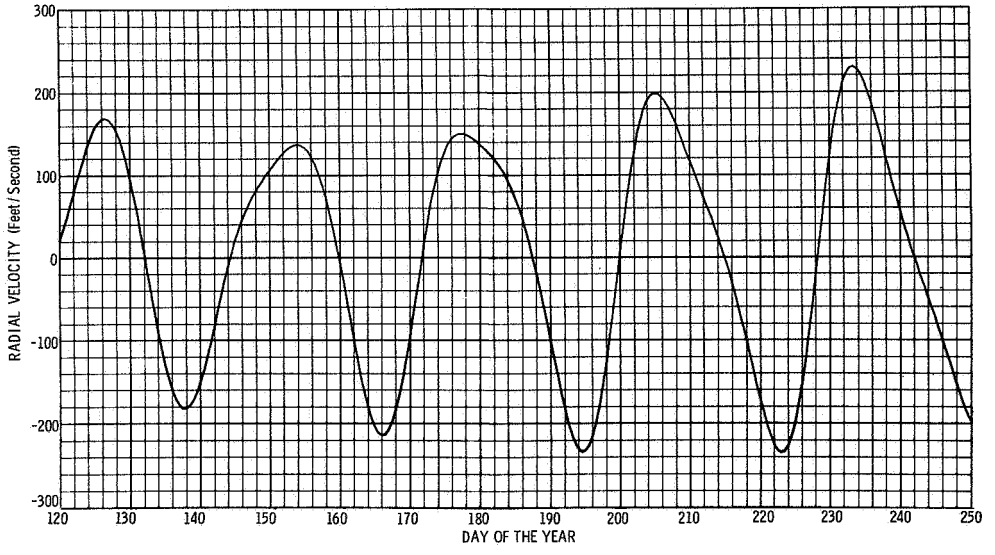


FIGURE B1970-3 DECLINATION OF THE MOON





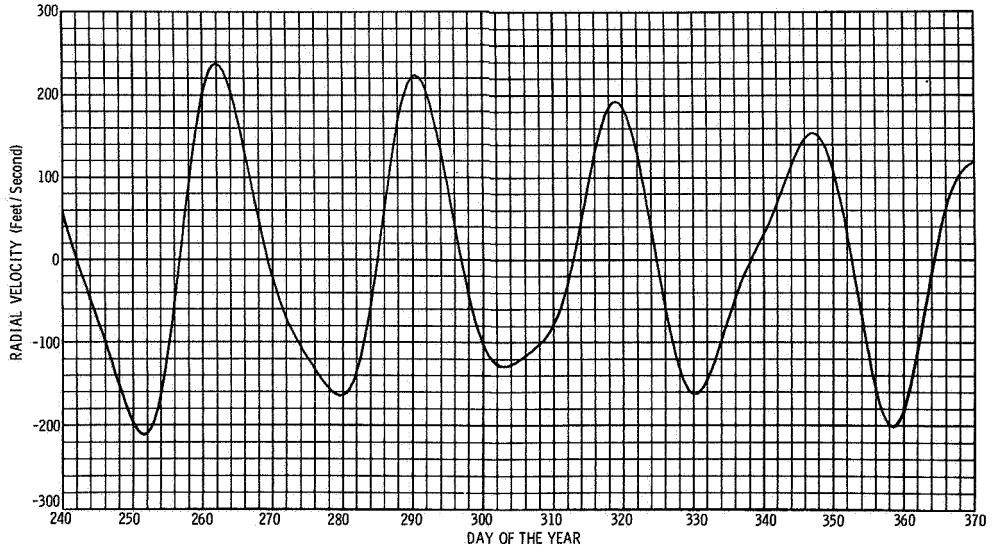
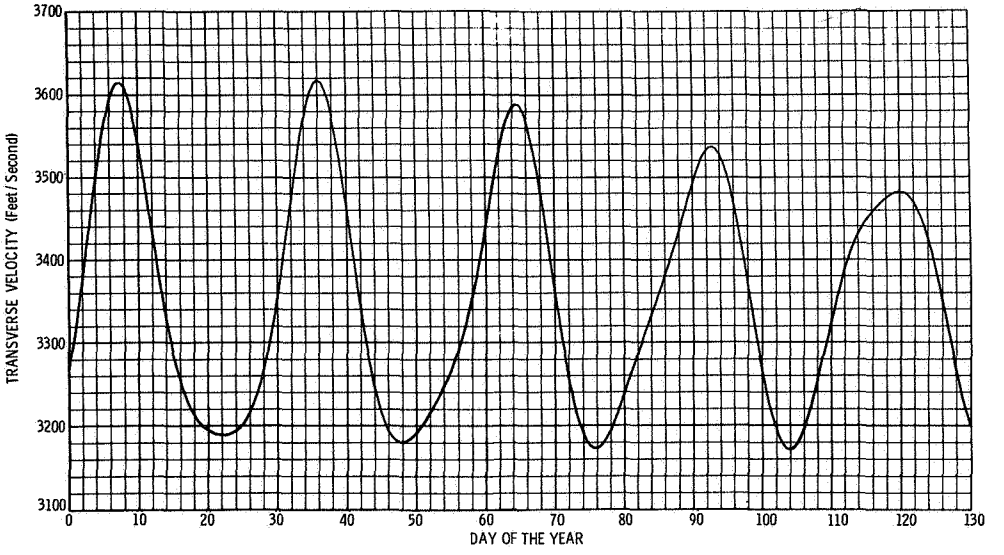
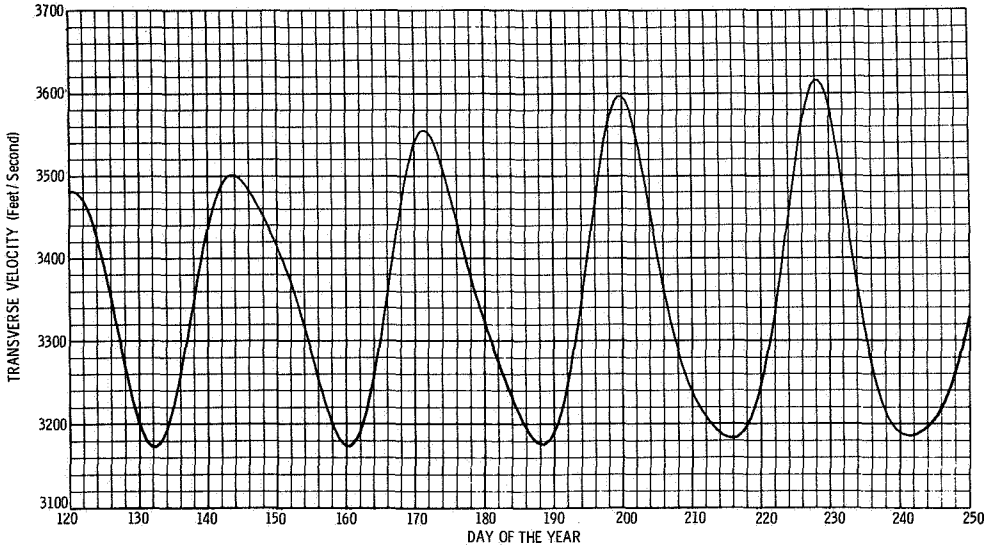


FIGURE B1970-4 RADIAL VELOCITY OF THE MOON





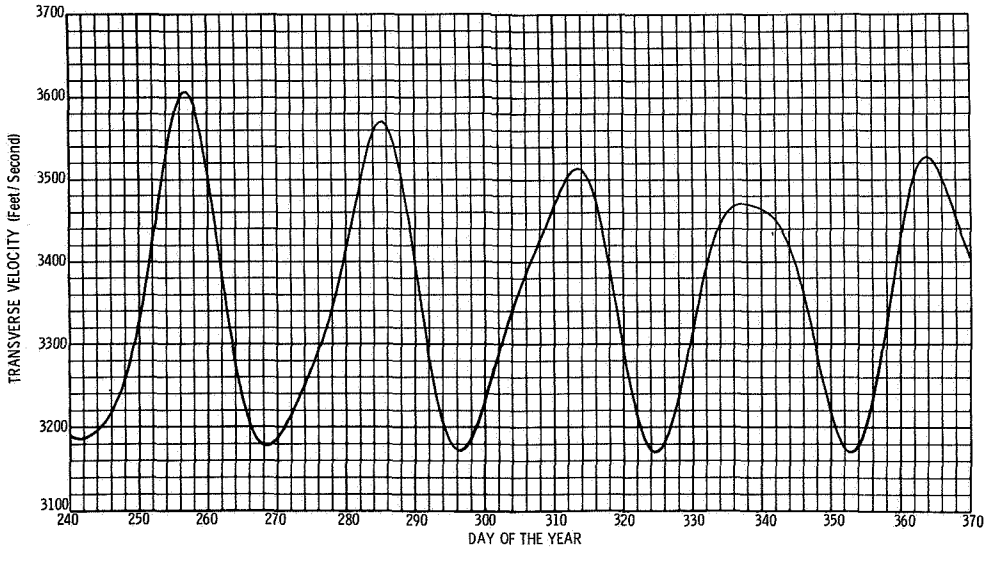
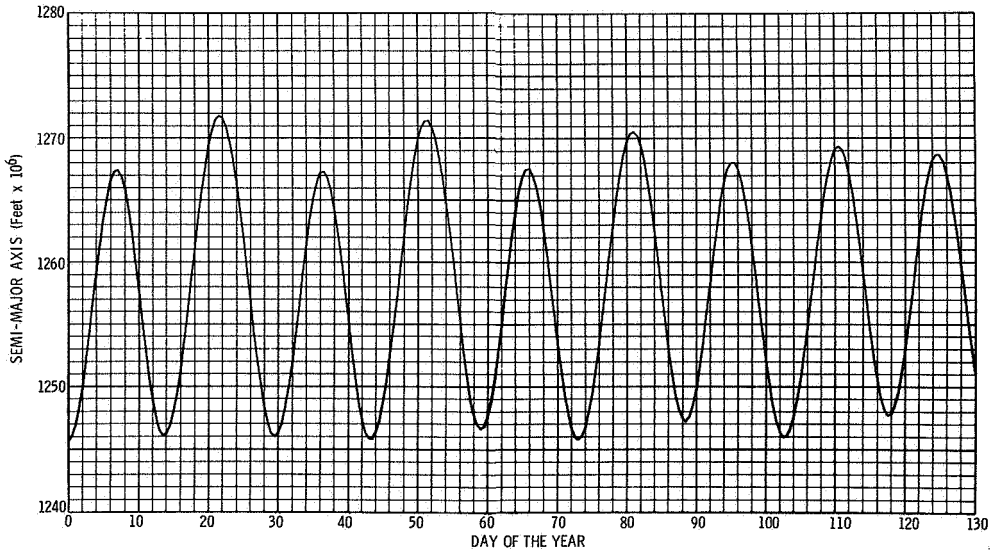
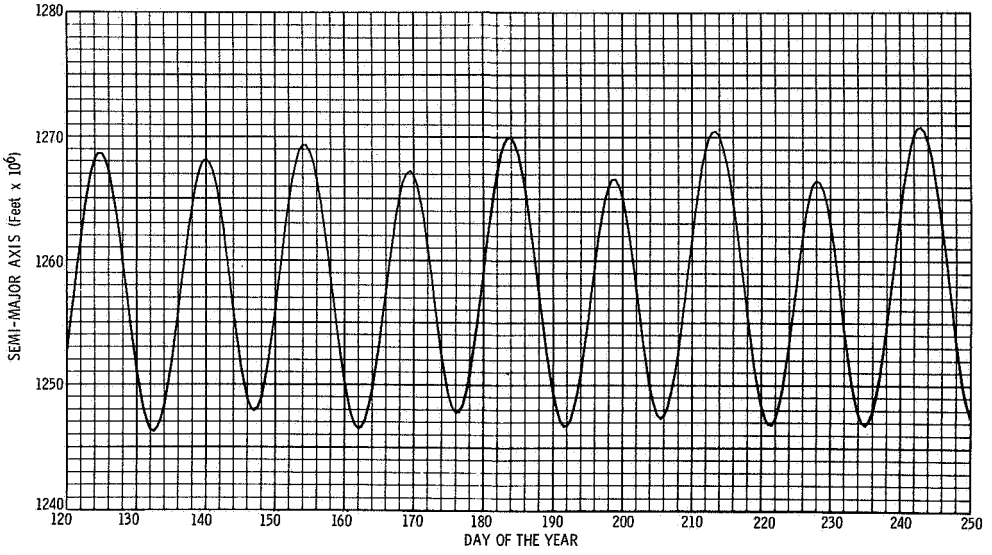


FIGURE B1970-5 TRANSVERSE VELOCITY OF THE MOON





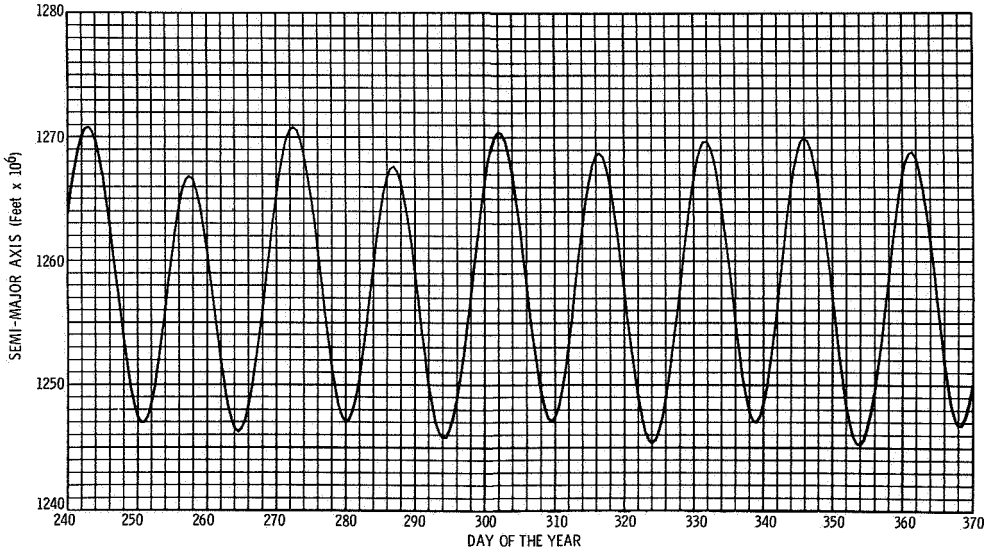


FIGURE B1970-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT

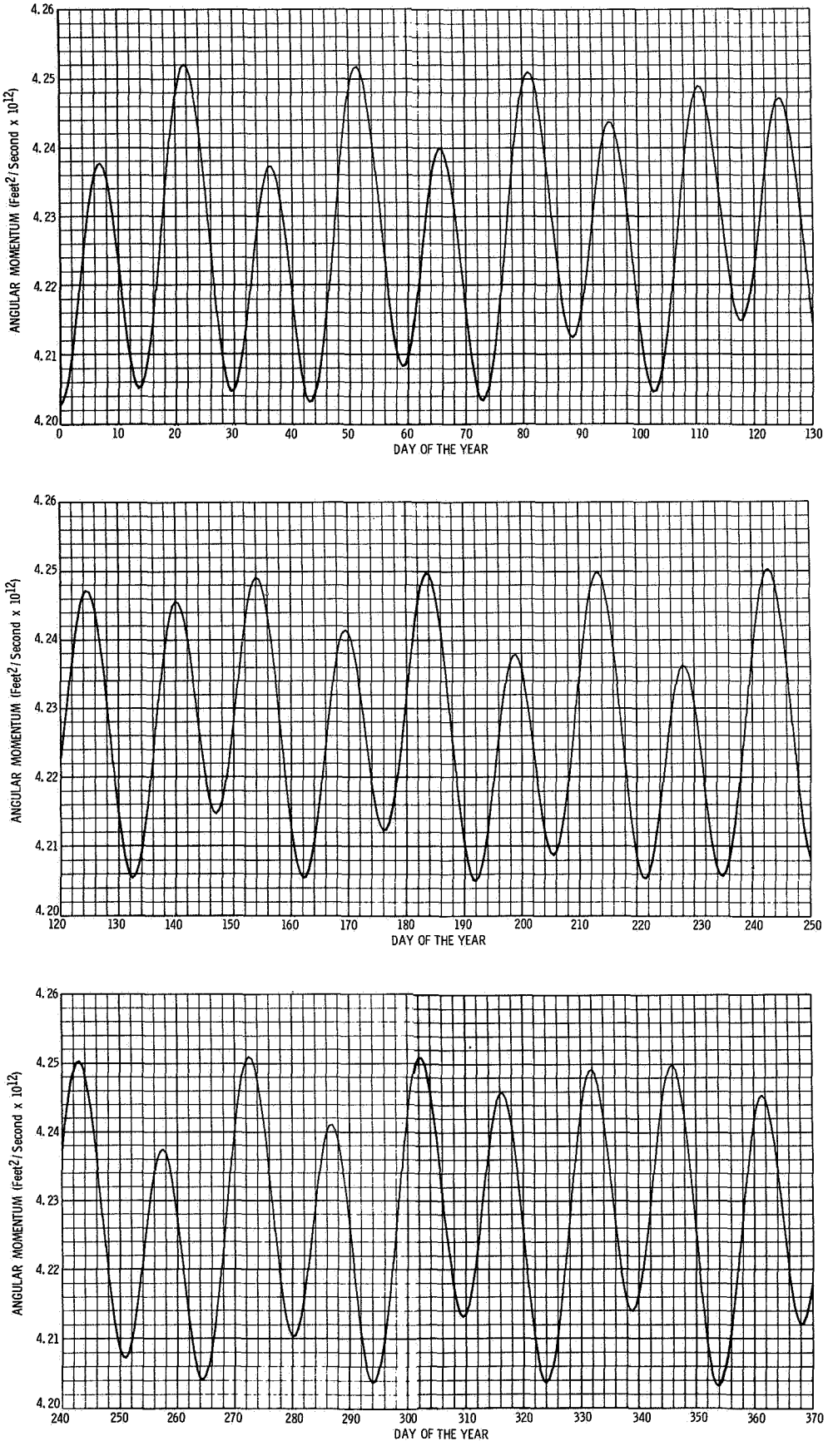


FIGURE B1970-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON

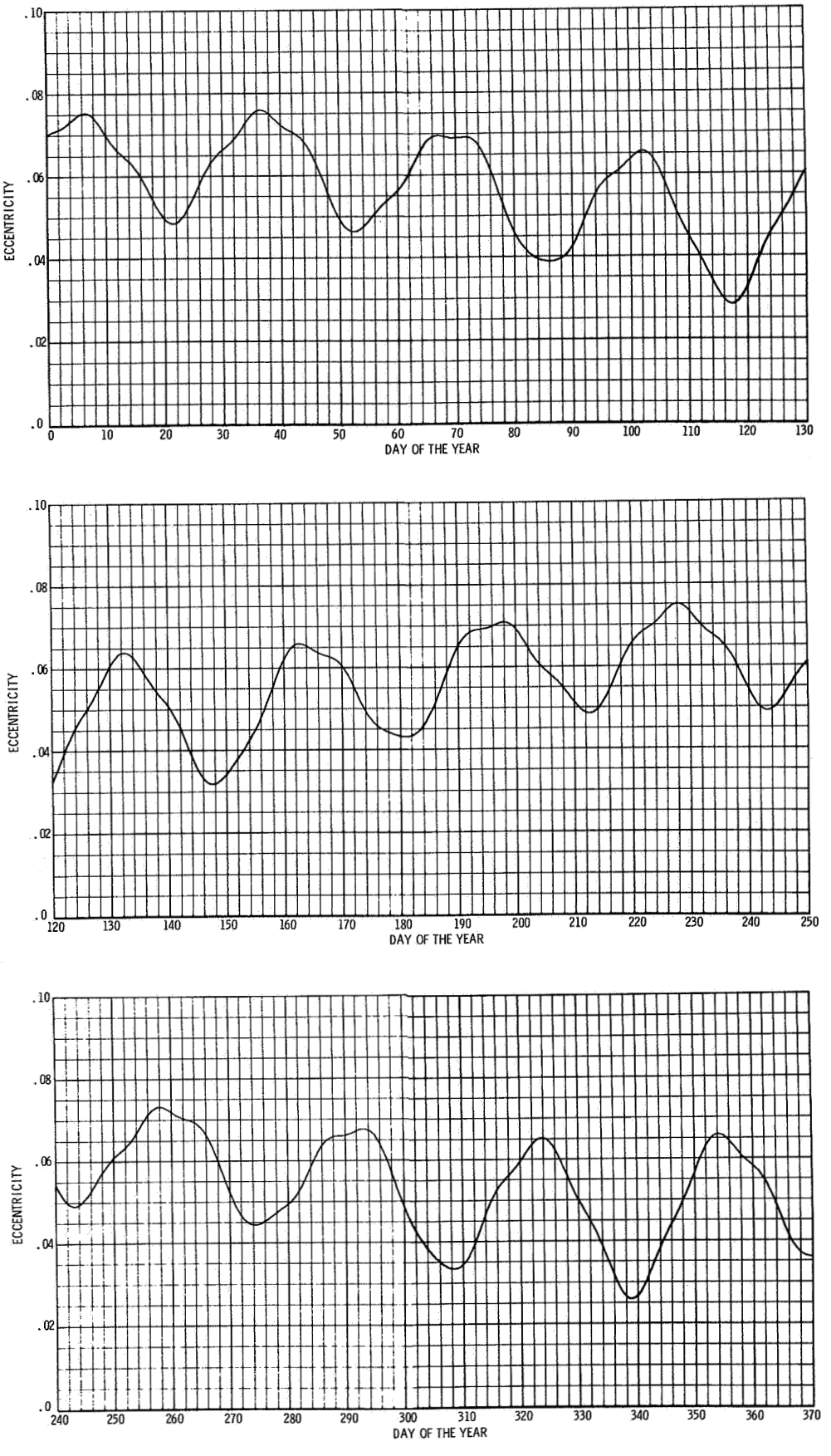
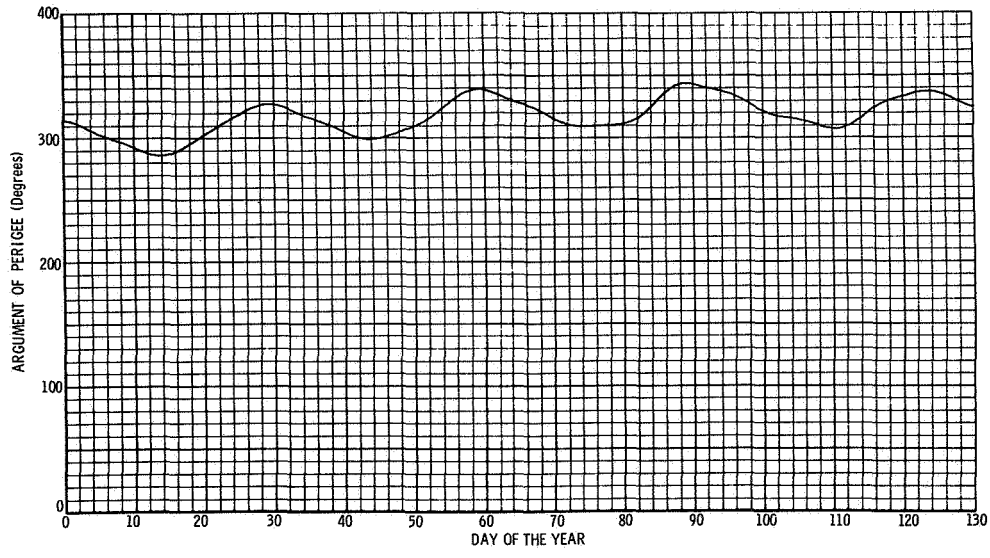
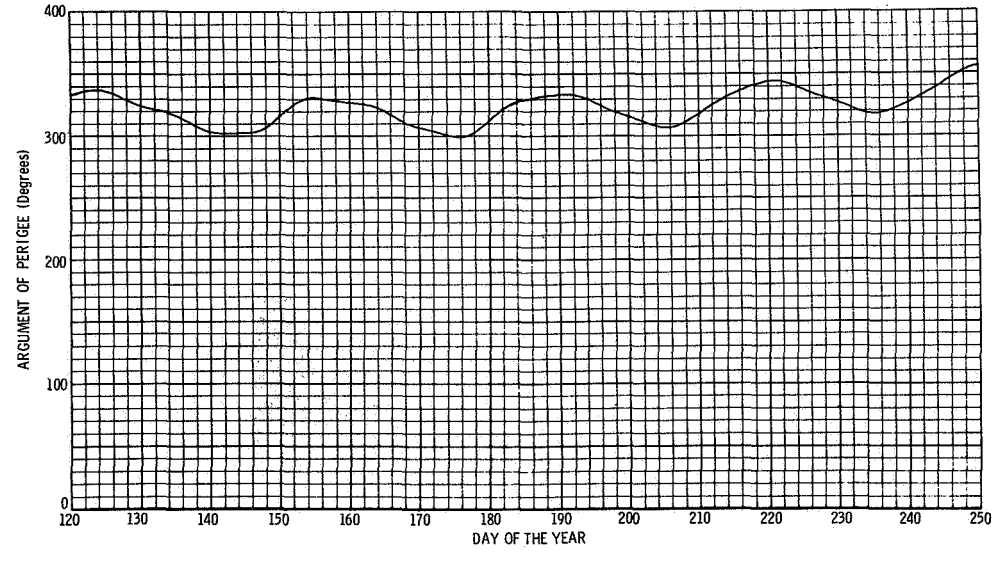


FIGURE B1970-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT





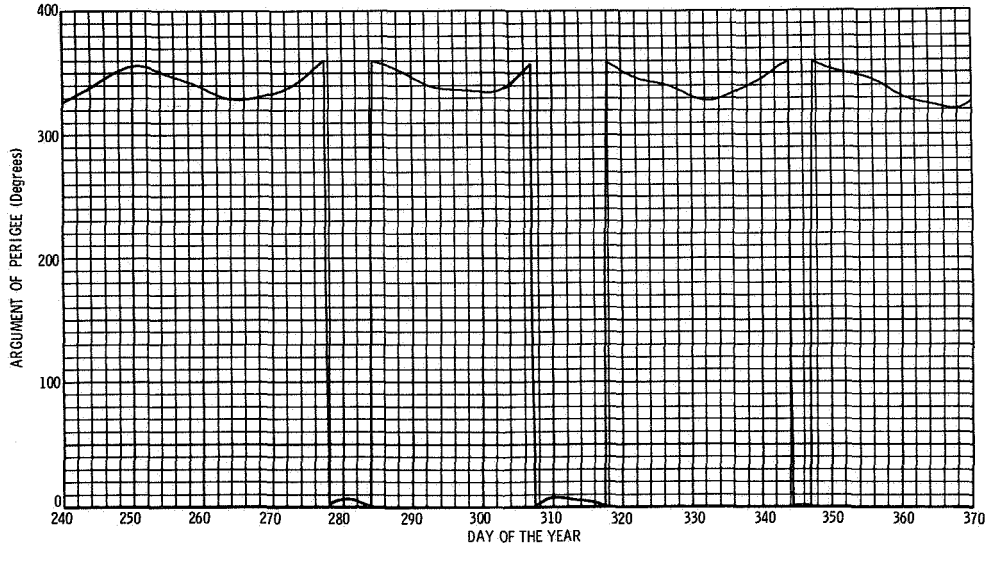
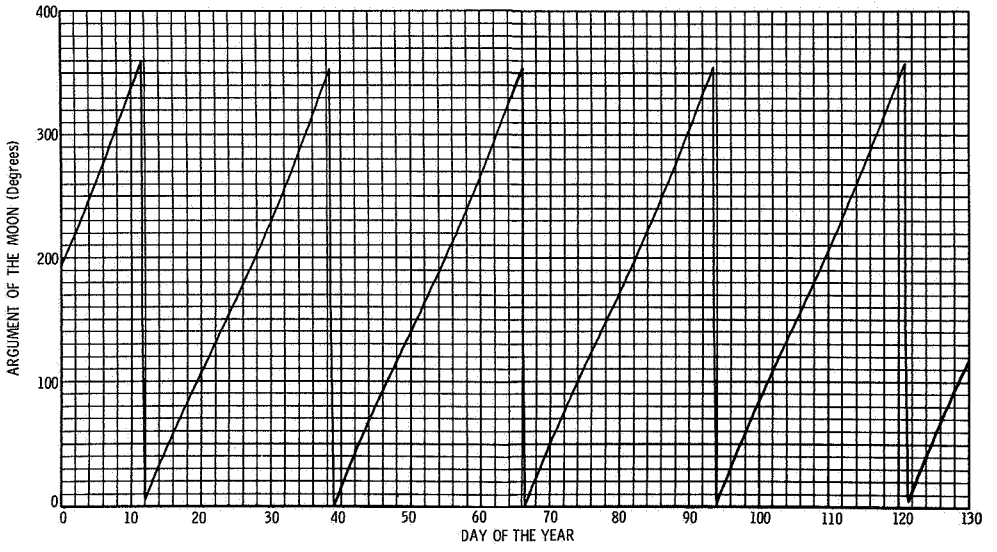
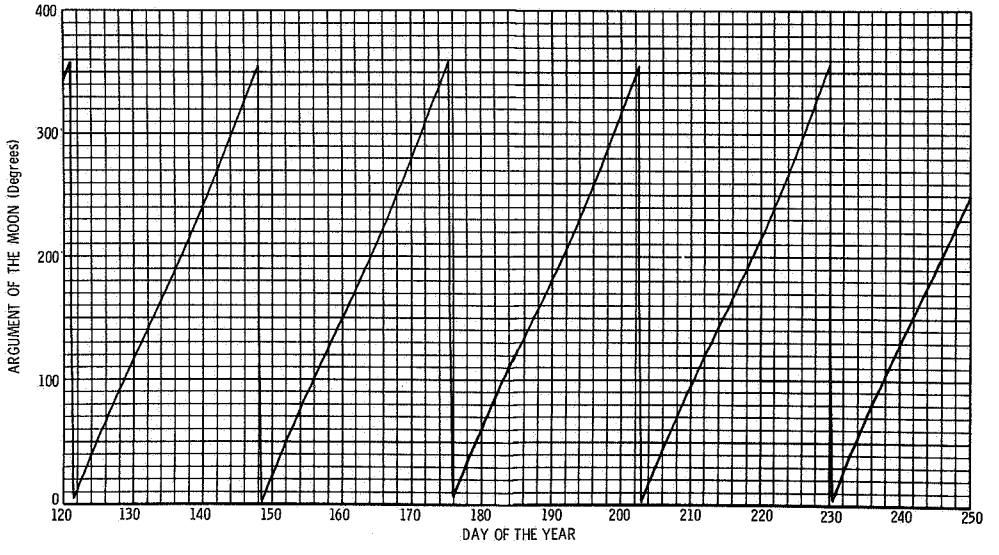


FIGURE B1970-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE





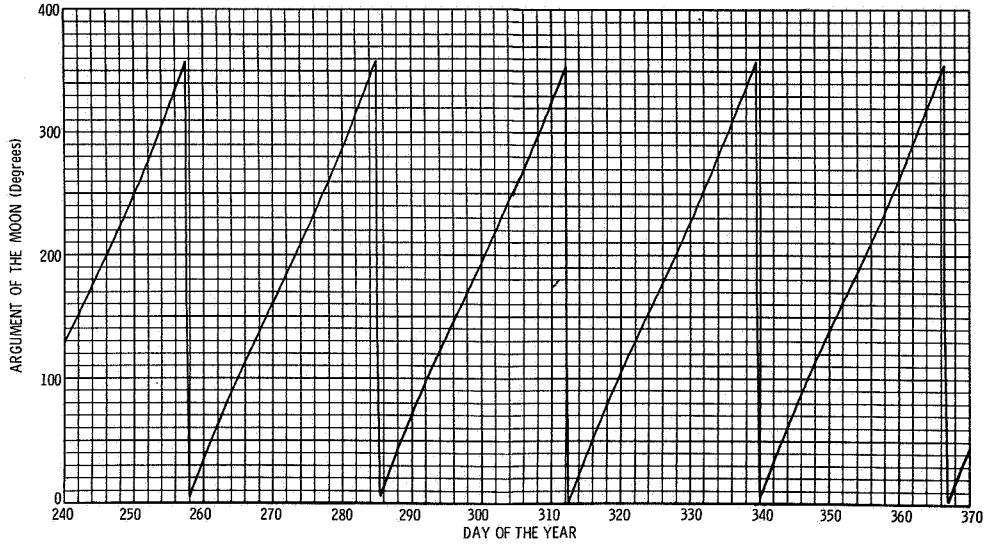
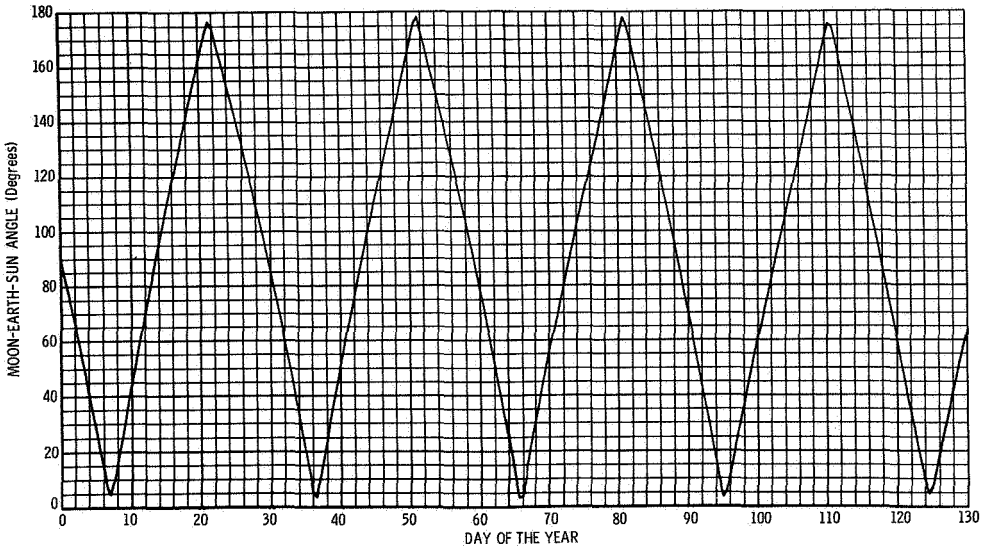
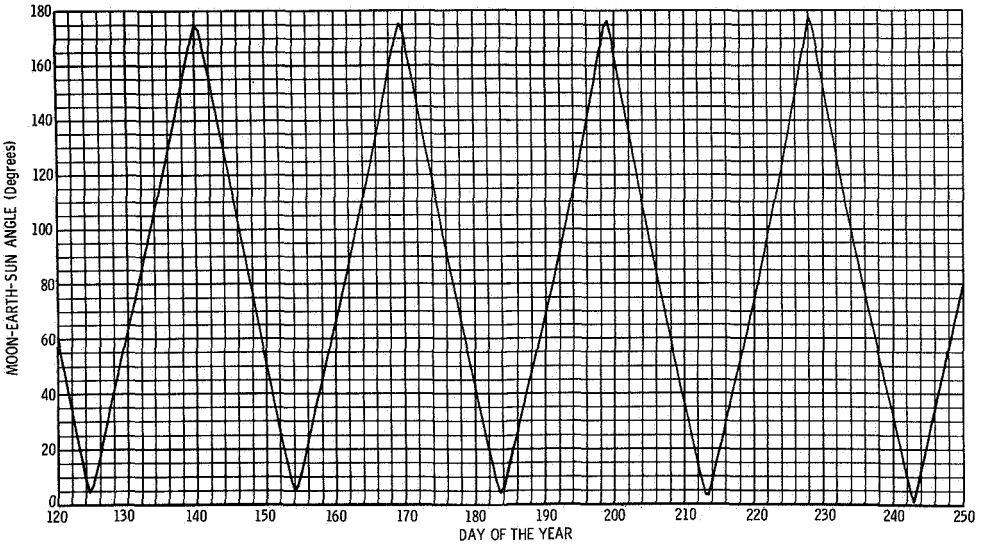


FIGURE B1970-10 ARGUMENT OF THE MOON'S POSITION





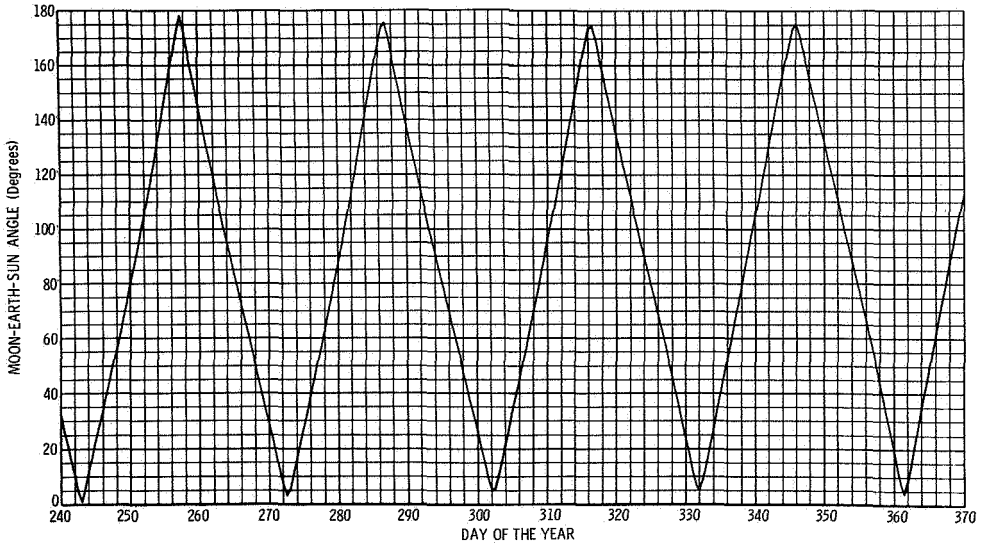
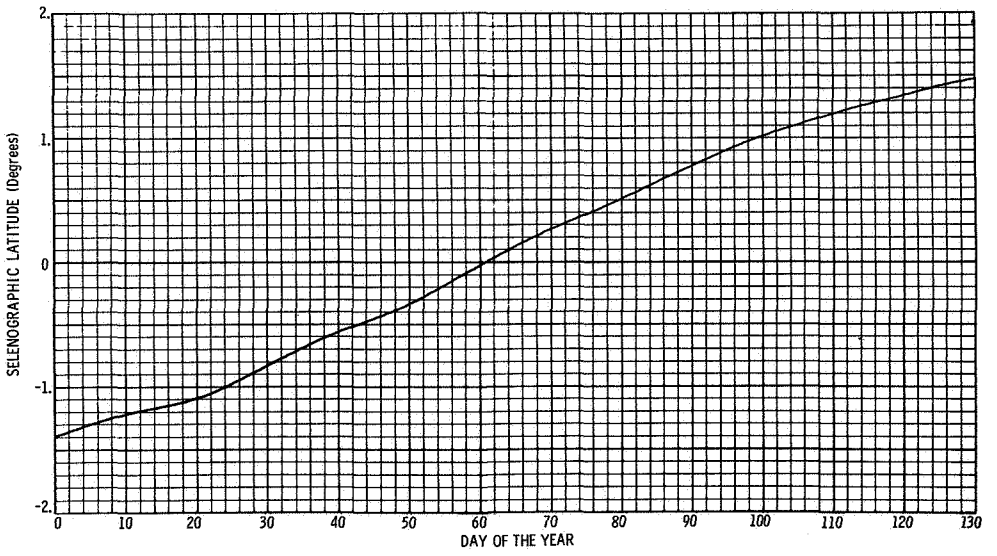
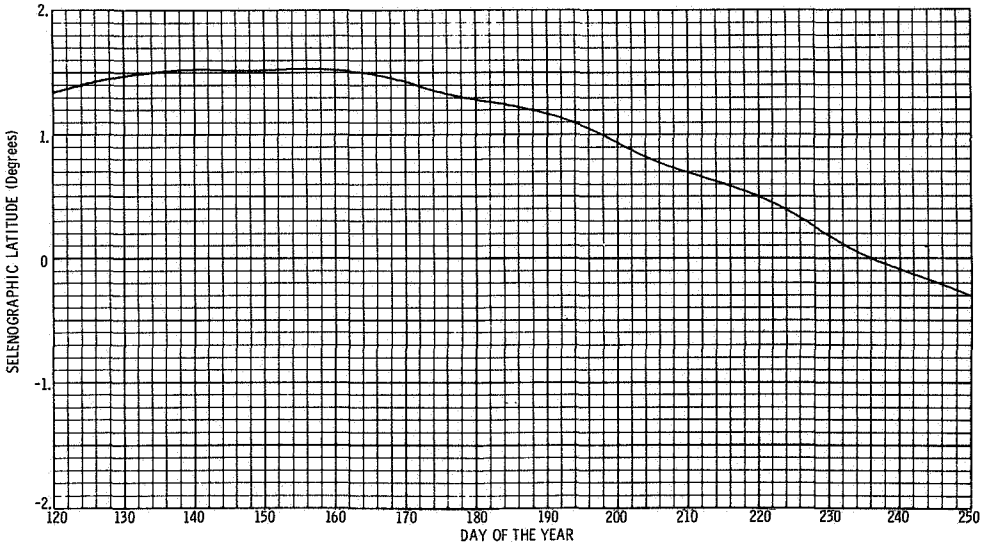


FIGURE B1970-11 MOON-EARTH-SUN ANGLE





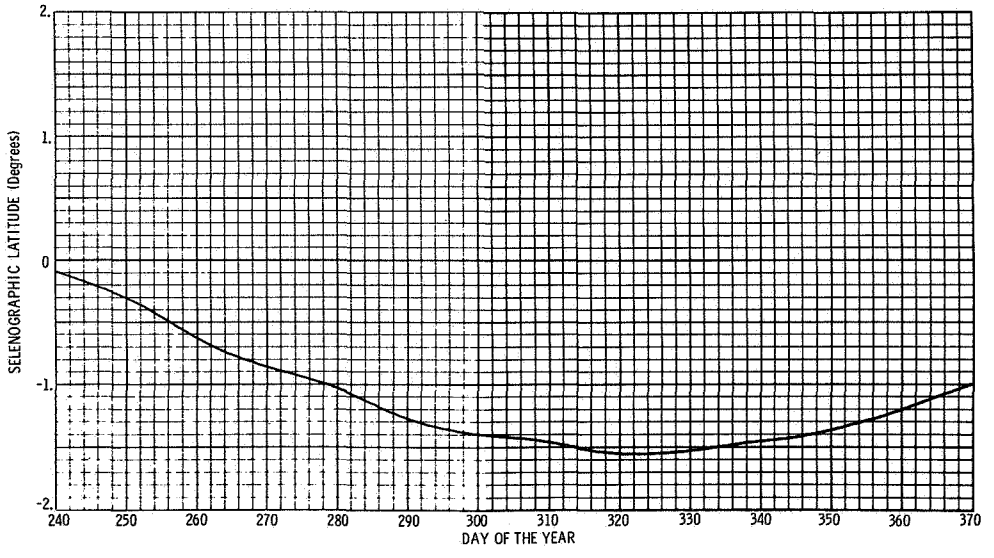
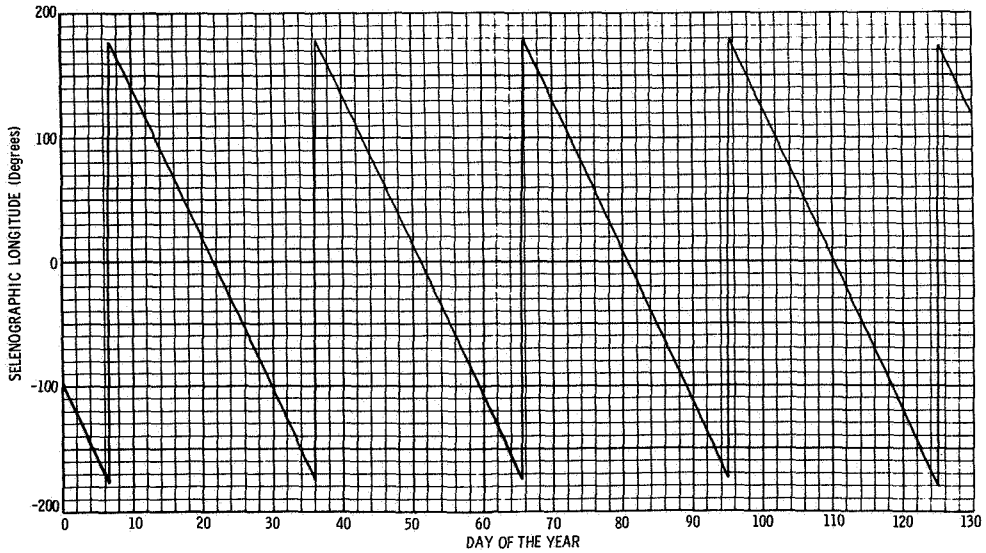
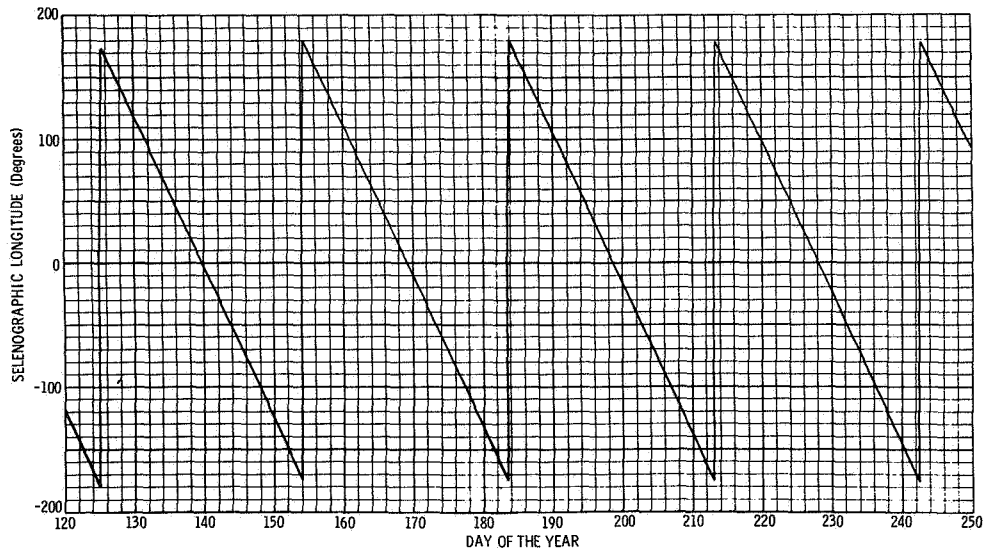


FIGURE B1970-12 SELENOGRAPHIC LATITUDE OF THE SUN





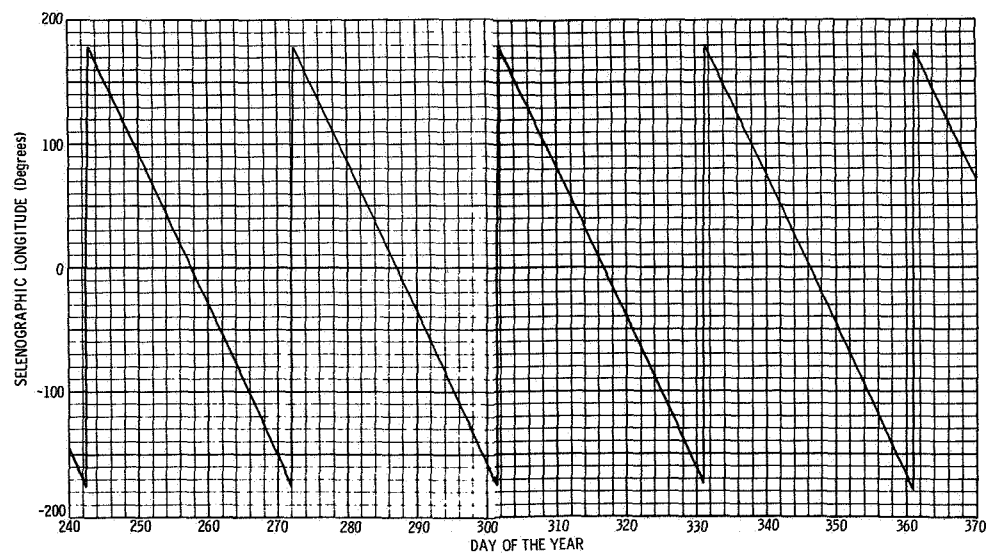
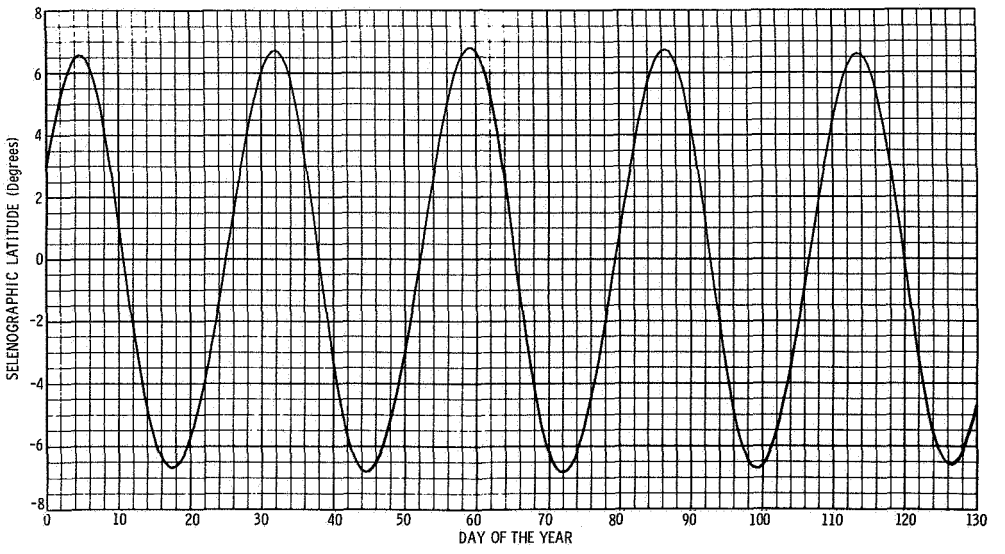
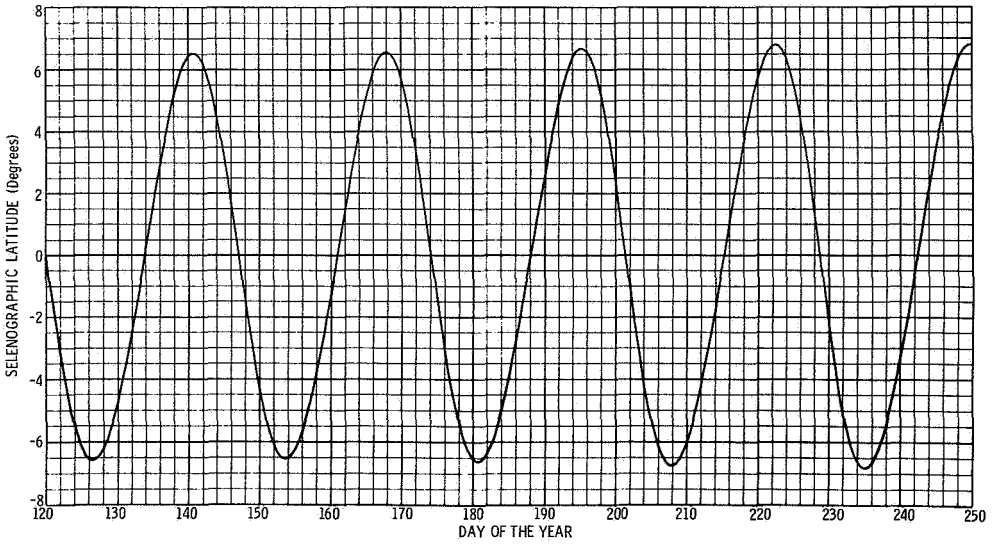


FIGURE B1970-13 SELENOGRAPHIC LONGITUDE OF THE SUN





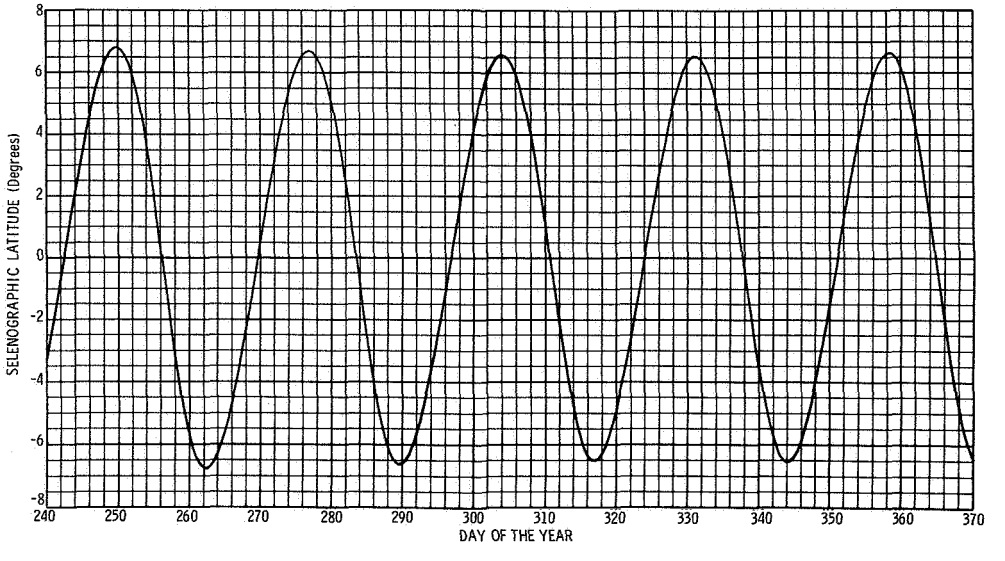


FIGURE B1970-14 SELENOGRAPHIC LATITUDE OF THE EARTH

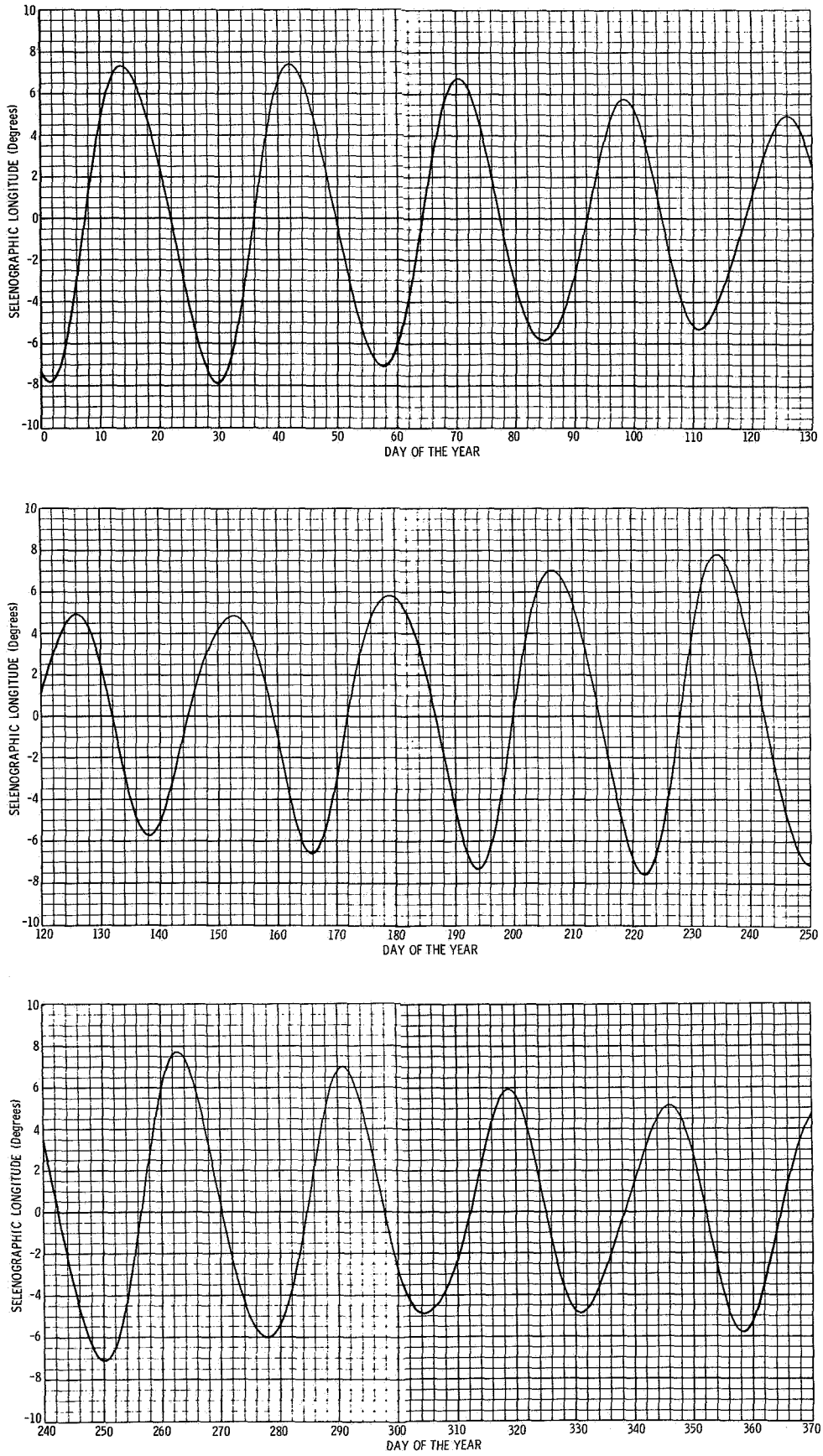
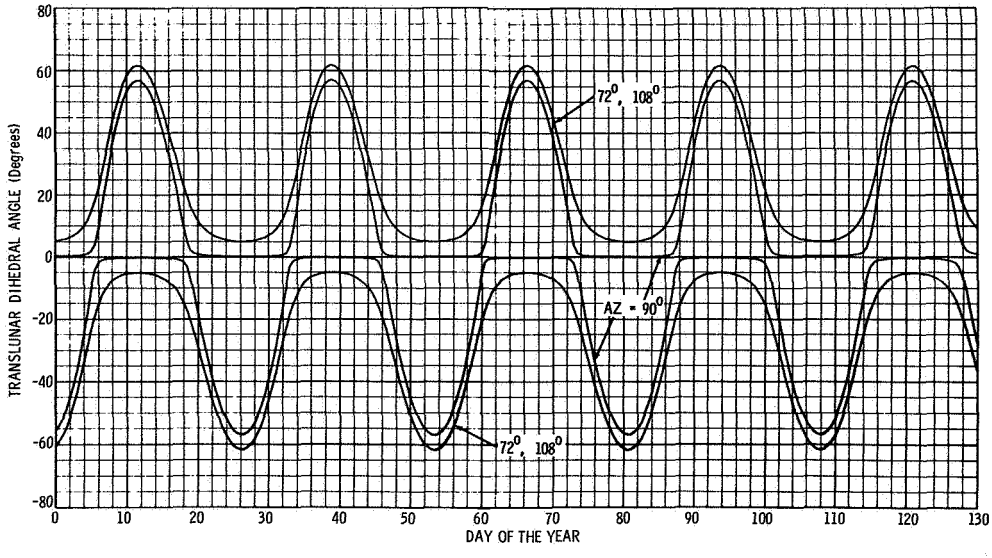
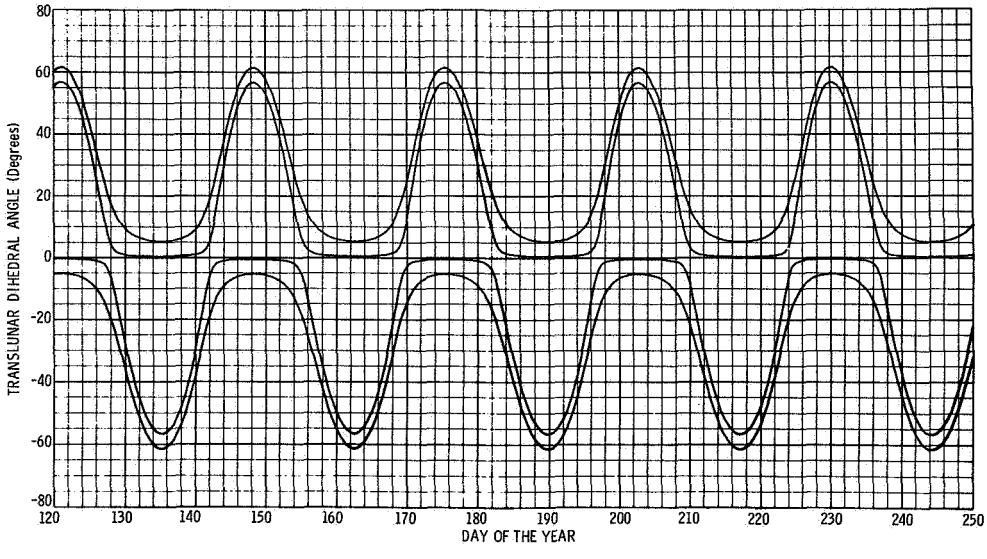


FIGURE B1970-15 SELENOGRAPHIC LONGITUDE OF THE EARTH





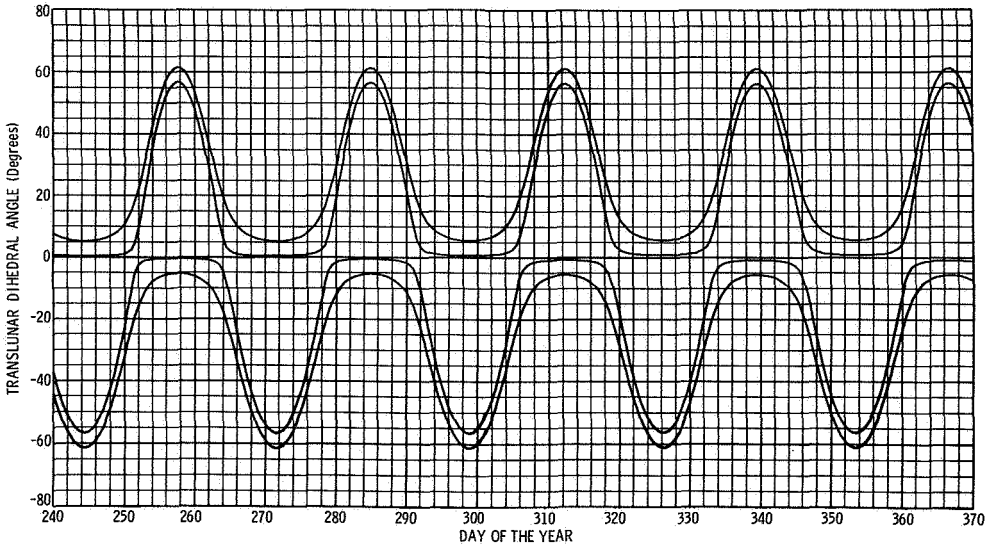


FIGURE B1970-16 TRANSLUNAR DIHEDRAL ANGLES

1971

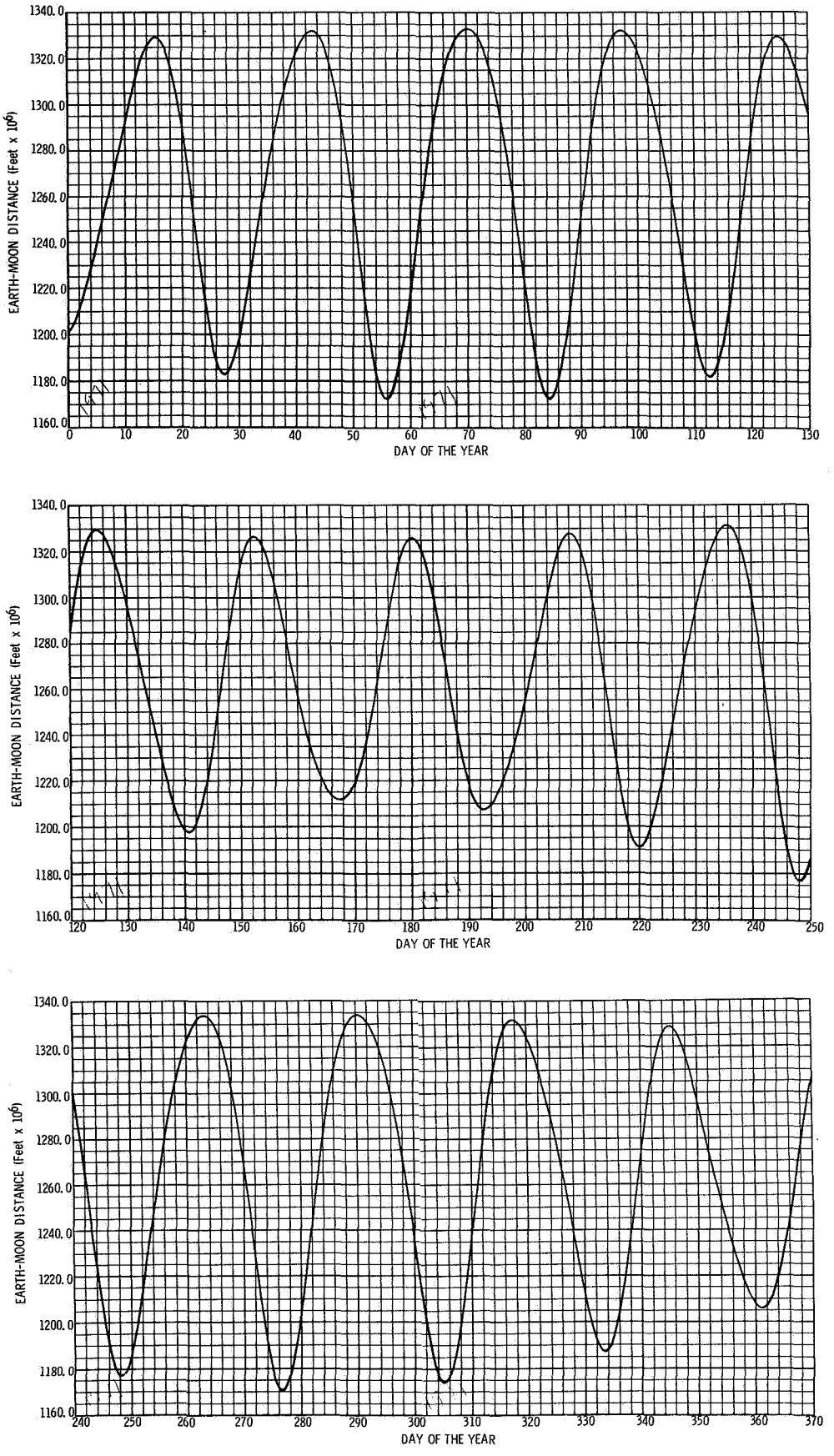
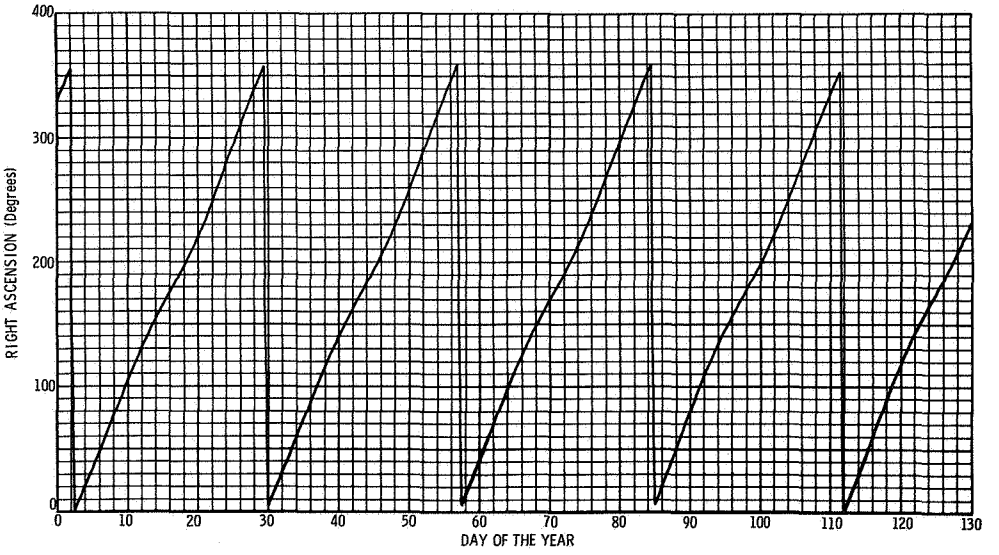
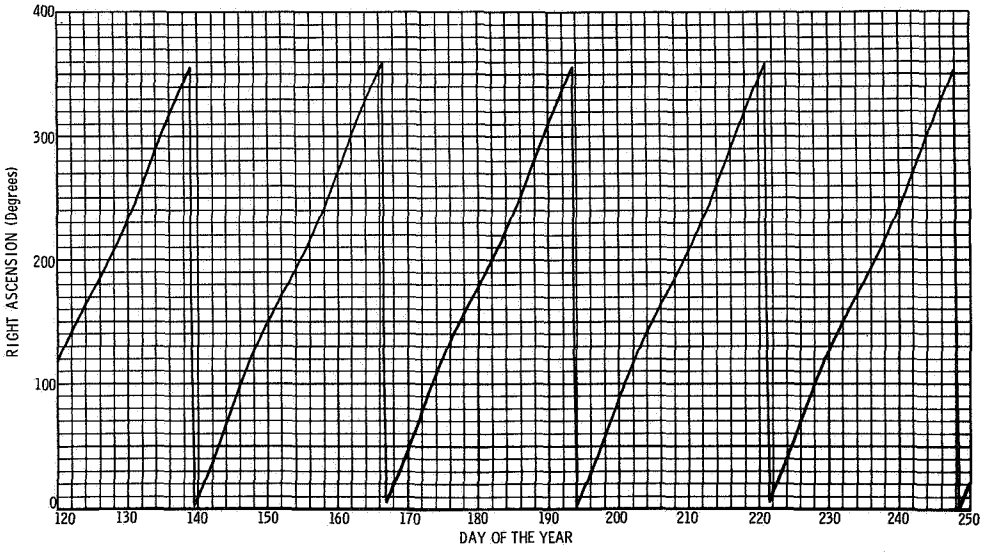


FIGURE B1971-1 EARTH-MOON DISTANCE





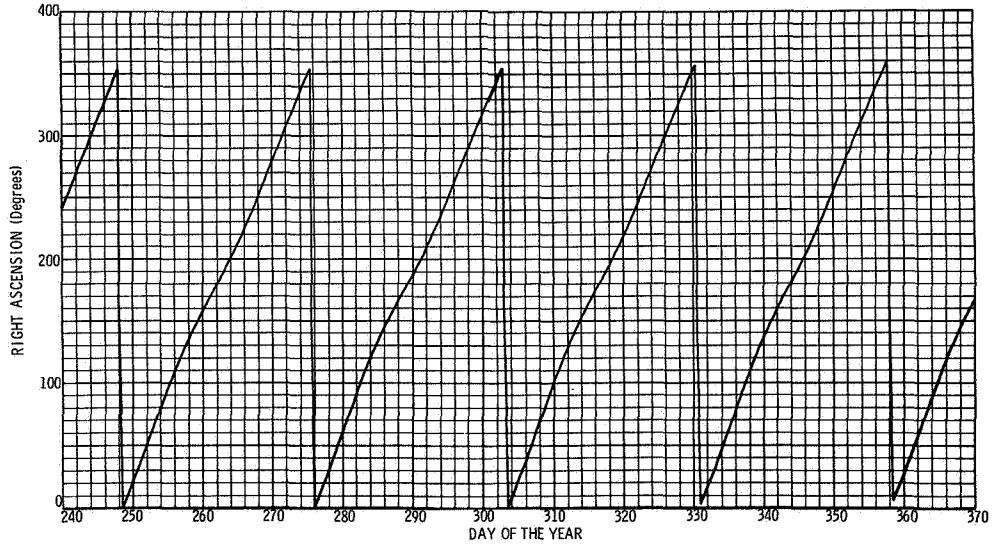


FIGURE B1971-2 RIGHT ASCENSION OF THE MOON

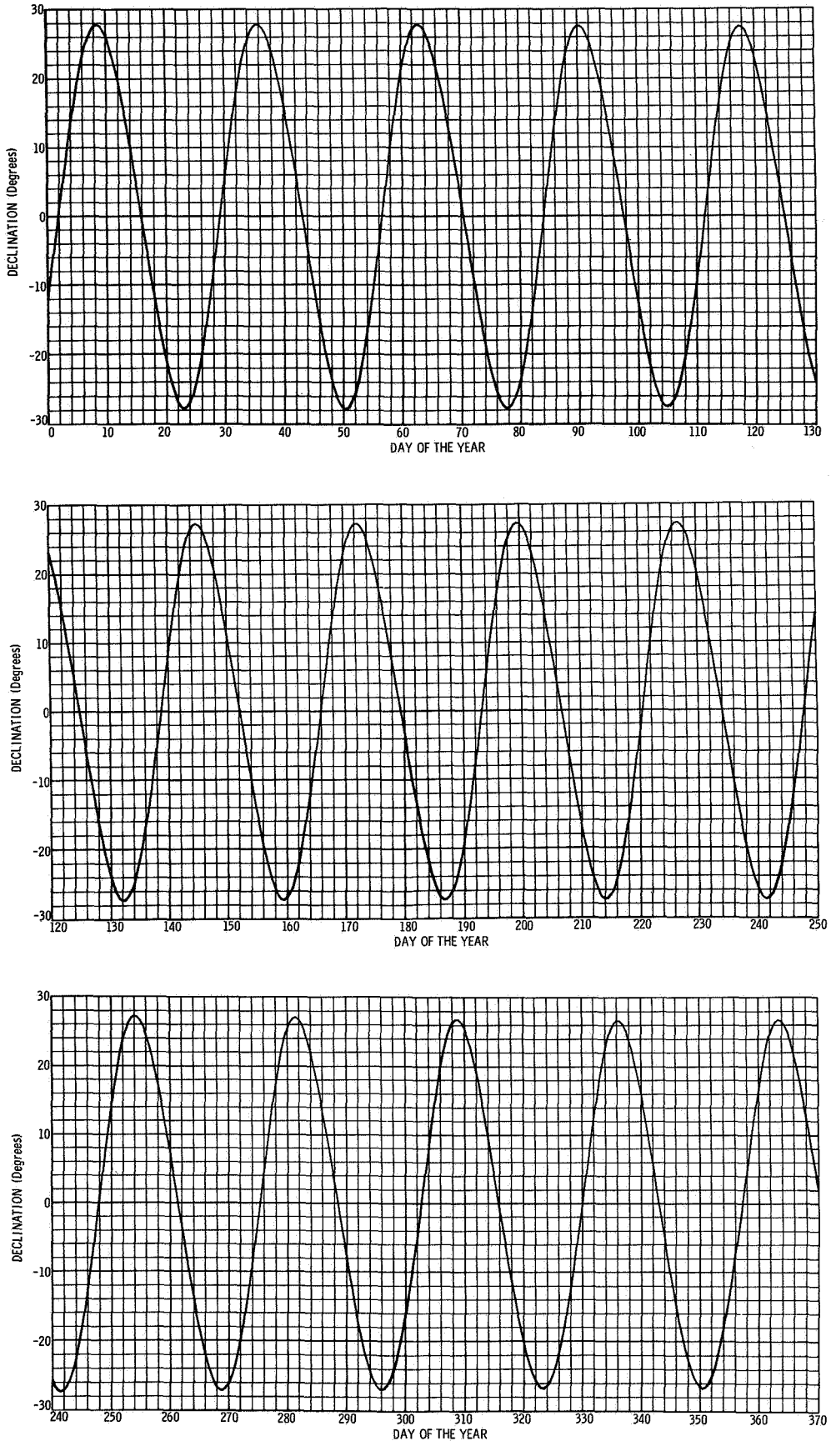
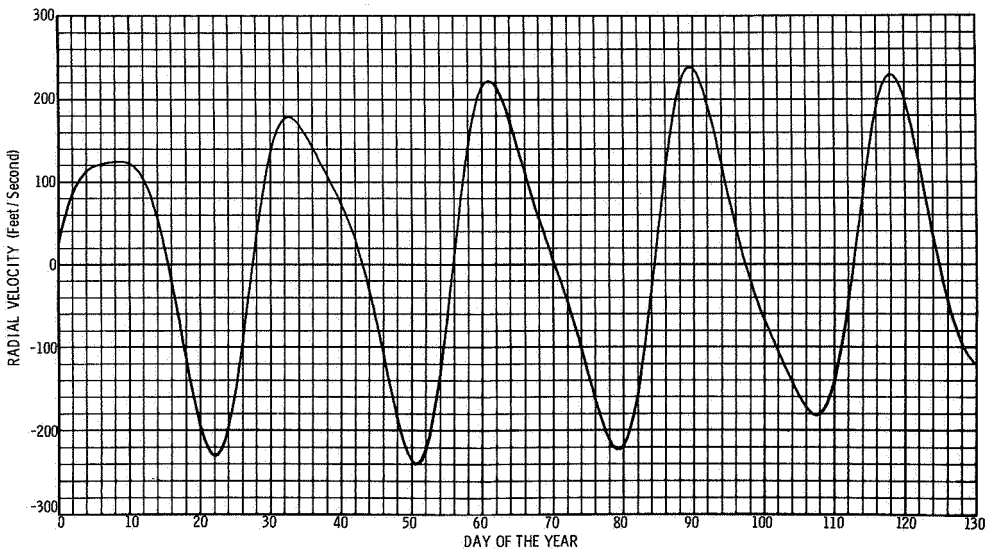
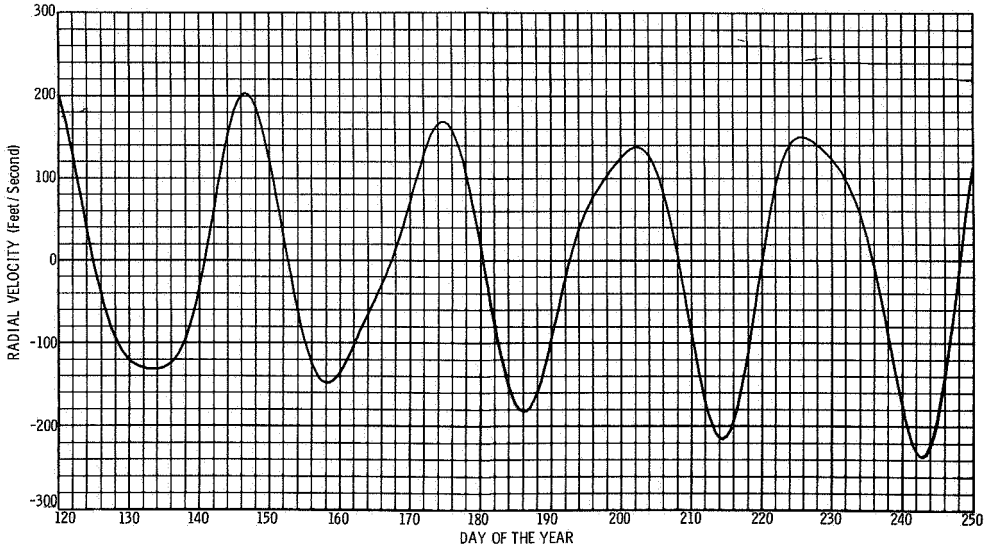


FIGURE B1971-3 DECLINATION OF THE MOON





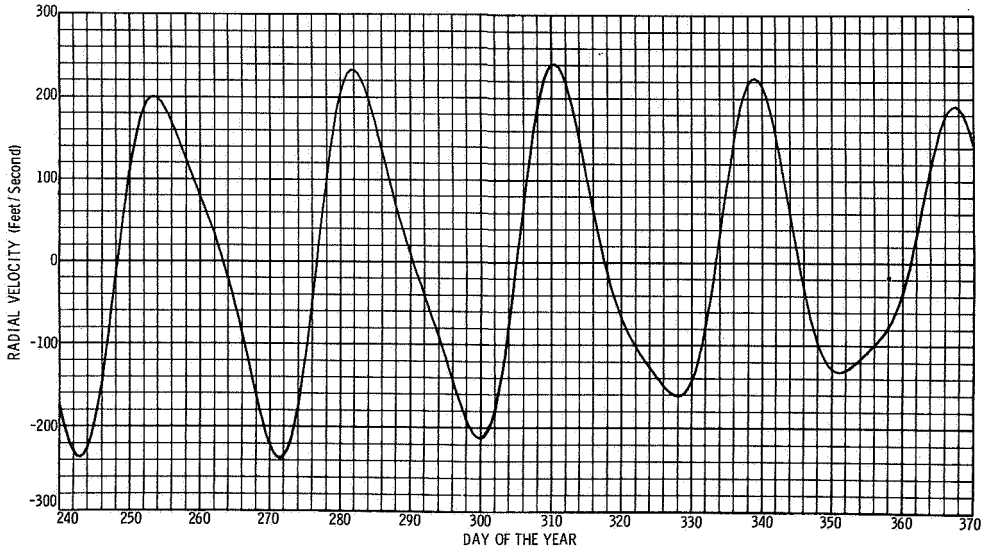


FIGURE B1971-4 RADIAL VELOCITY OF THE MOON

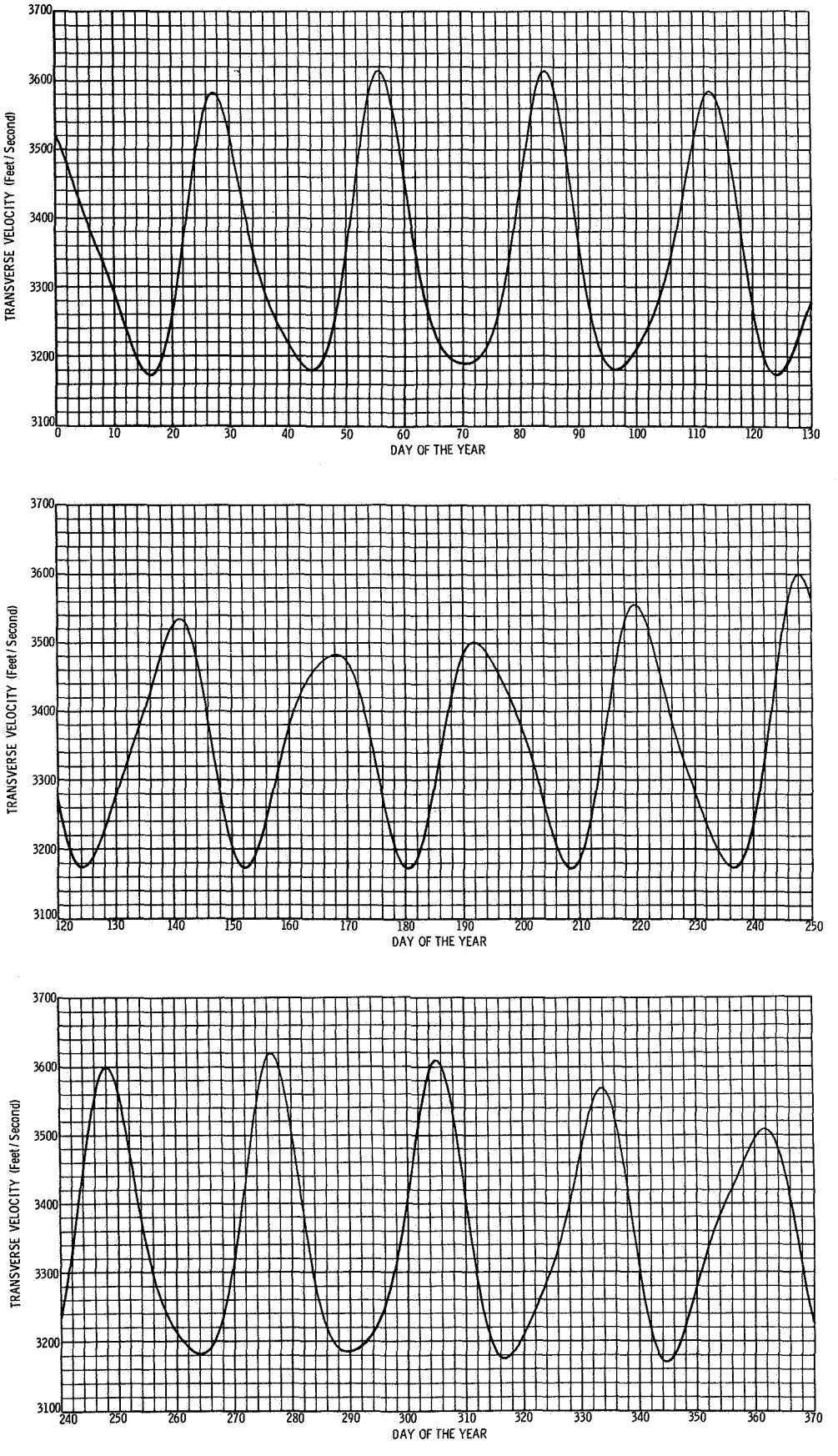
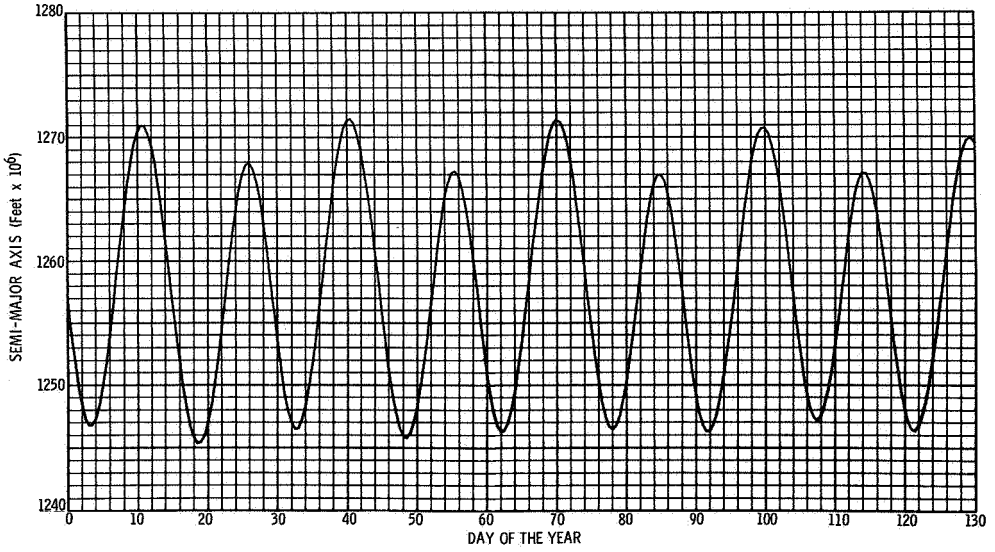
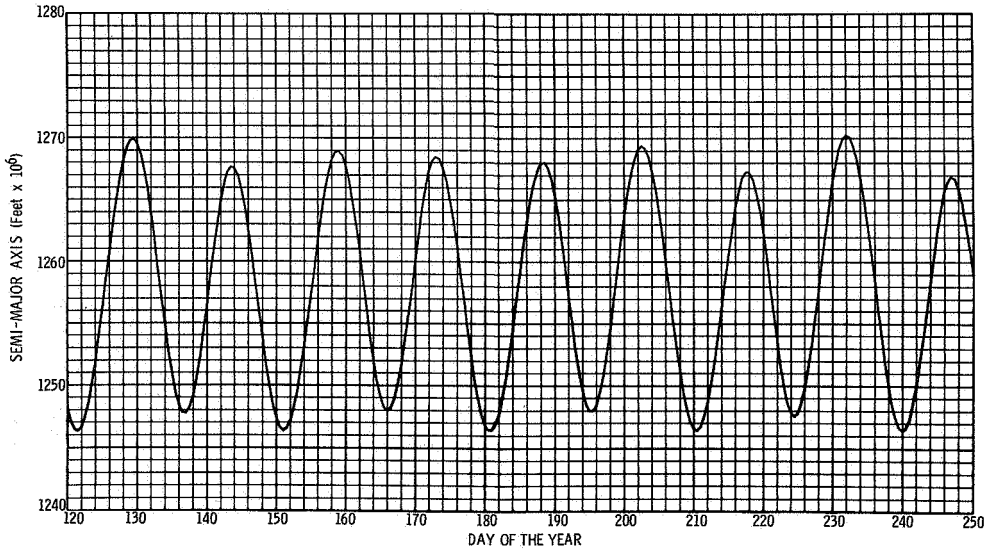


FIGURE B1971-5 TRANSVERSE VELOCITY OF THE MOON





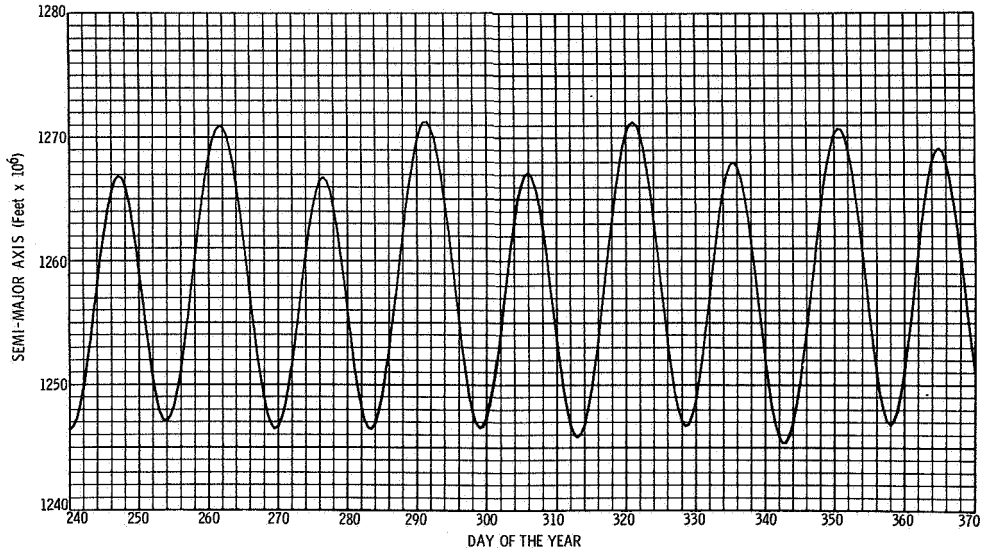


FIGURE B1971-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT

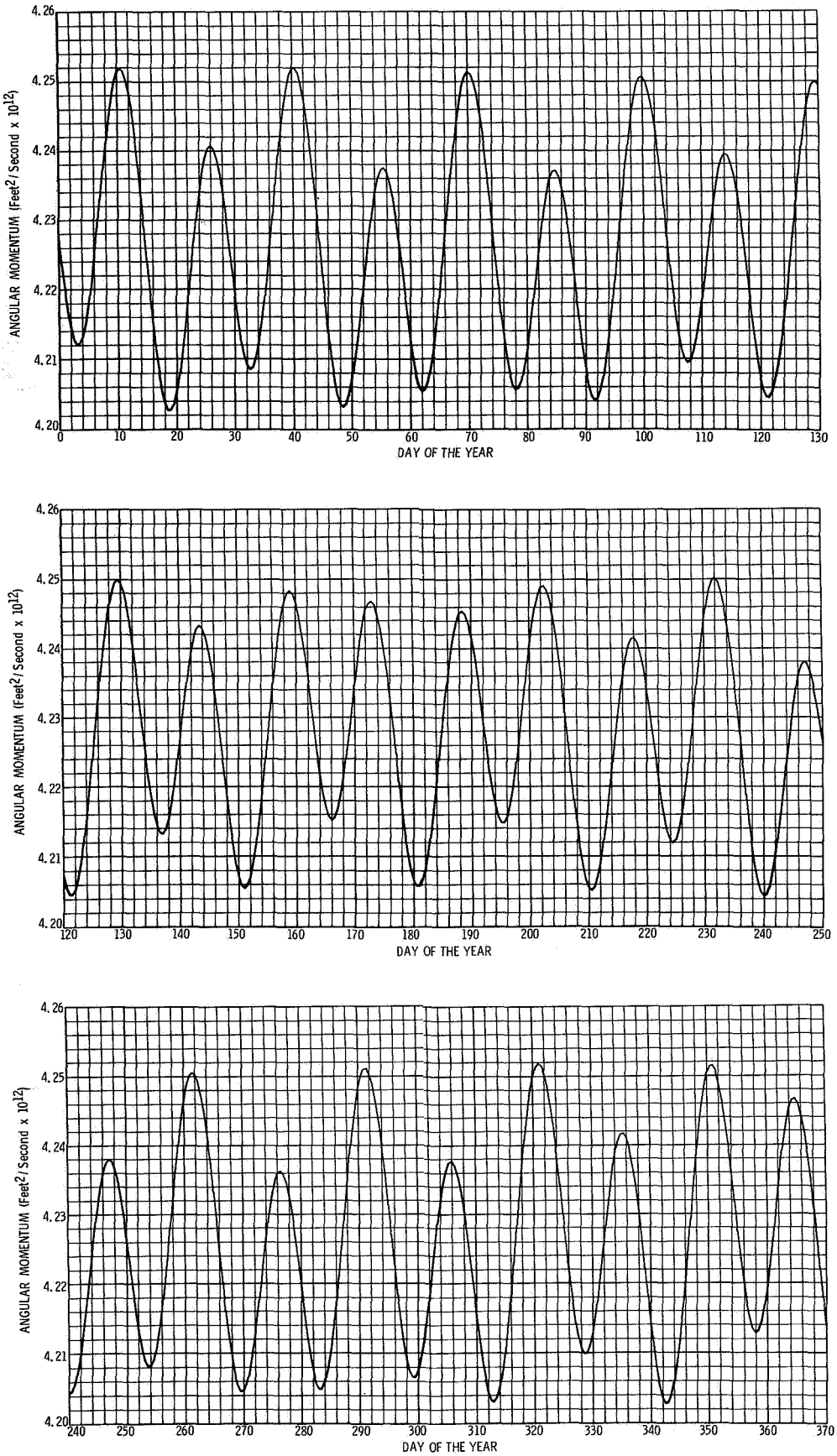
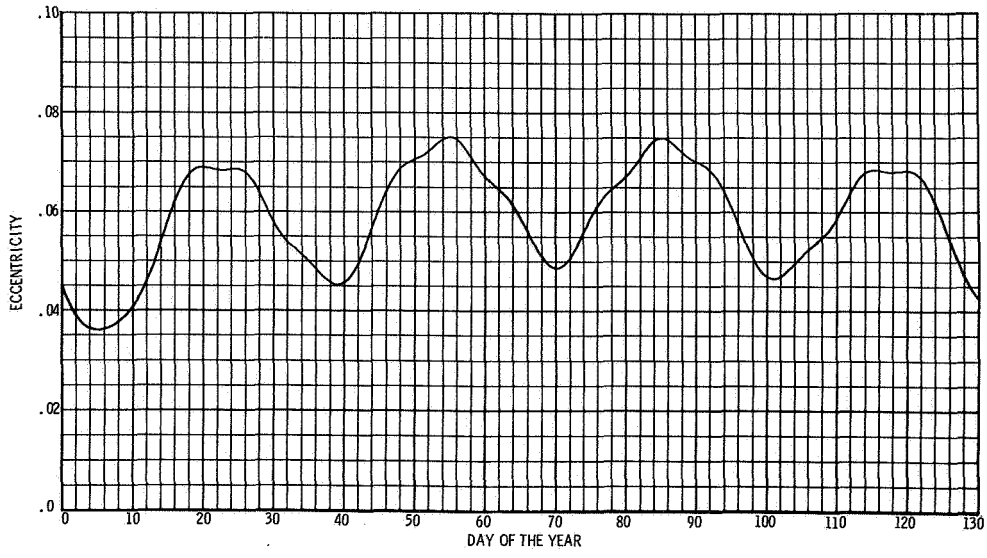
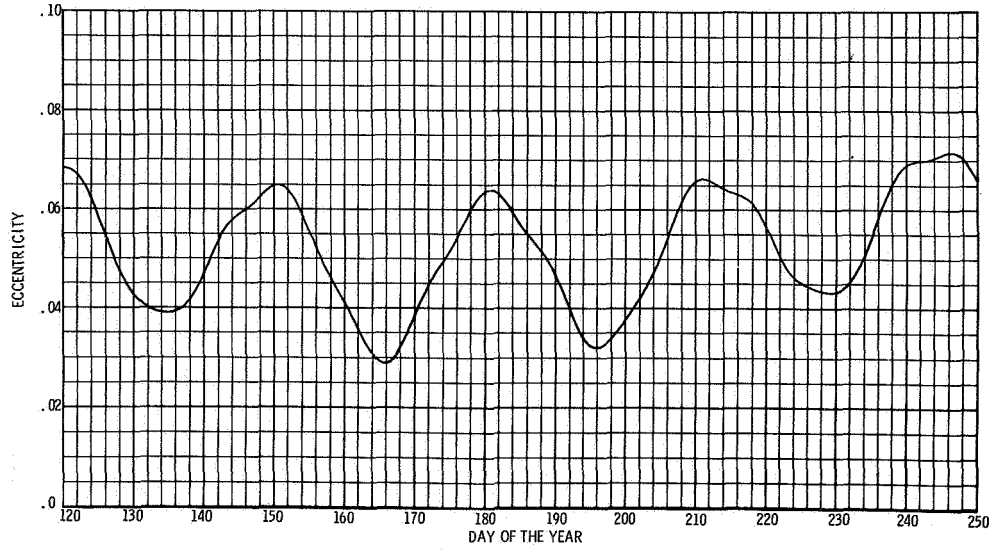


FIGURE B1971-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON





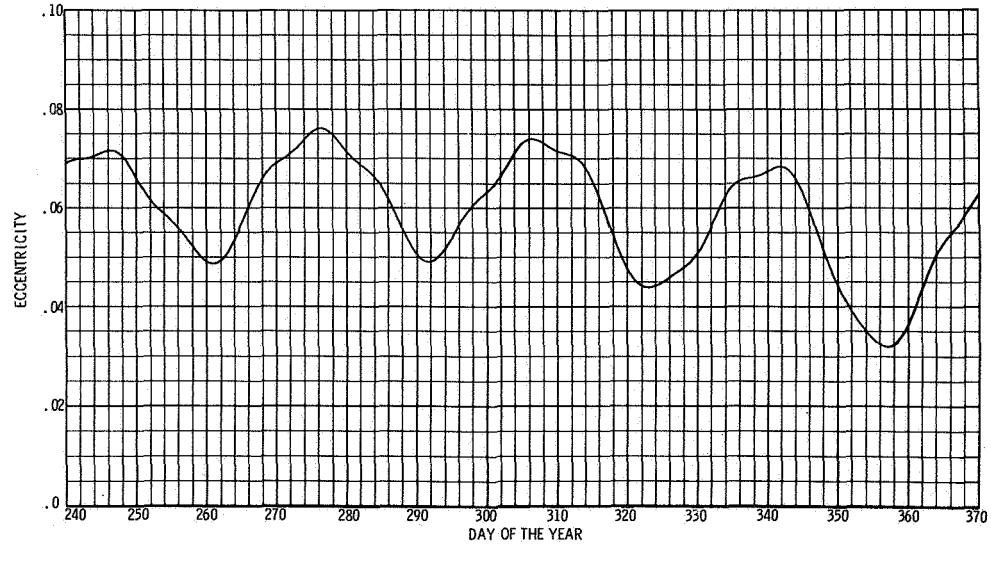


FIGURE B1971-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT

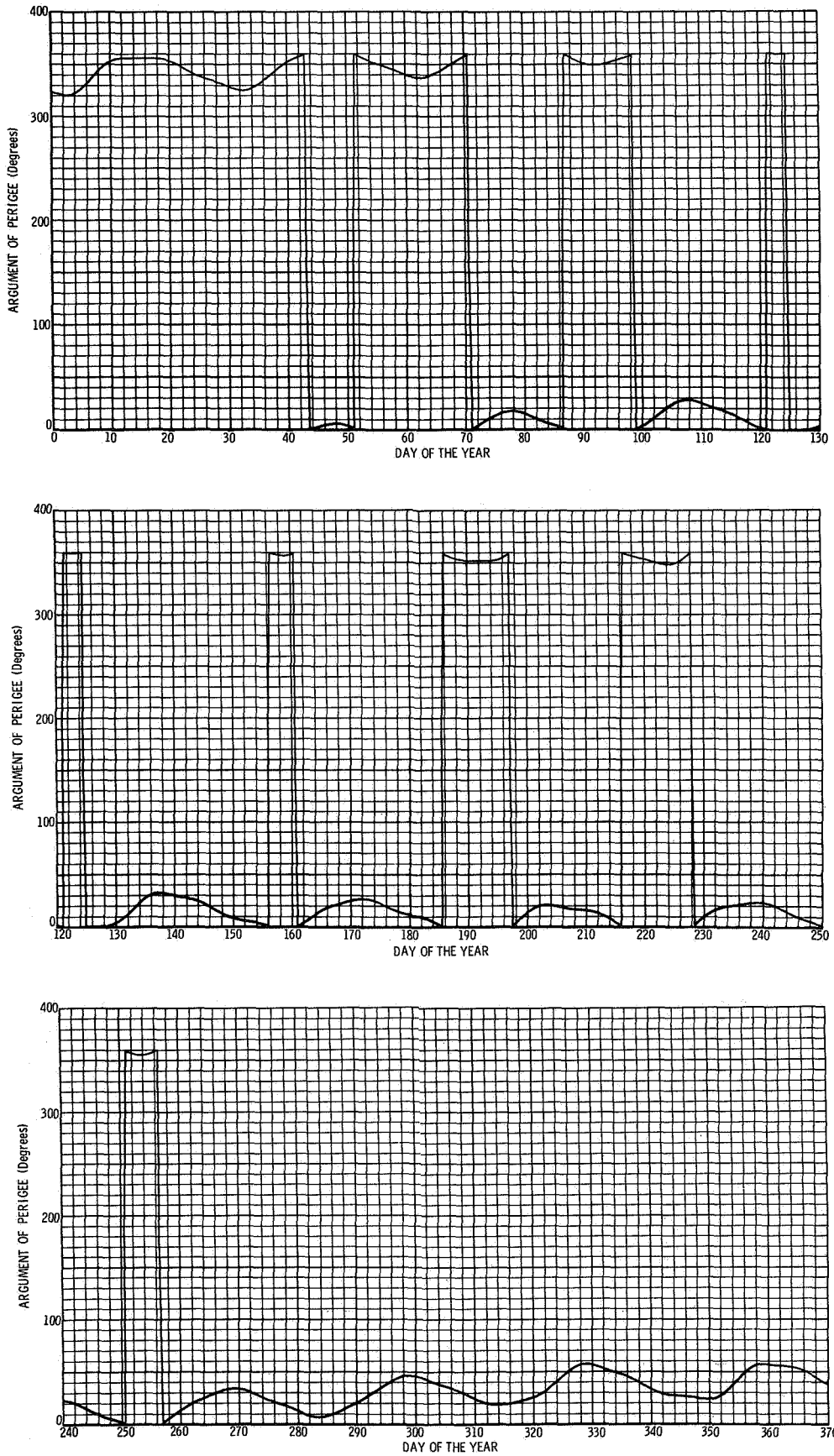
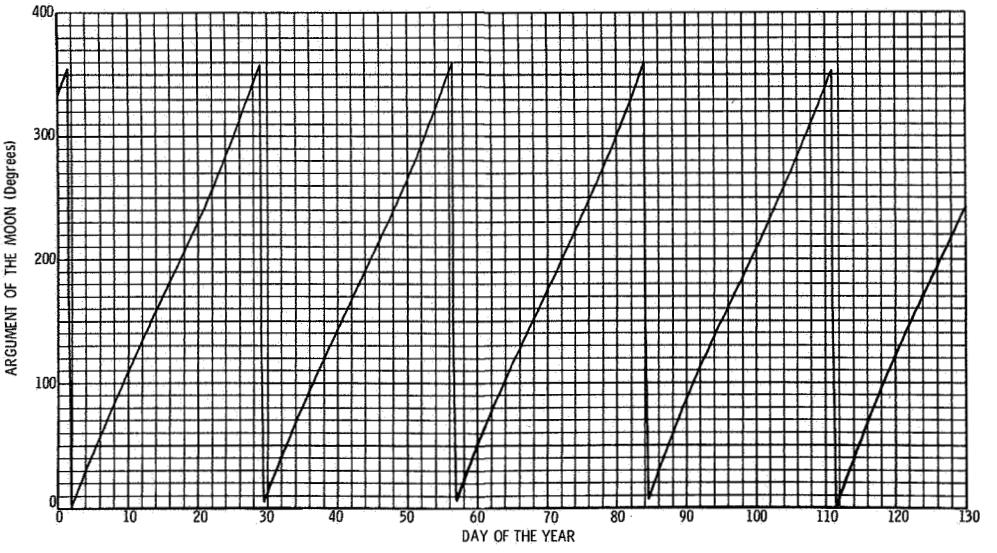
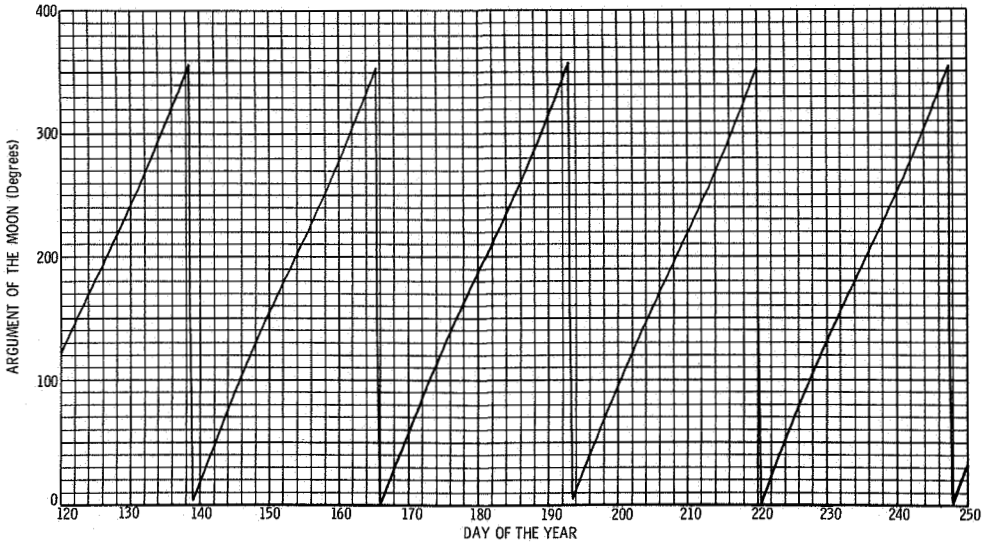


FIGURE B1971-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE





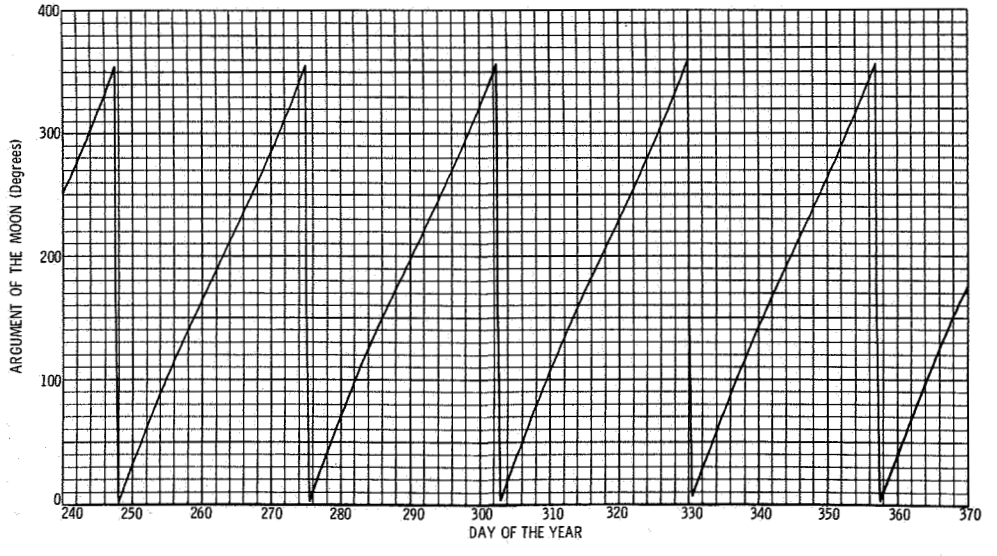
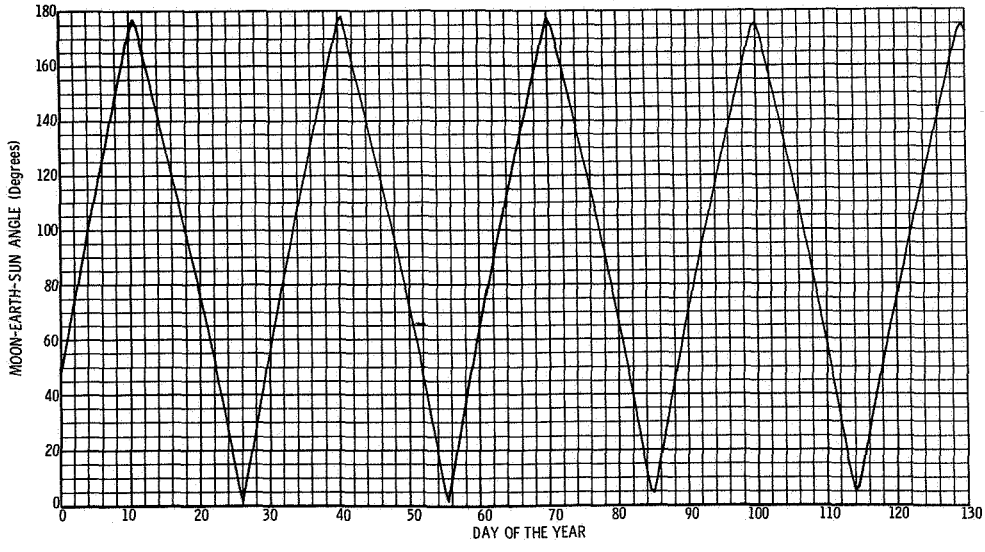
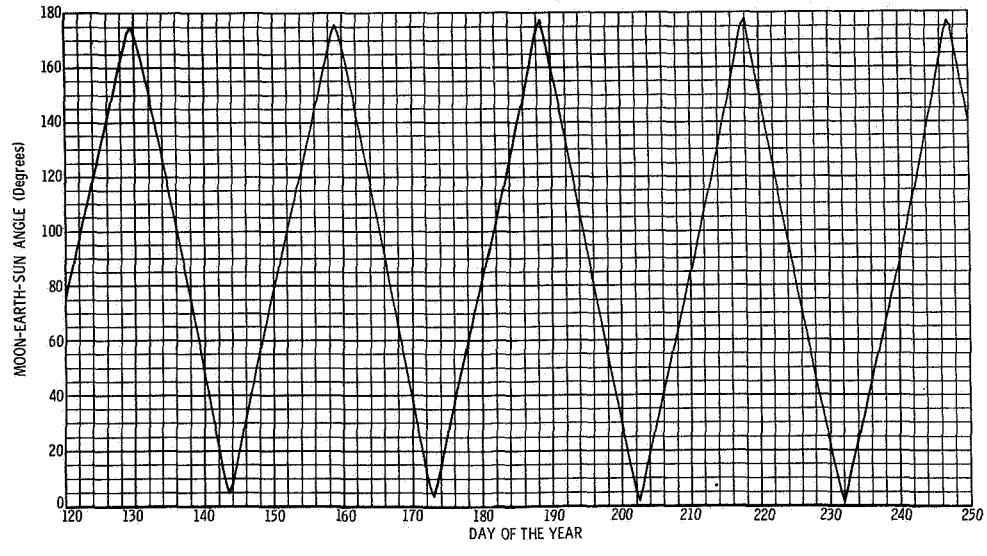


FIGURE B1971-10 ARGUMENT OF THE MOON'S POSITION





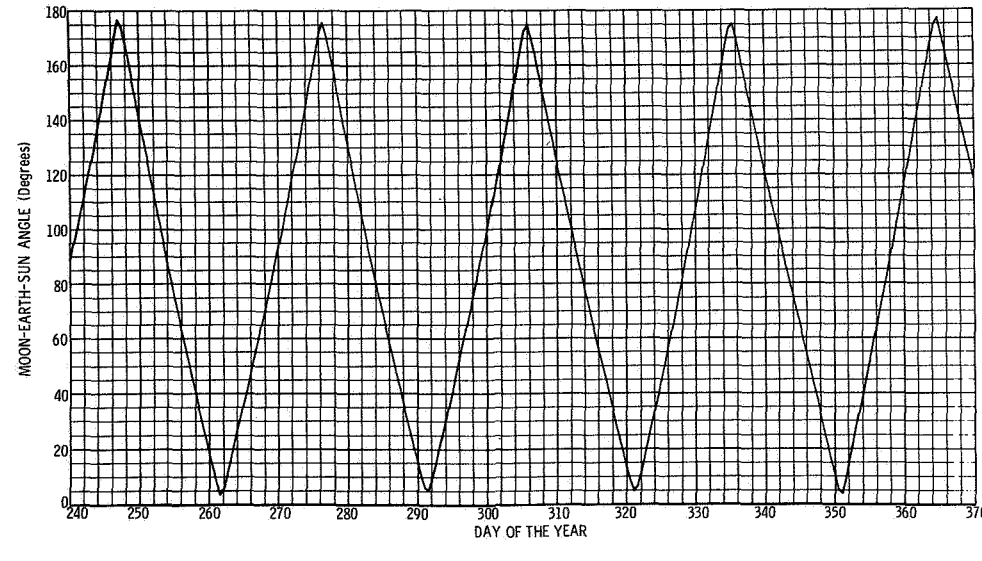
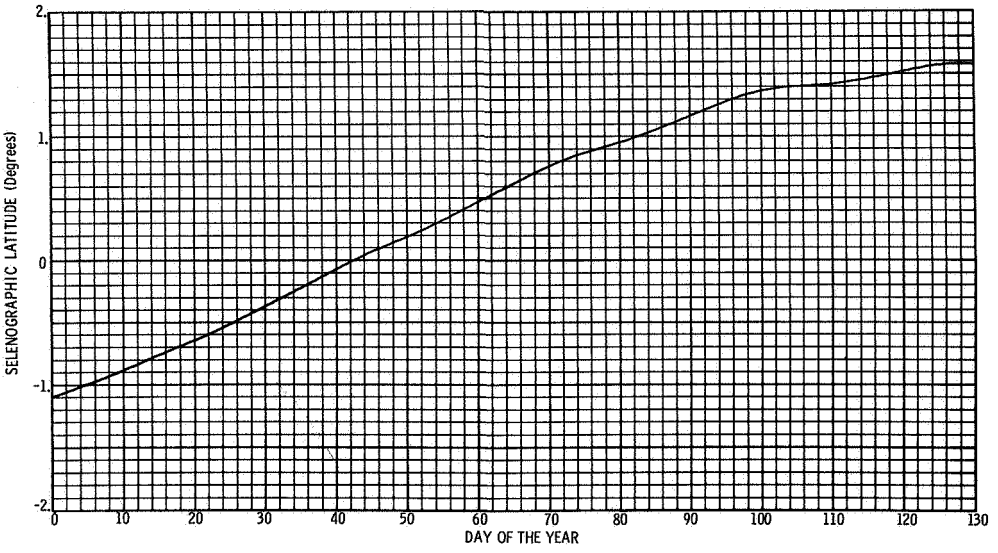
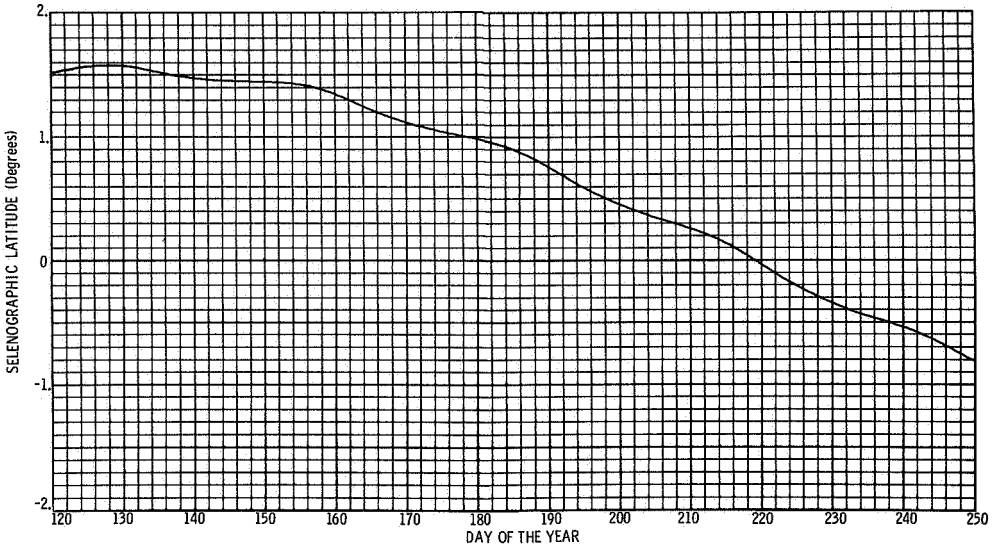


FIGURE B1971-11 MOON-EARTH-SUN ANGLE





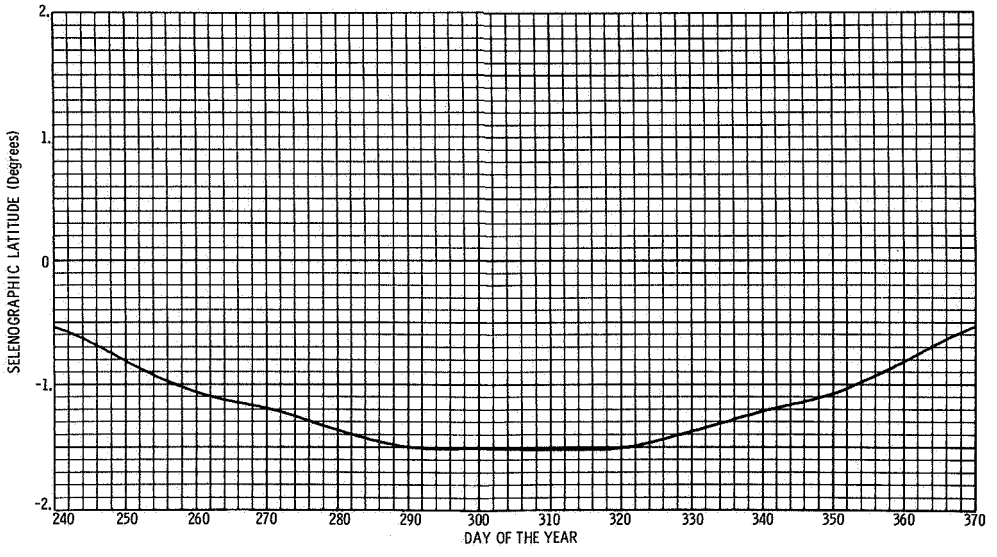


FIGURE B1971-12 SELENOGRAPHIC LATITUDE OF THE SUN

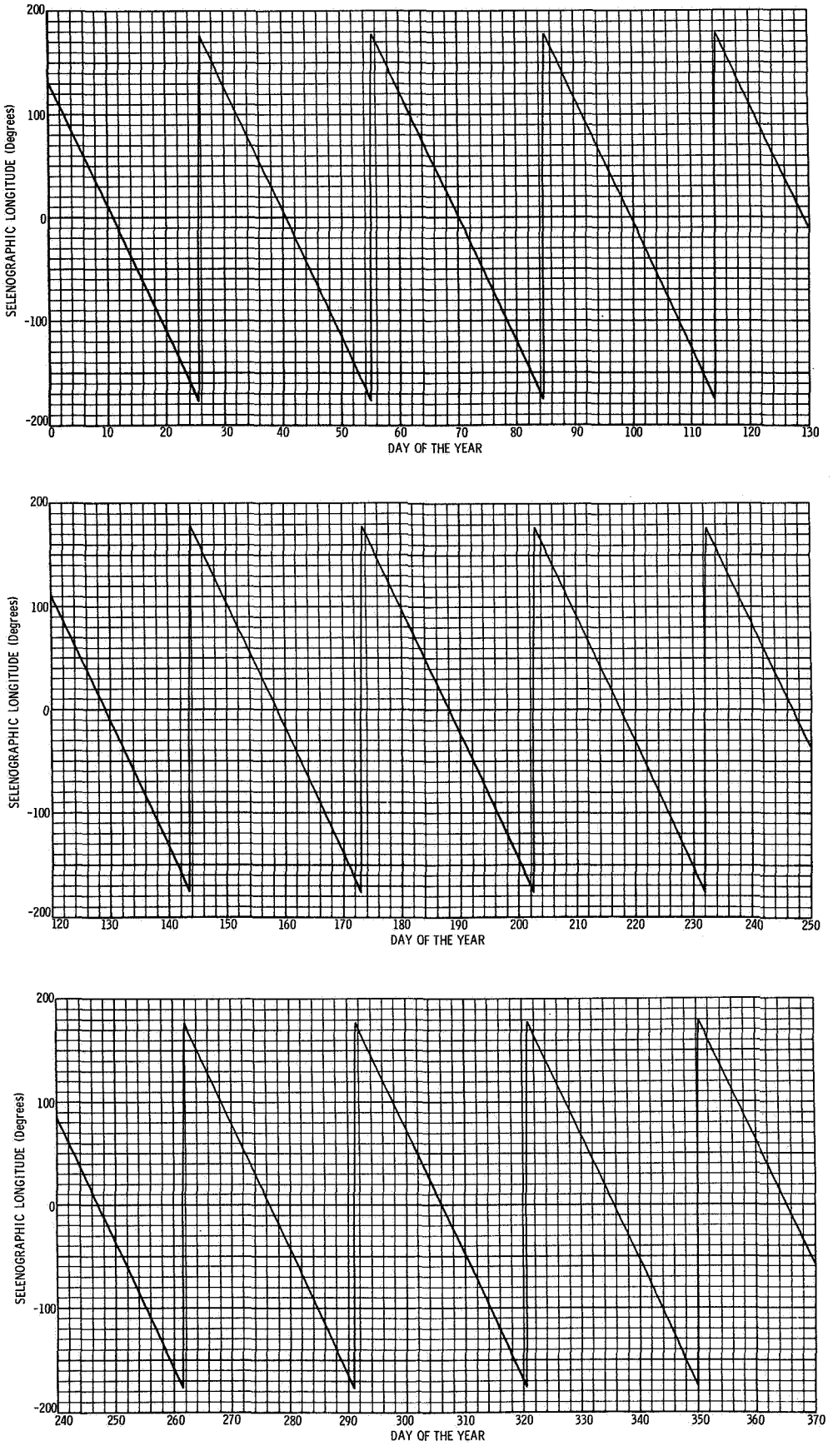
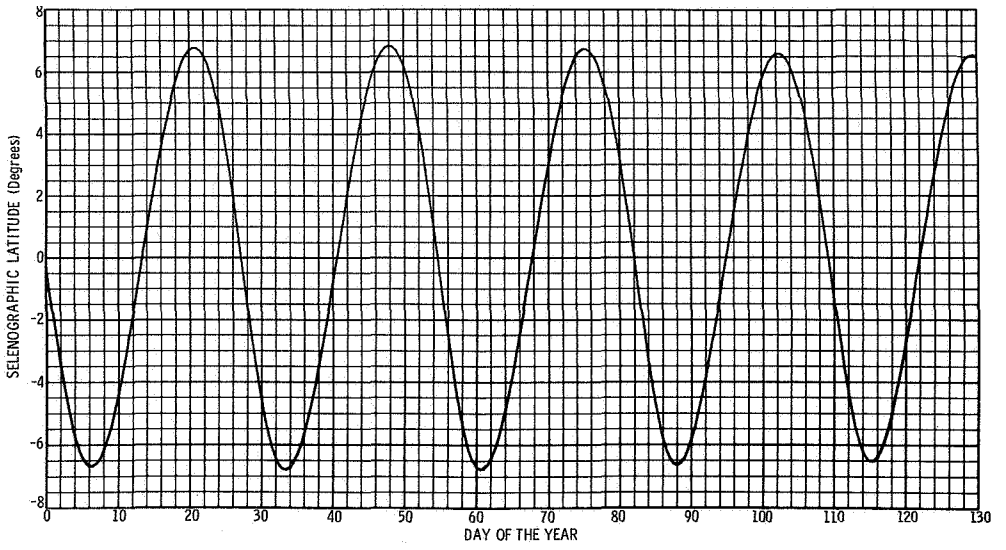
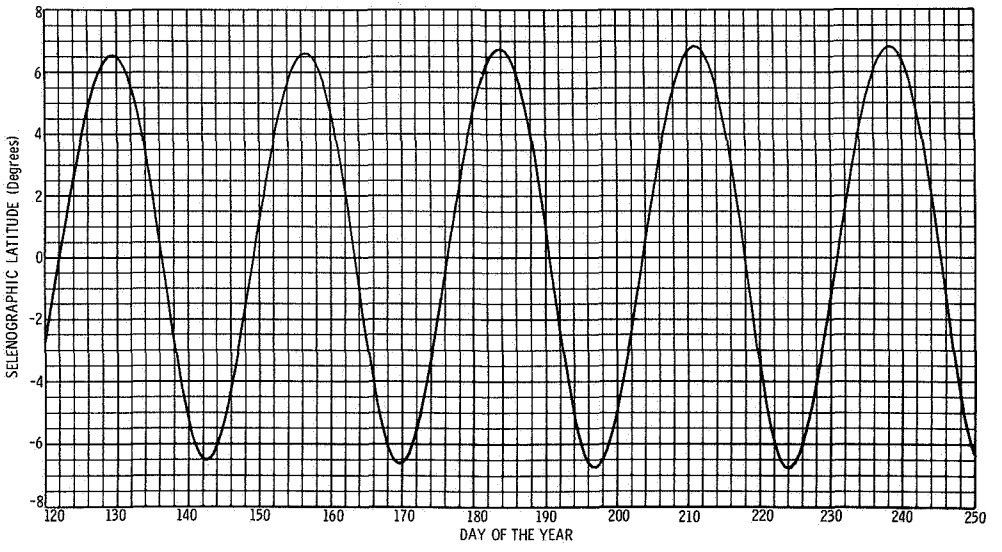


FIGURE B1971-13 SELENOGRAPHIC LONGITUDE OF THE SUN





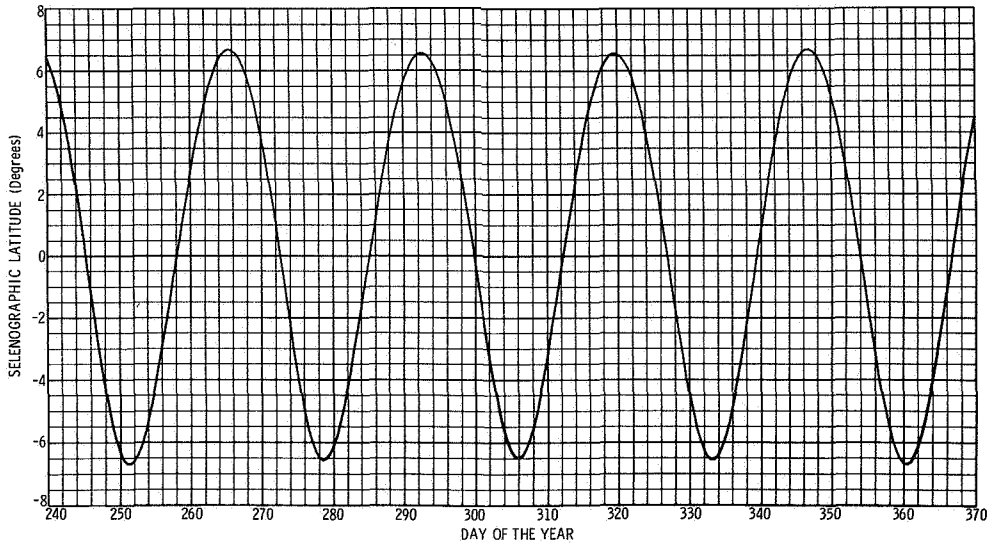
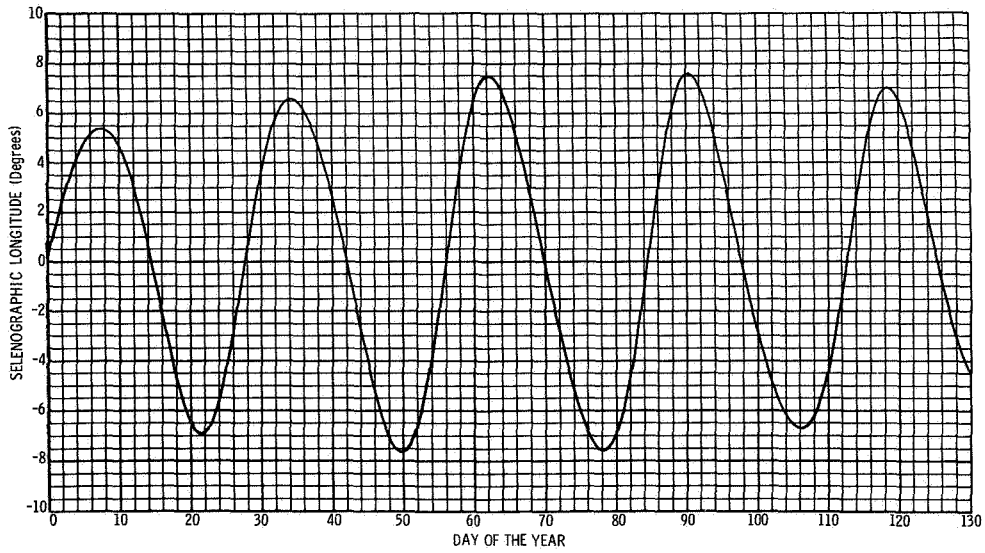
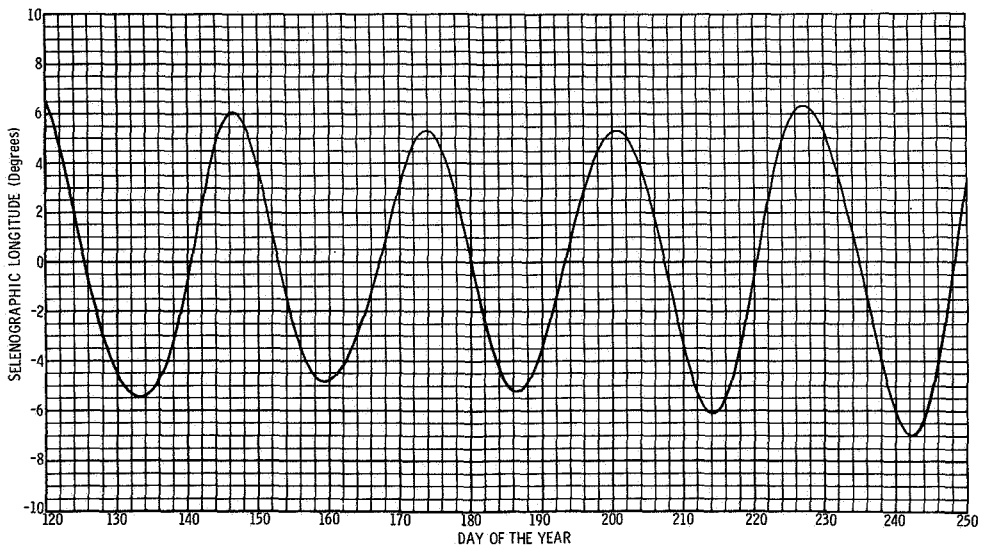


FIGURE B1971-14 SELENOGRAPHIC LATITUDE OF THE EARTH





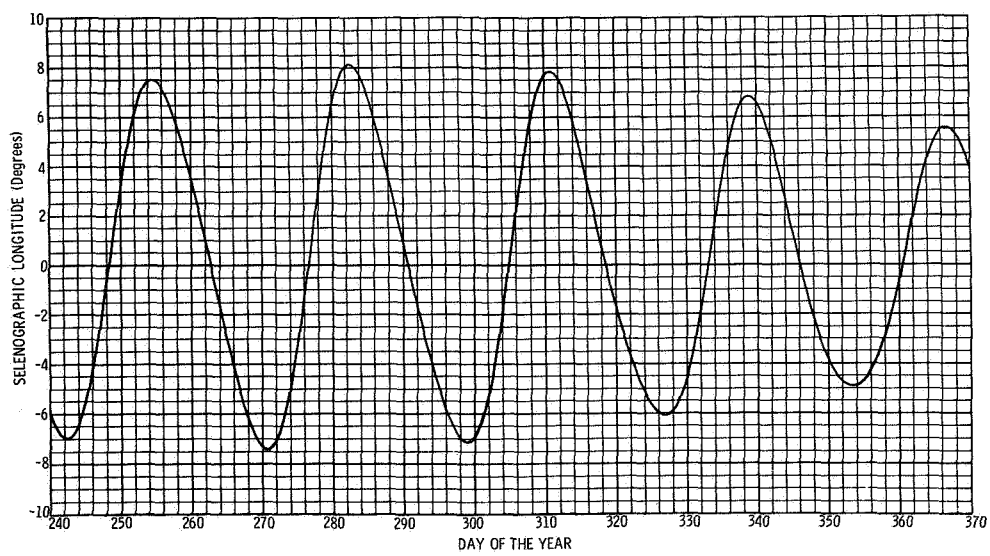


FIGURE B1971-15 SELENOGRAPHIC LONGITUDE OF THE EARTH

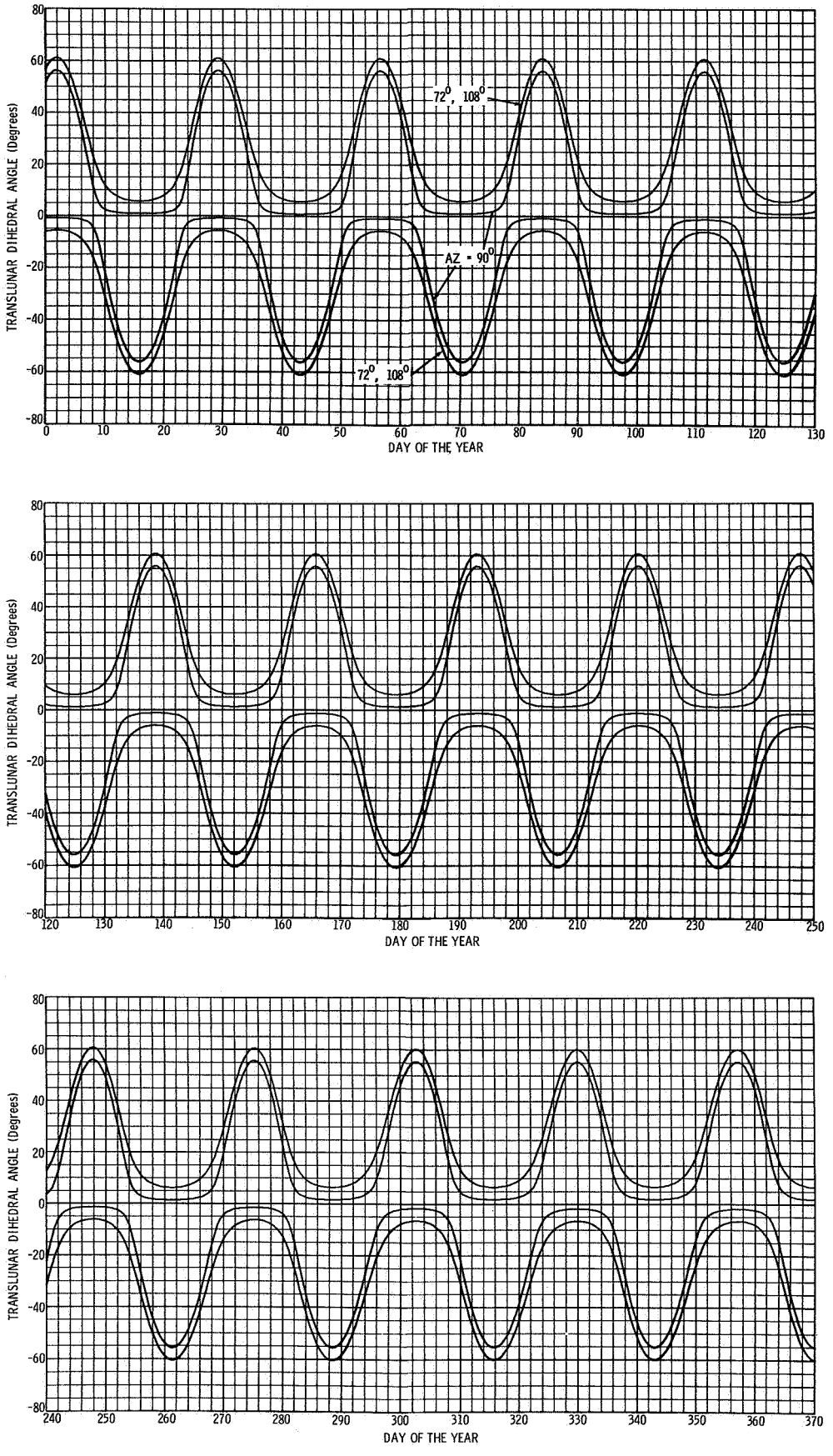


FIGURE B1971-16 TRANSLUNAR DIHEDRAL ANGLES

1972

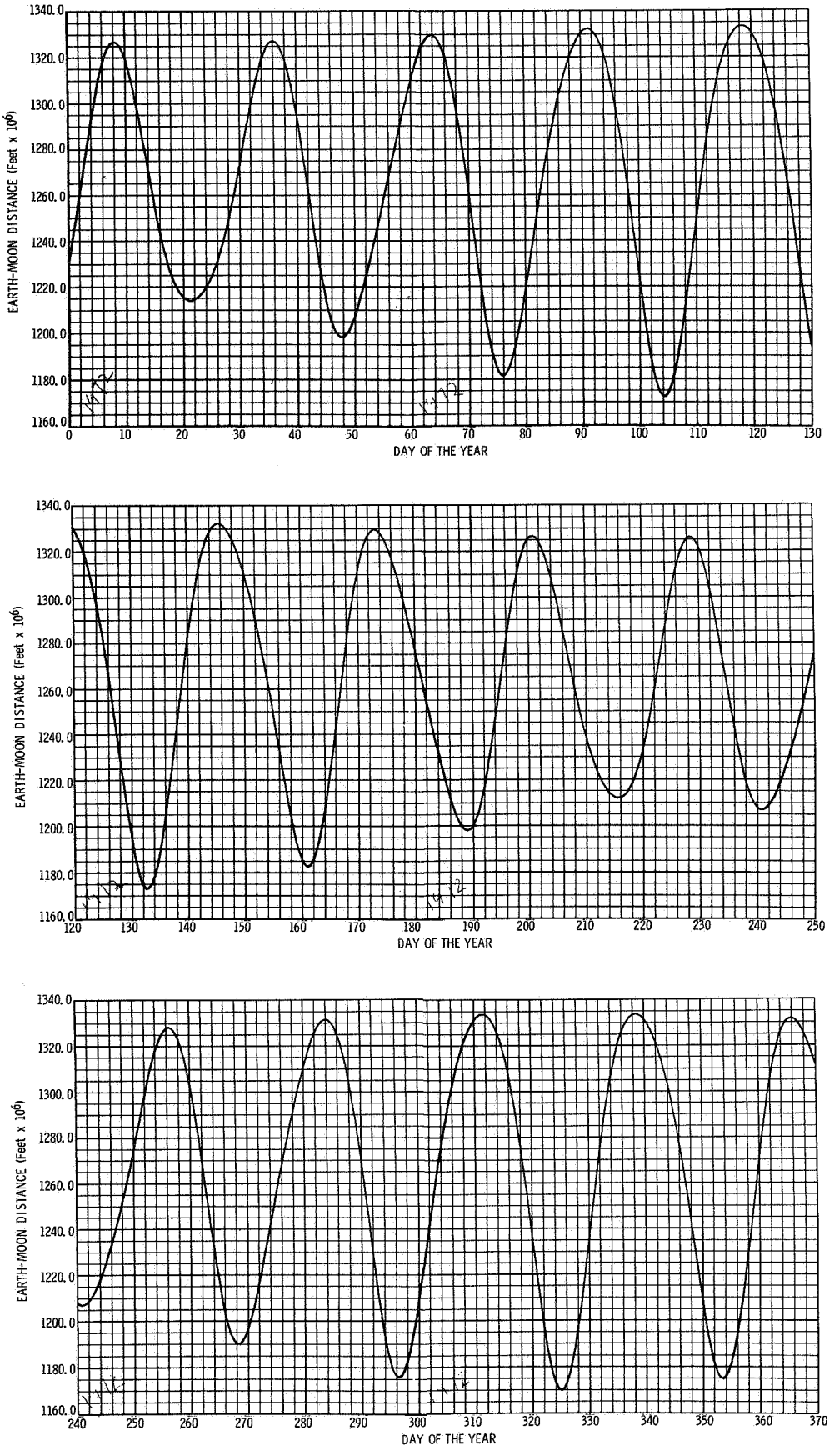
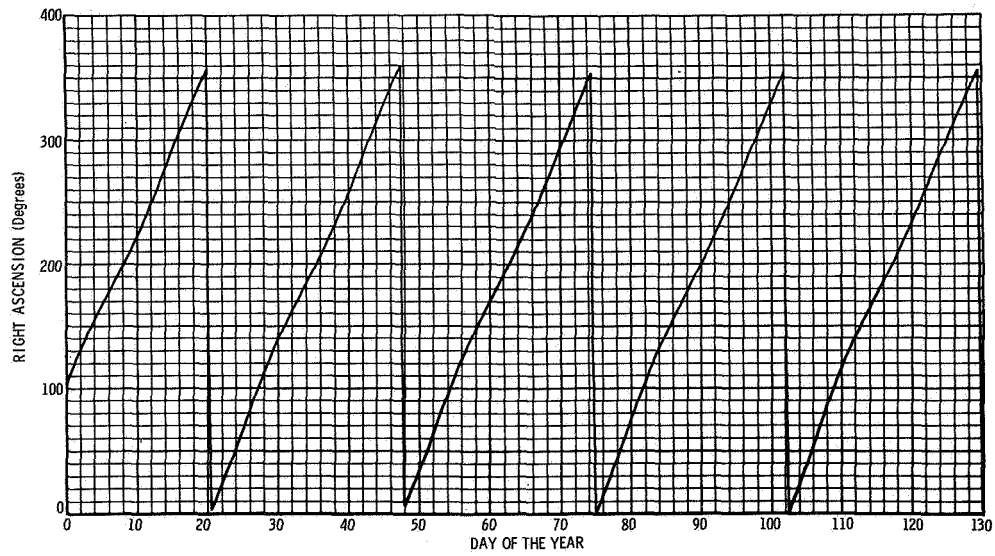
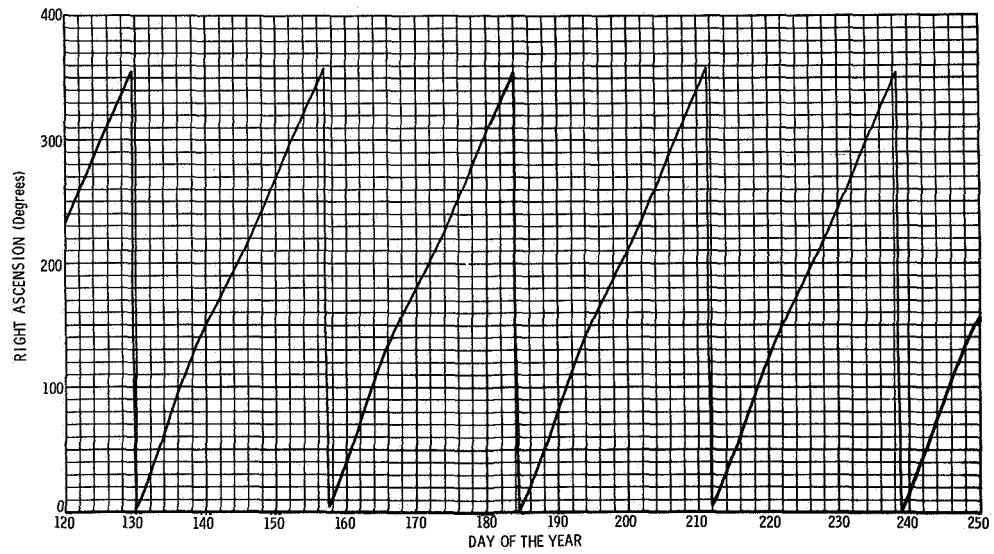


FIGURE B1972-1 EARTH-MOON DISTANCE





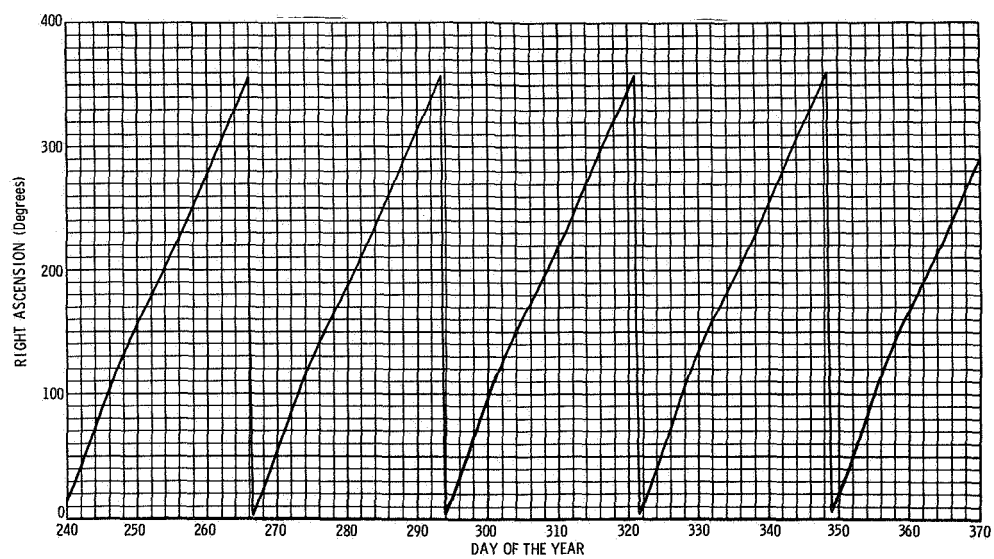
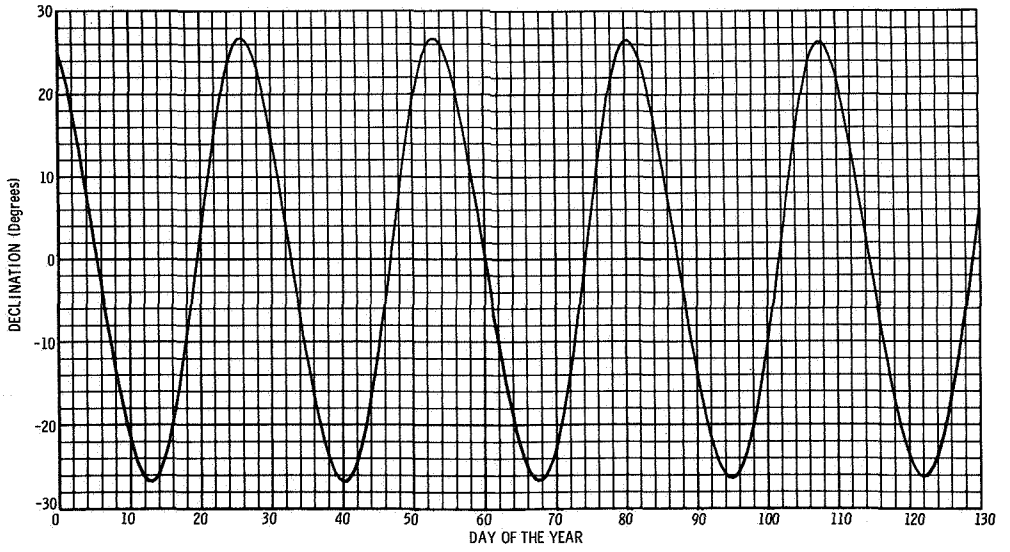
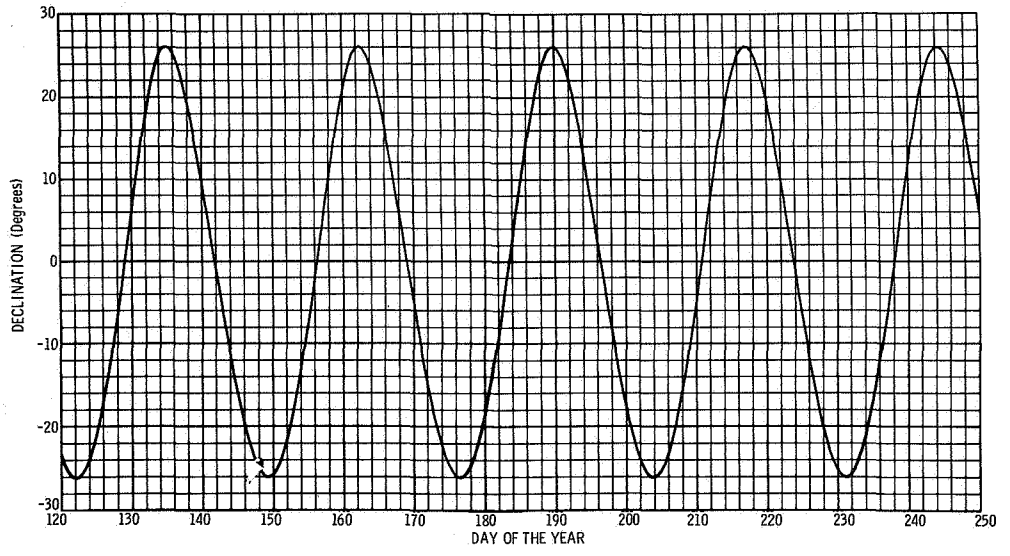


FIGURE B1972-2 RIGHT ASCENSION OF THE MOON





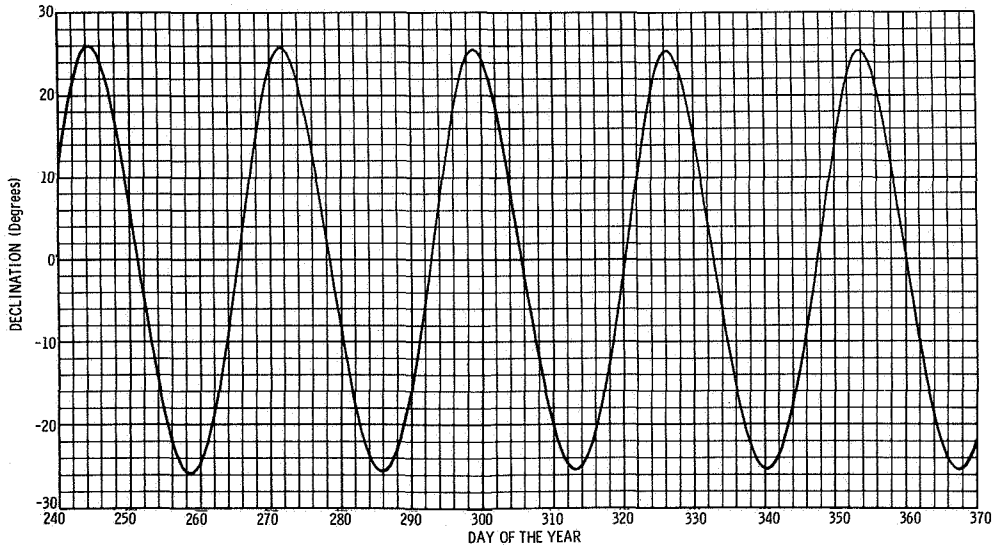
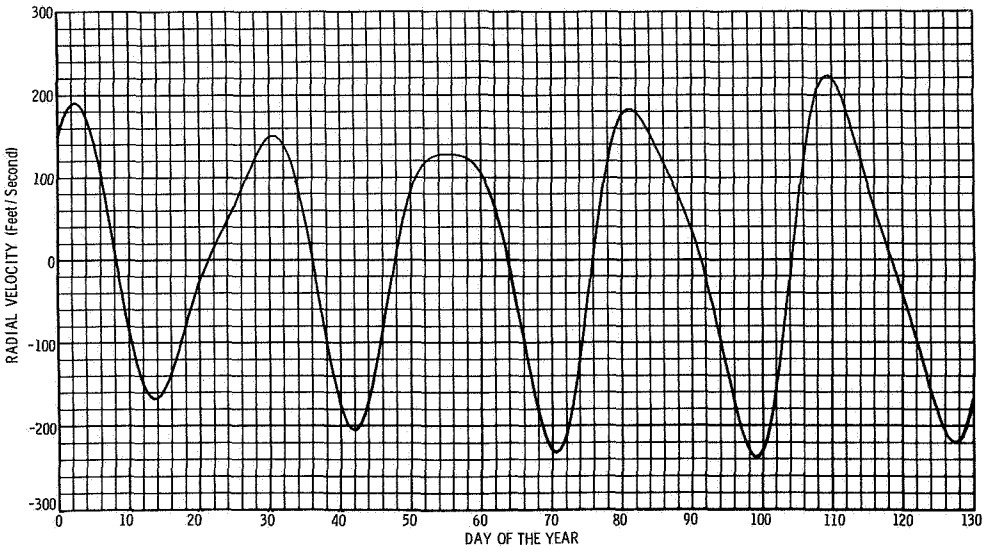
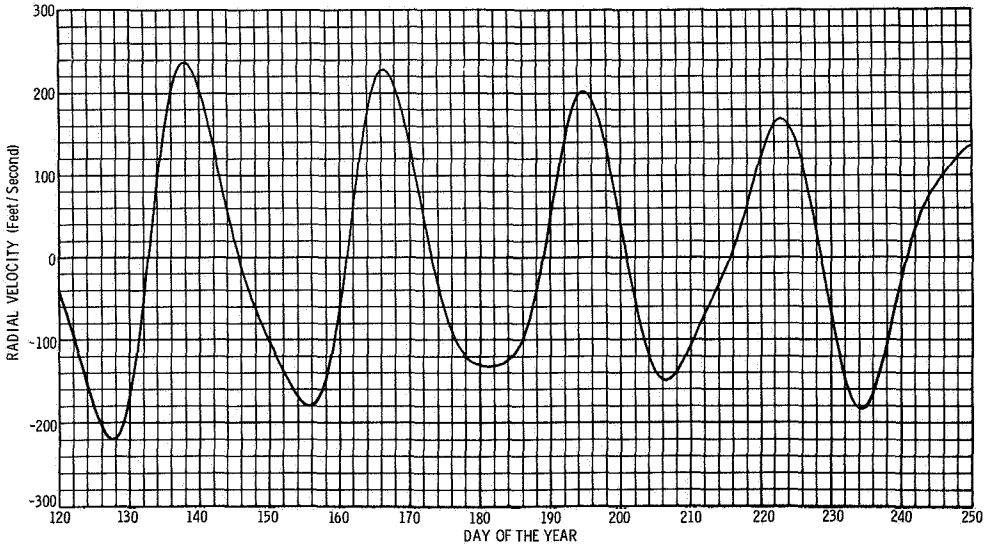


FIGURE B1972-3 DECLINATION OF THE MOON





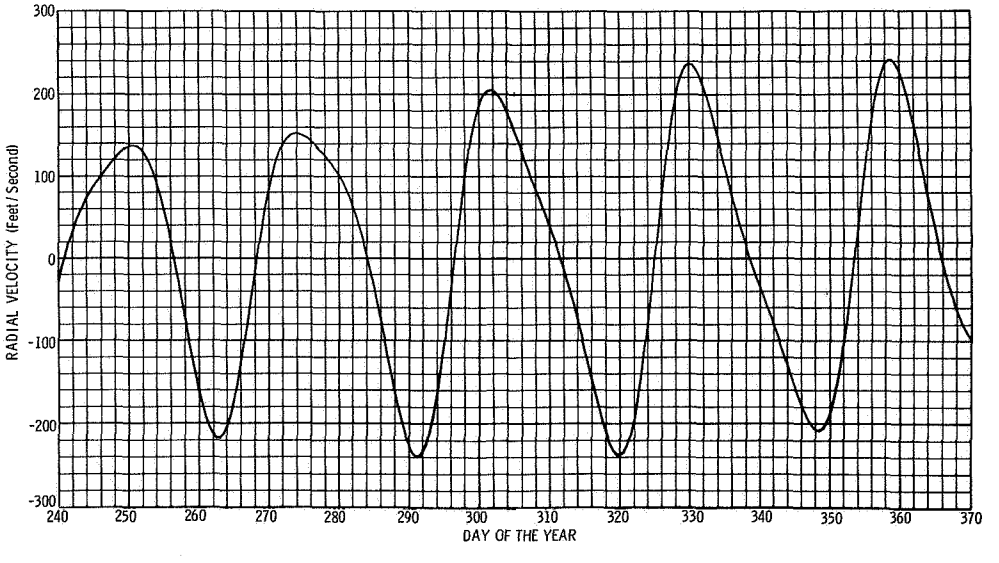


FIGURE B1972-4 RADIAL VELOCITY OF THE MOON

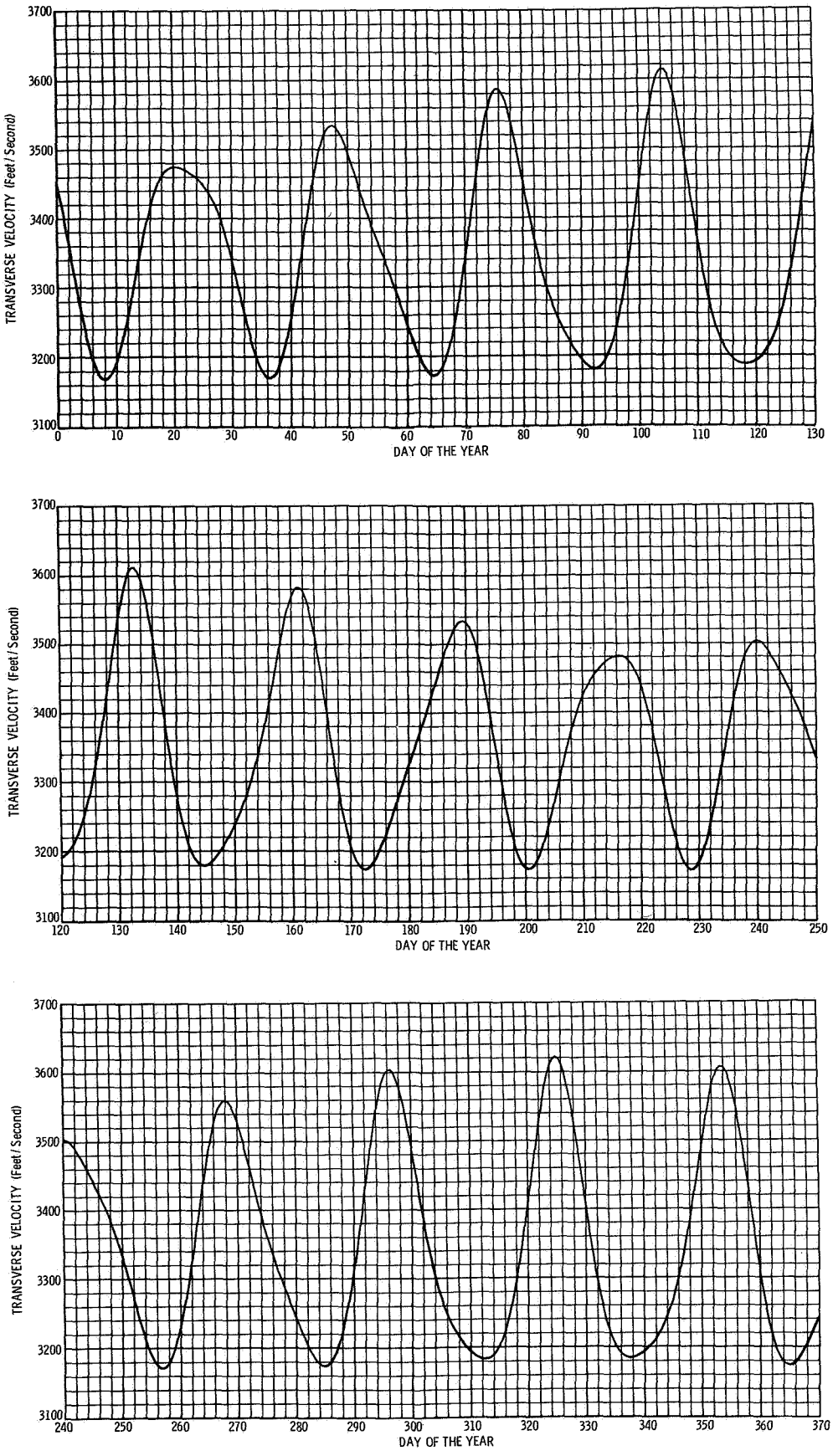


FIGURE B1972-5 TRANSVERSE VELOCITY OF THE MOON

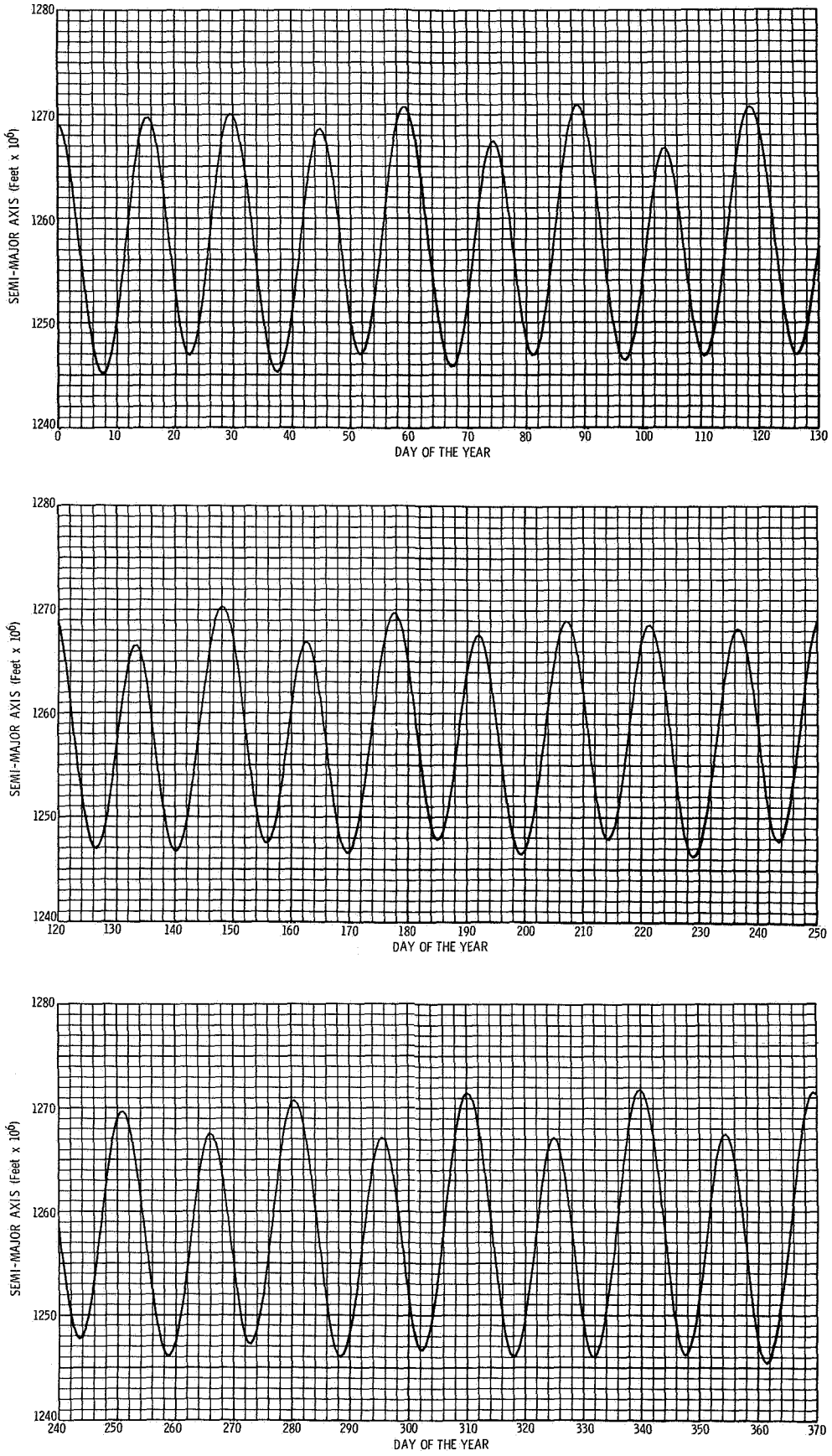
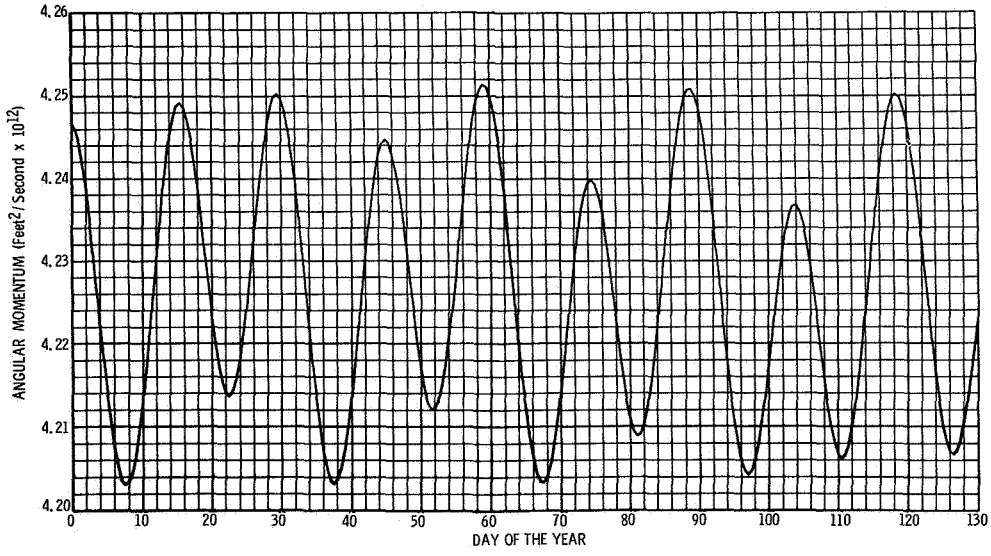
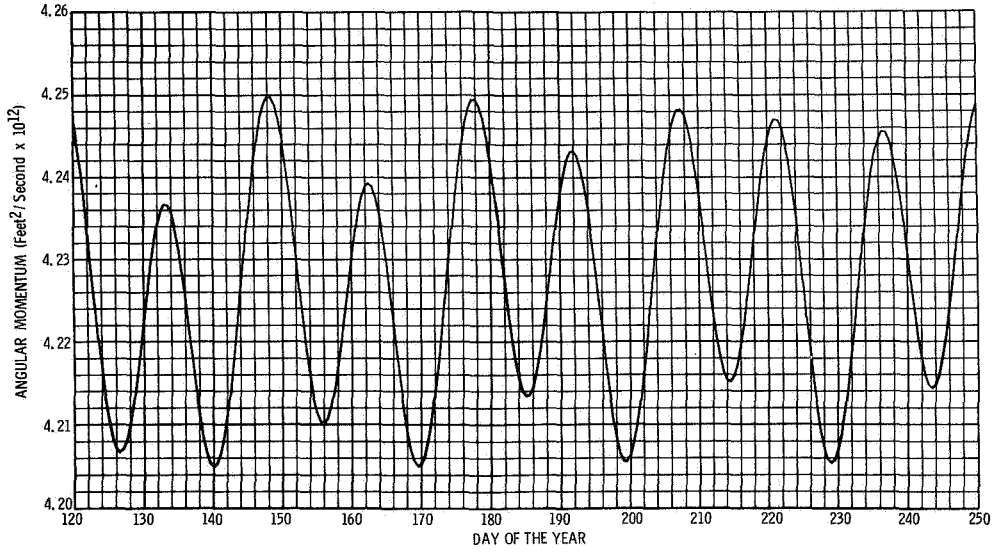


FIGURE B1972-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT





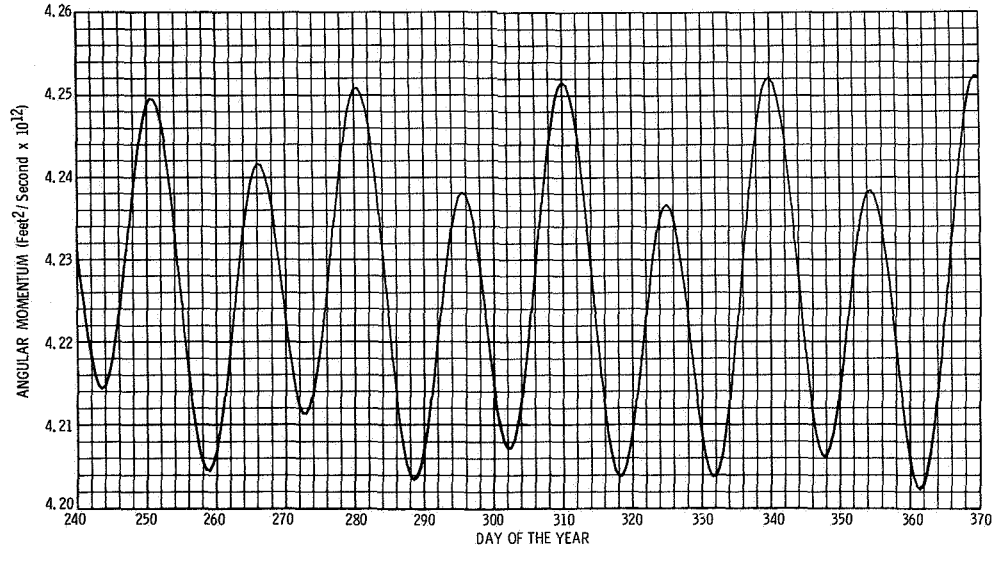
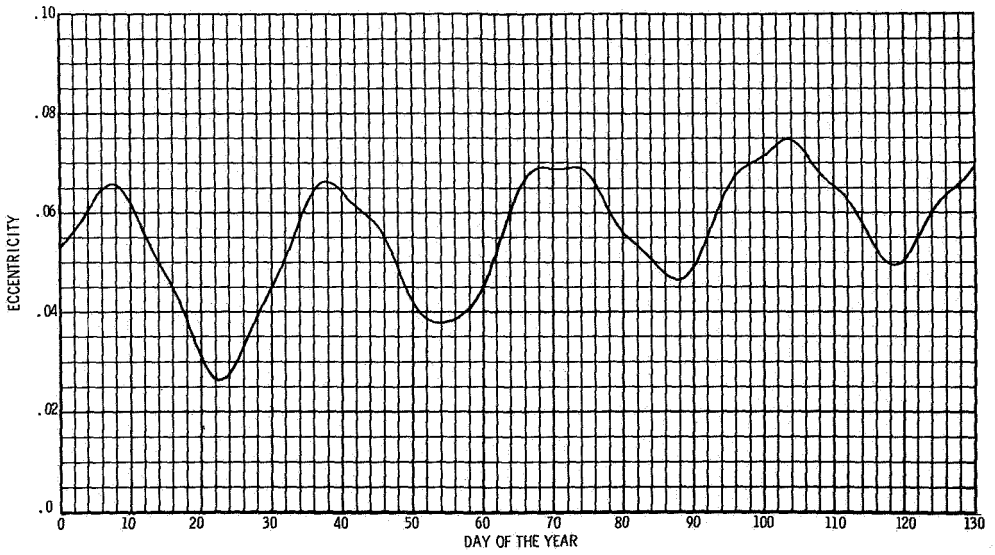
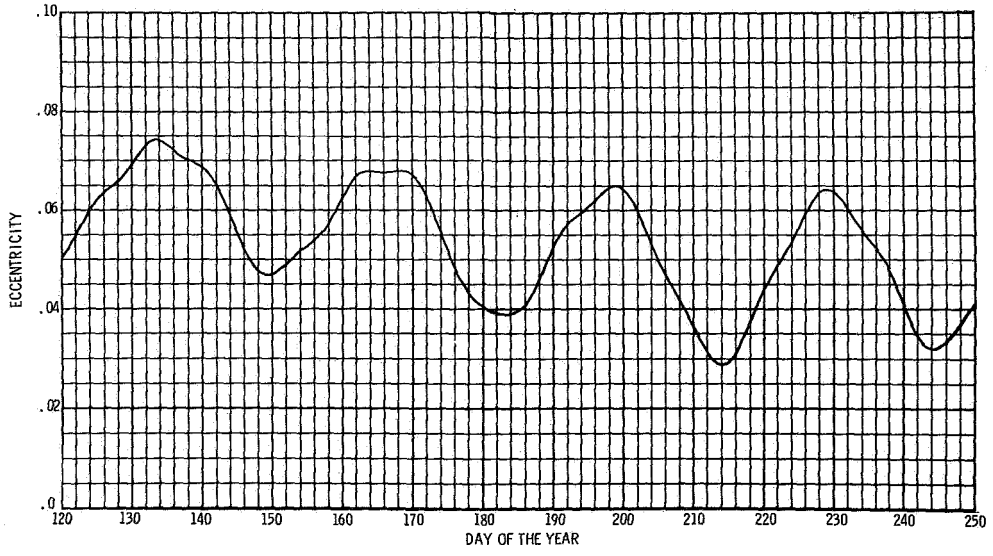


FIGURE B1972-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON





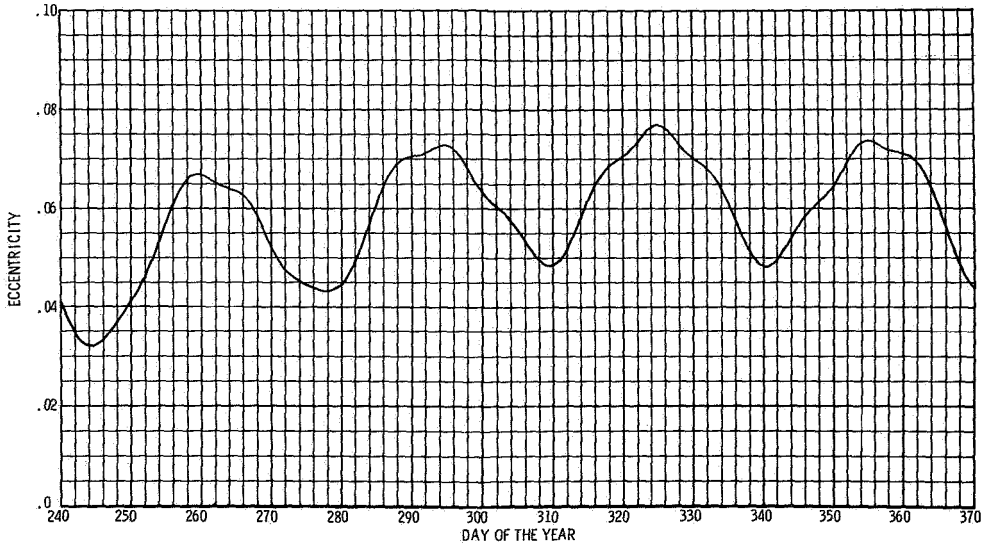


FIGURE B1972-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT

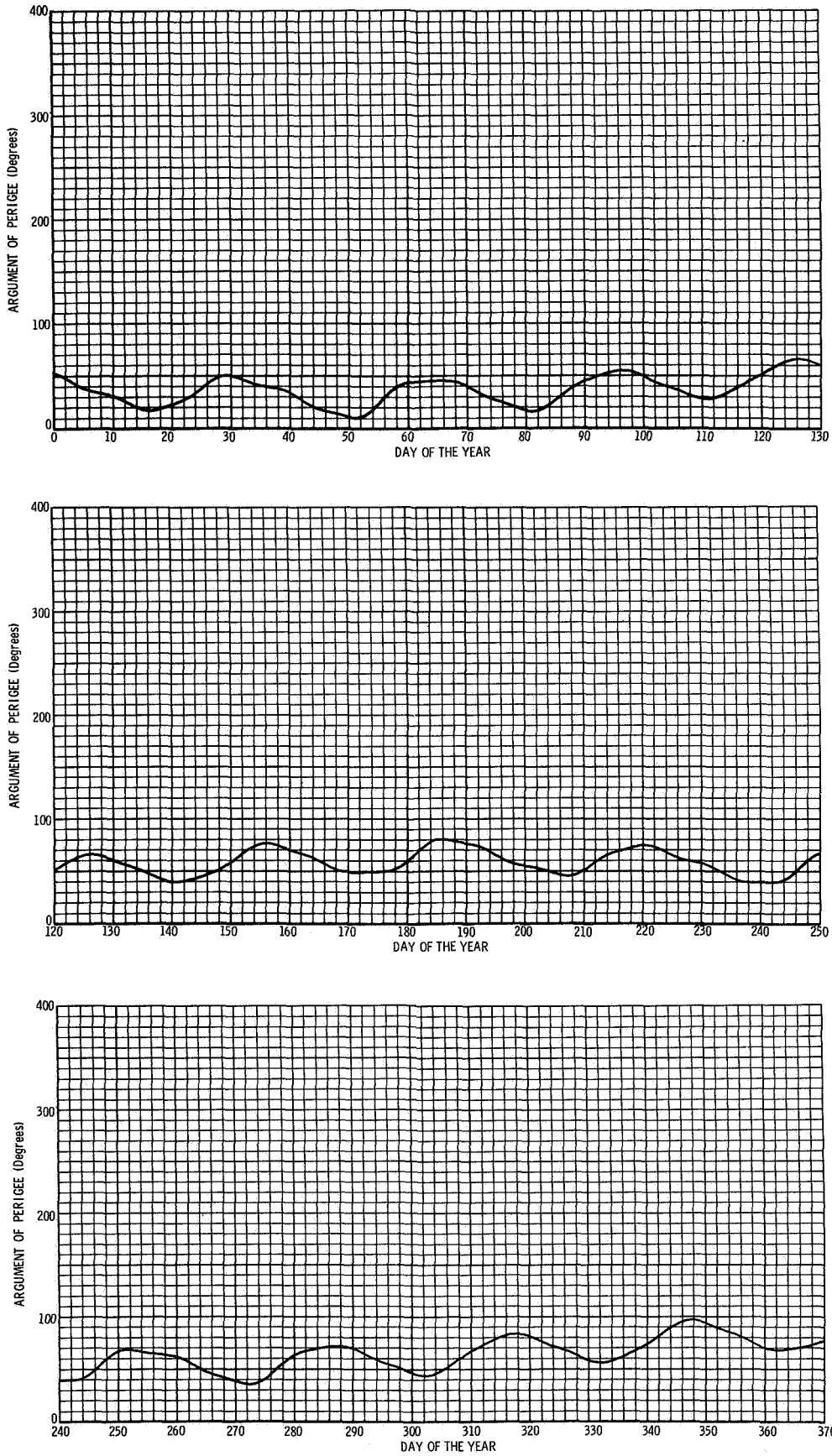
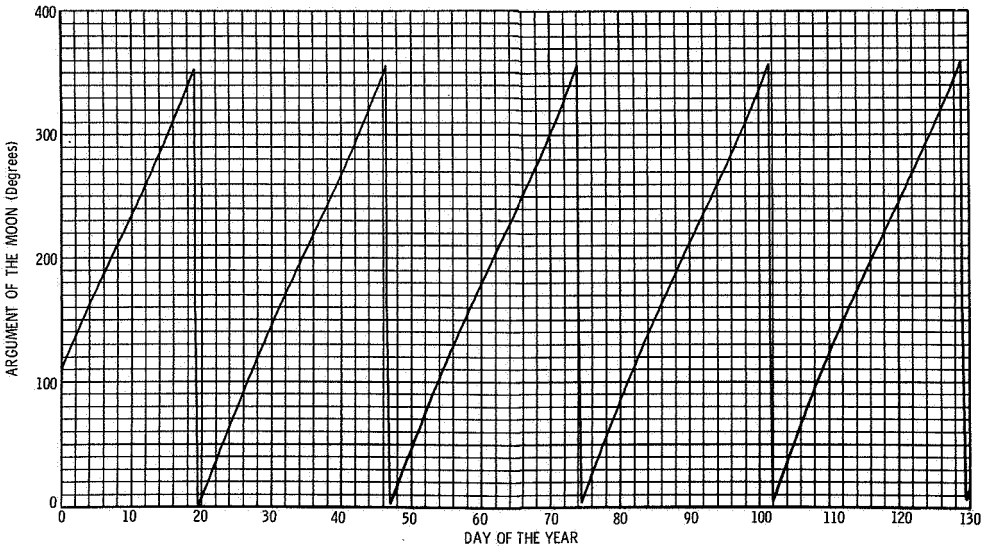
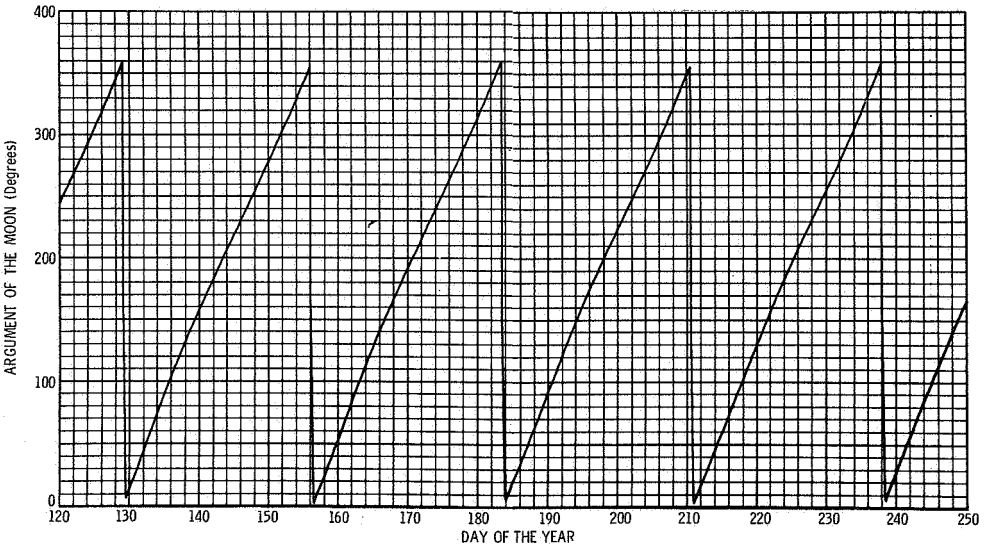


FIGURE B1972-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE





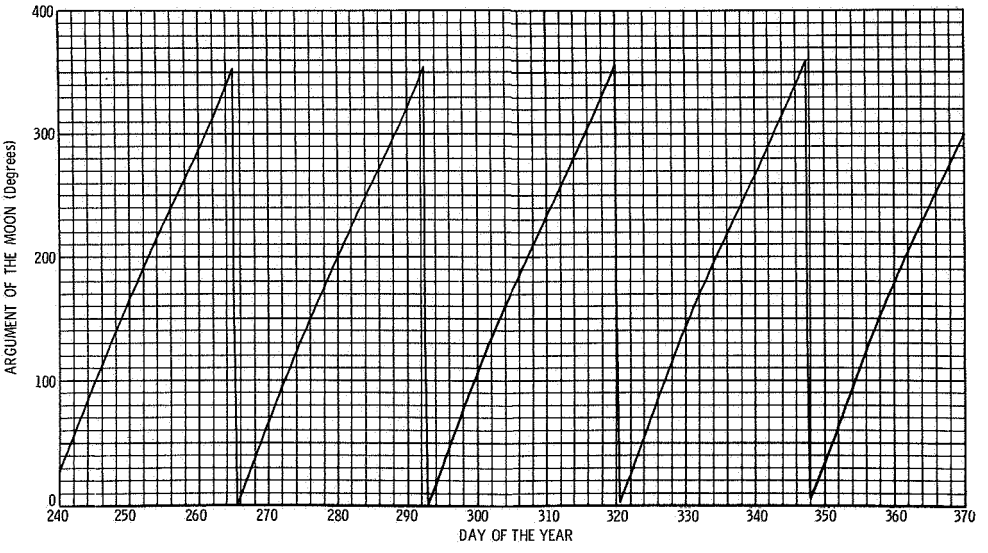
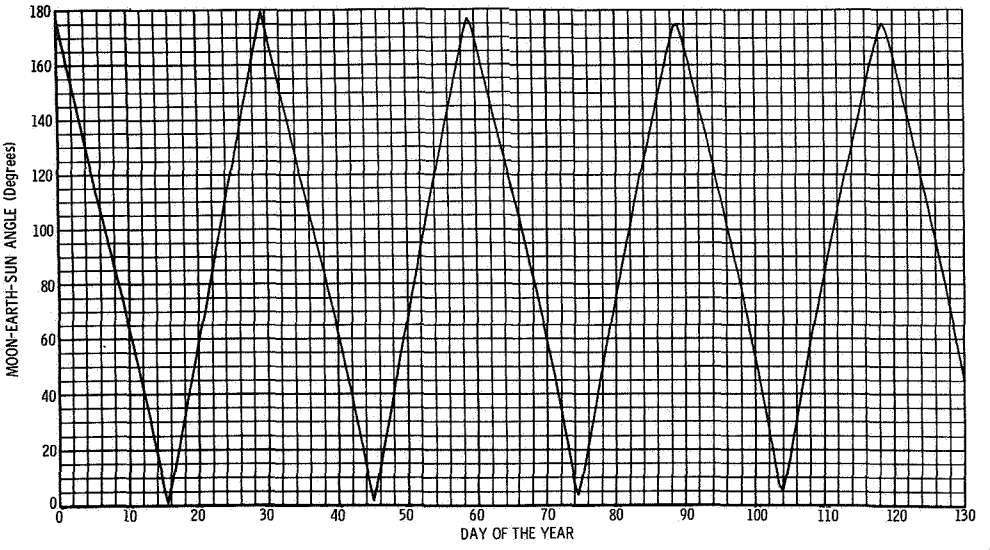
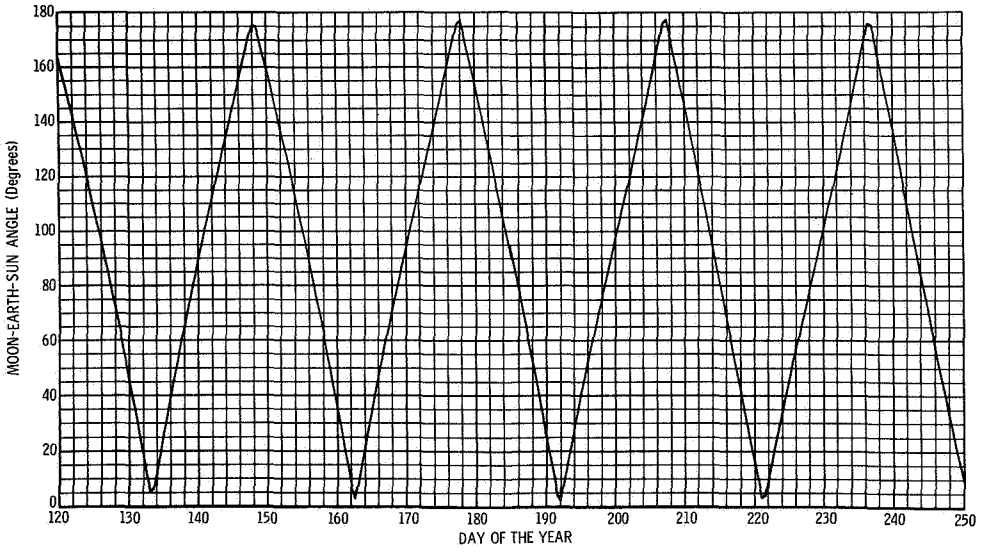


FIGURE B1972-10 ARGUMENT OF THE MOON'S POSITION





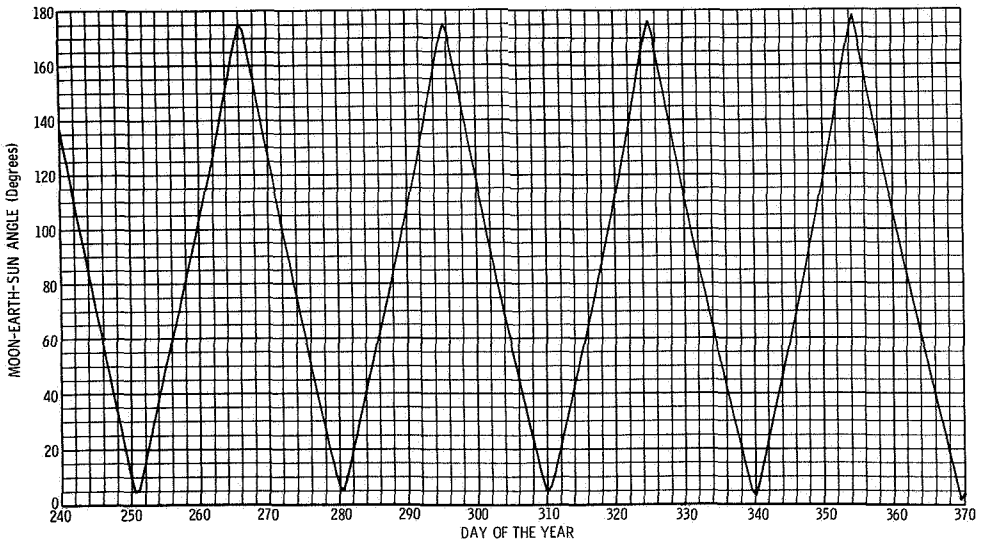
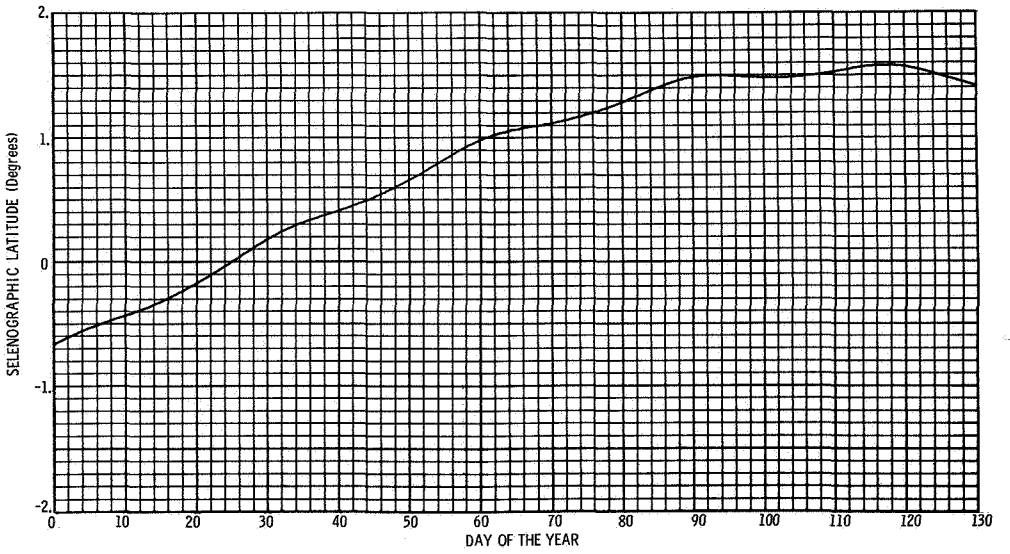
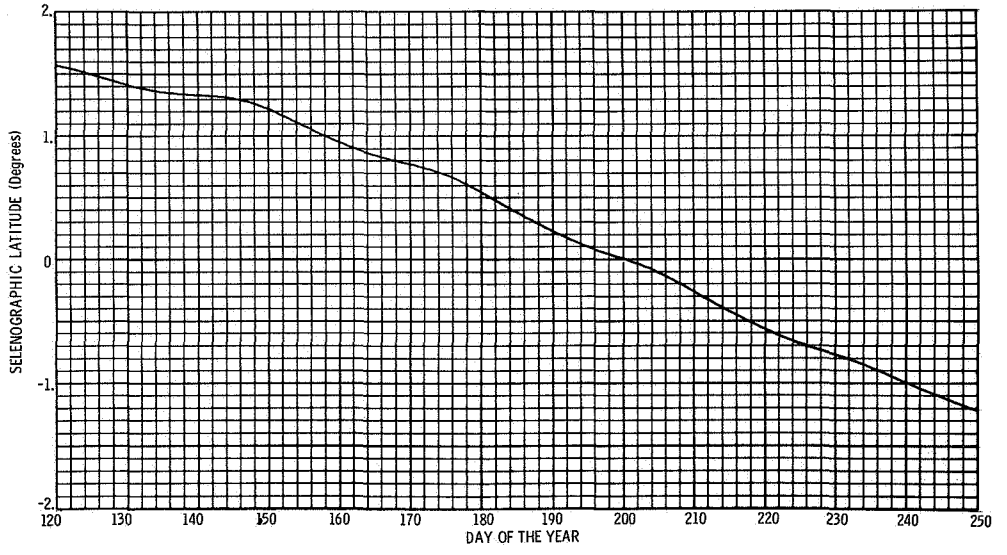


FIGURE B1972-11 MOON-EARTH-SUN ANGLE





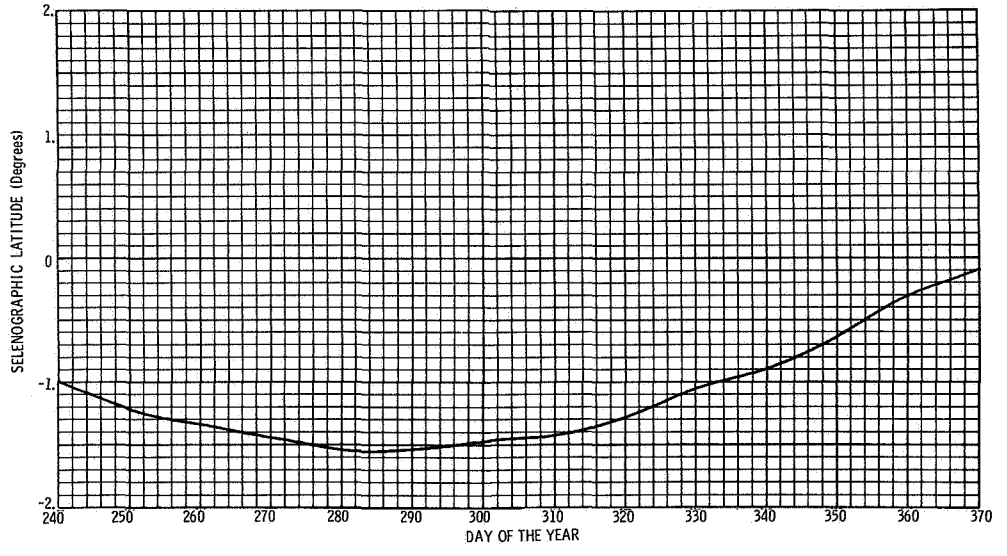
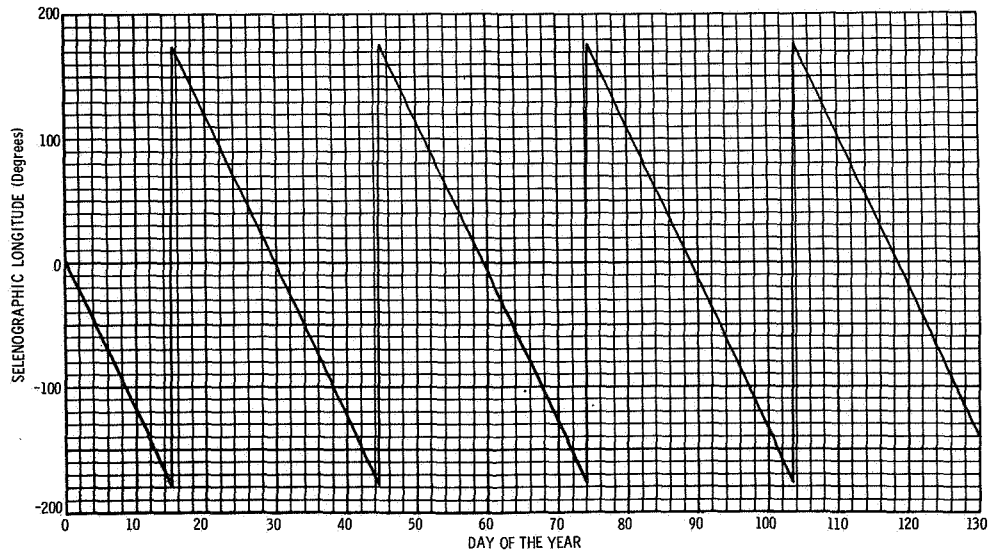
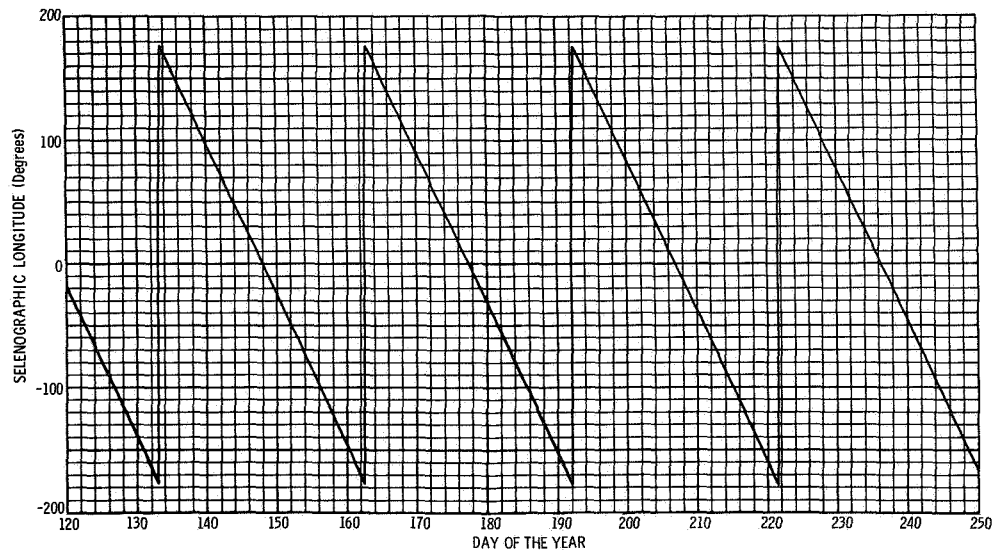


FIGURE B1972-12 SELENOGRAPHIC LATITUDE OF THE SUN





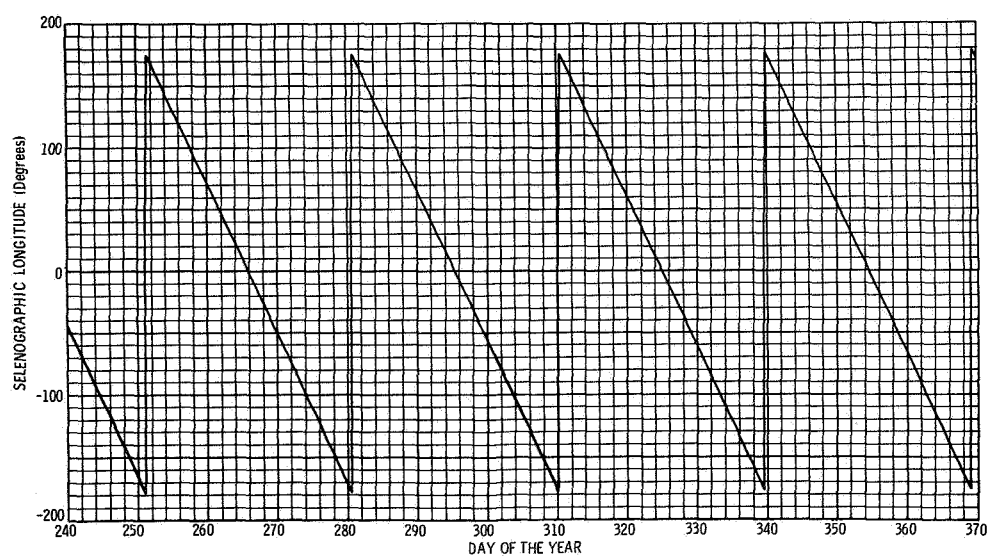
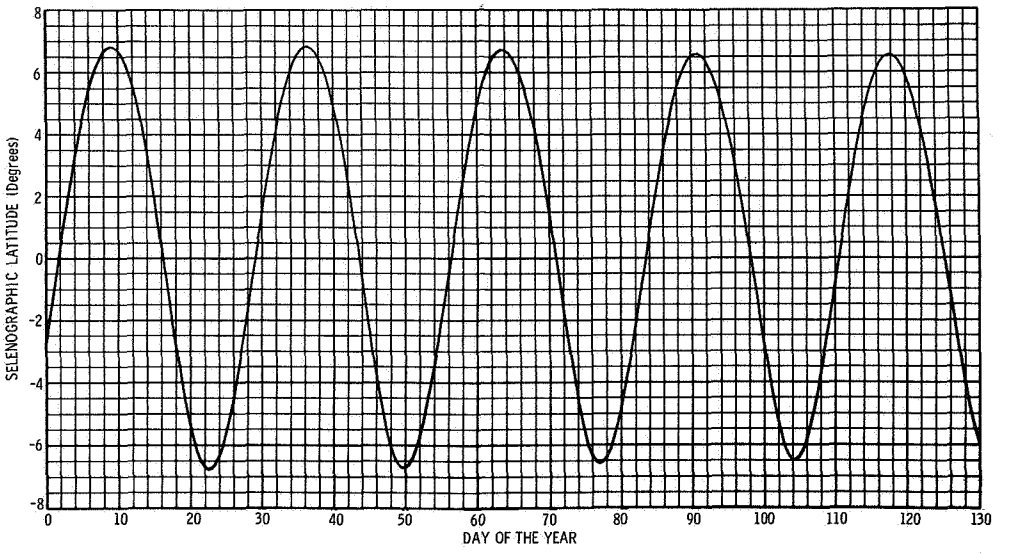
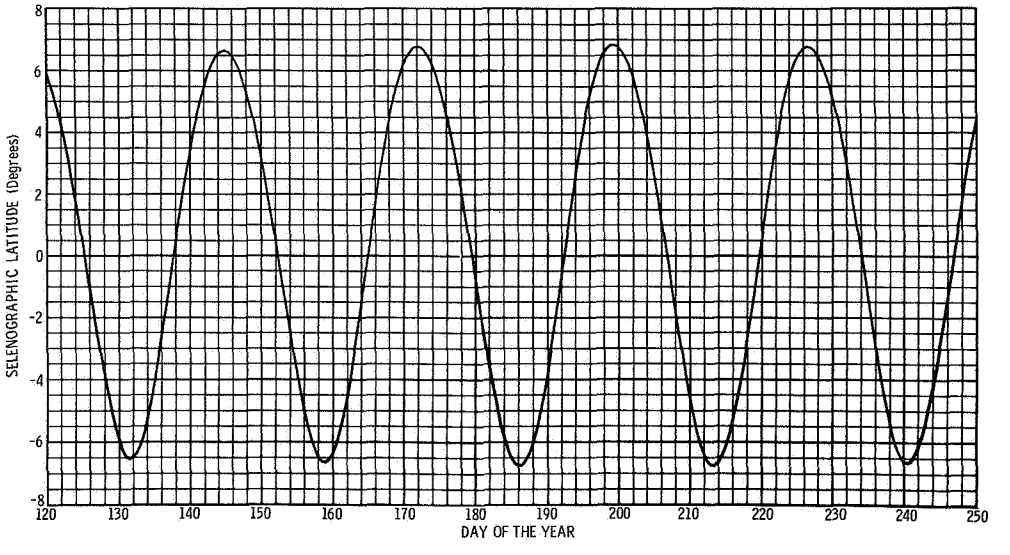


FIGURE B1972-13 SELENOGRAPHIC LONGITUDE OF THE SUN





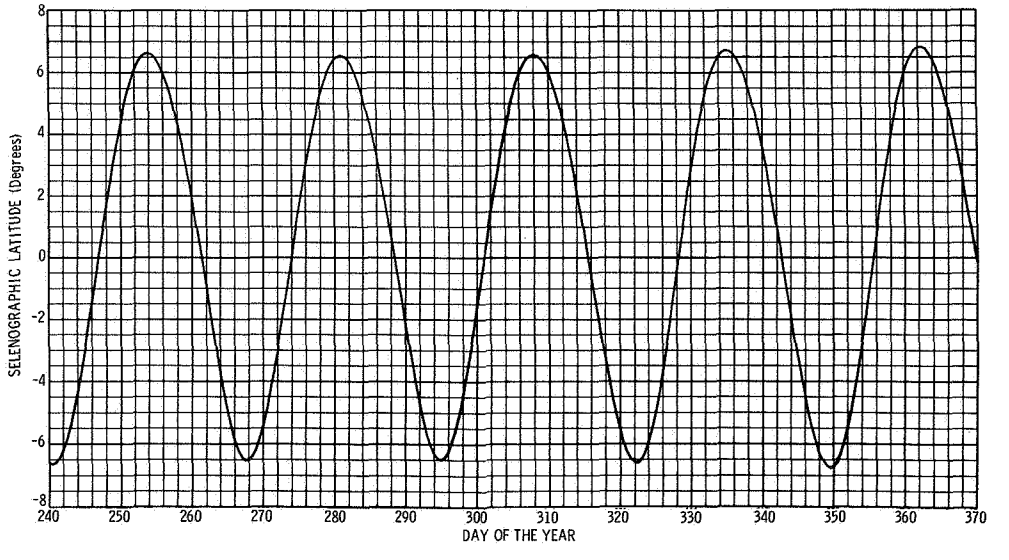


FIGURE B1972-14 SELENOGRAPHIC LATITUDE OF THE EARTH

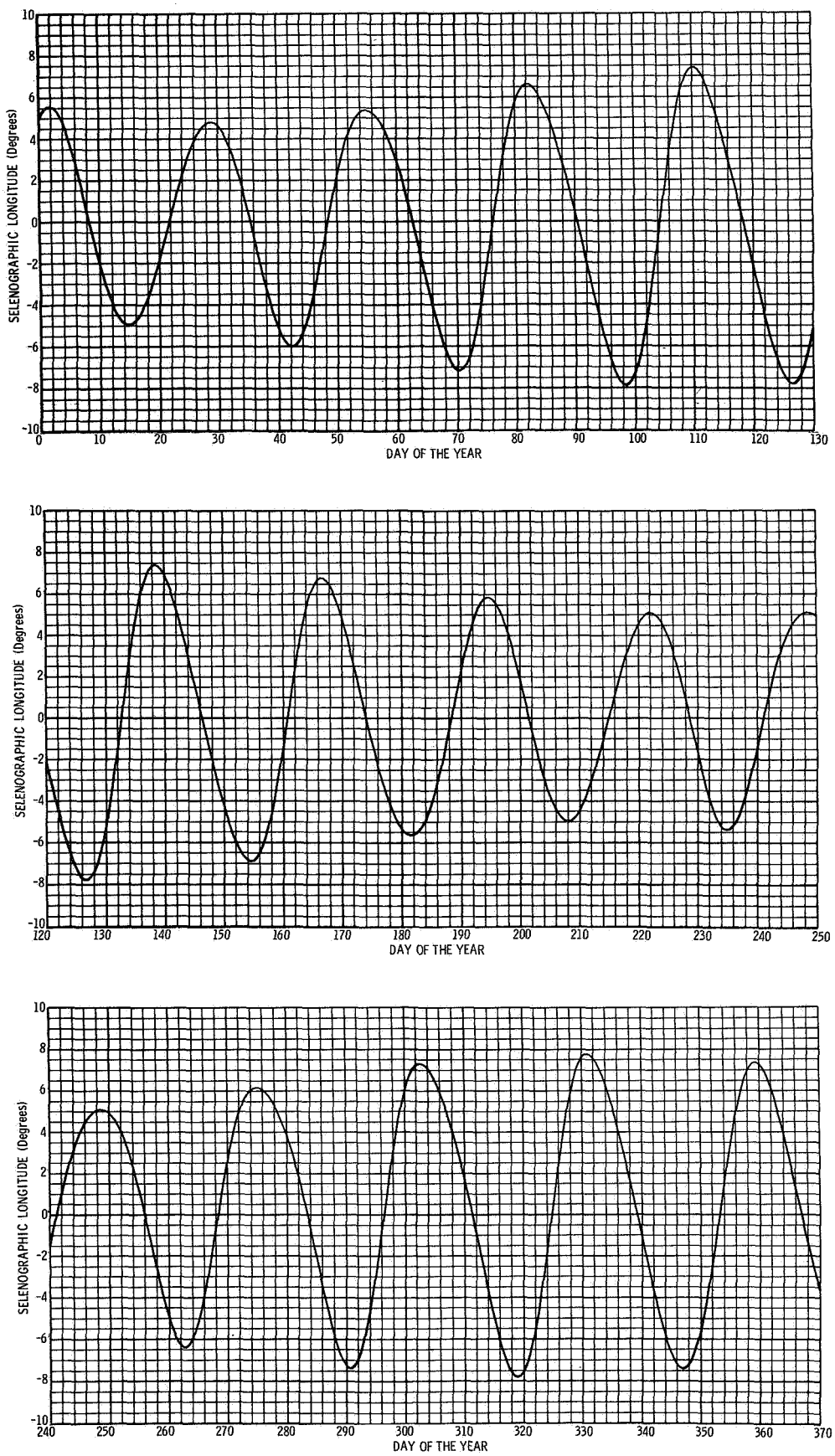


FIGURE B1972-15 SELENOGRAPHIC LONGITUDE OF THE EARTH

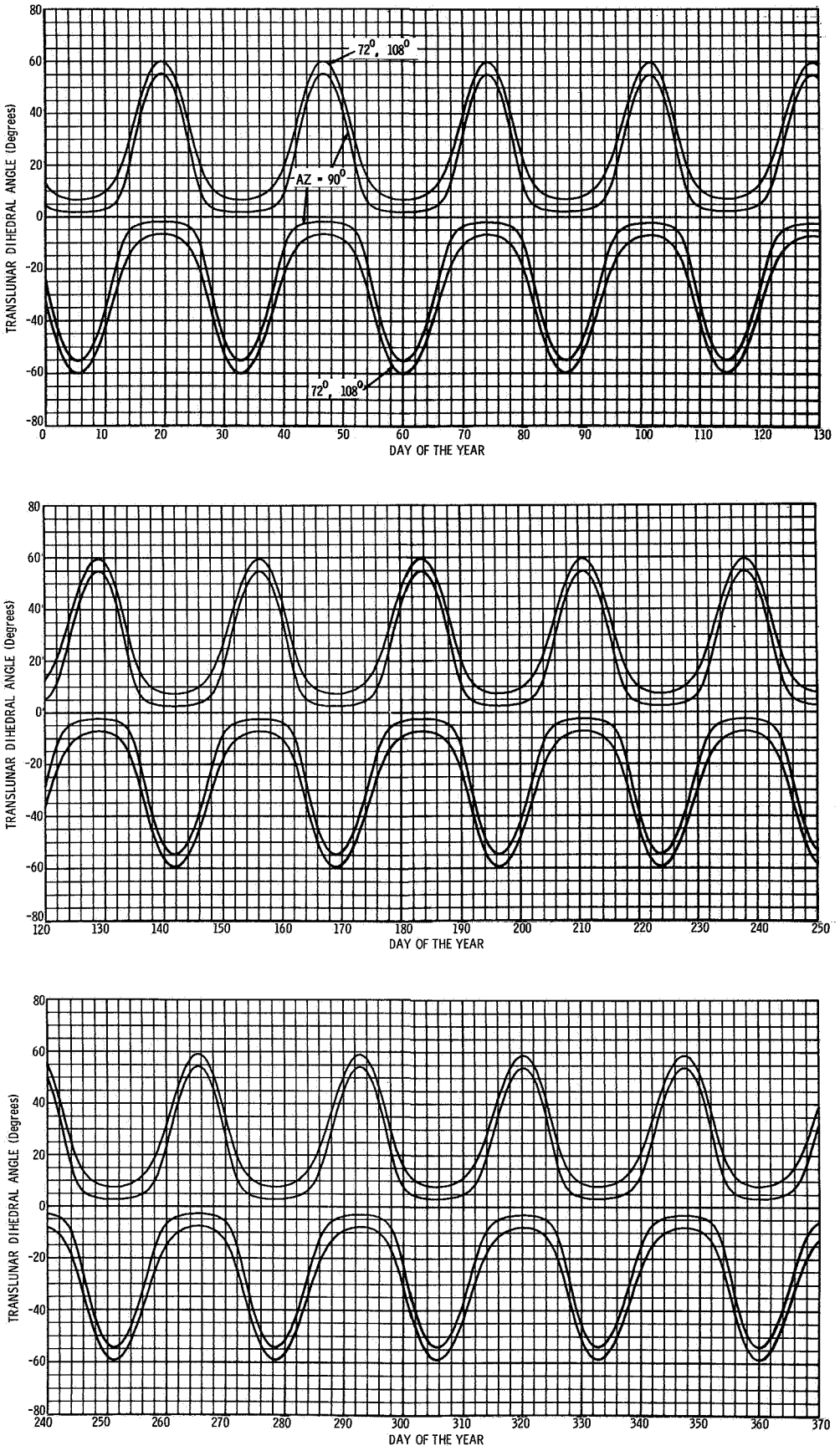
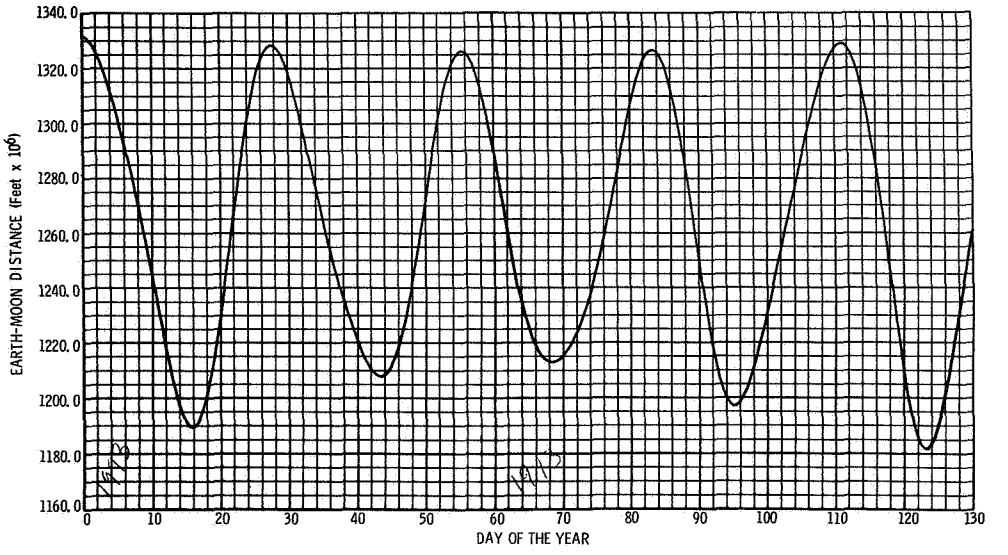
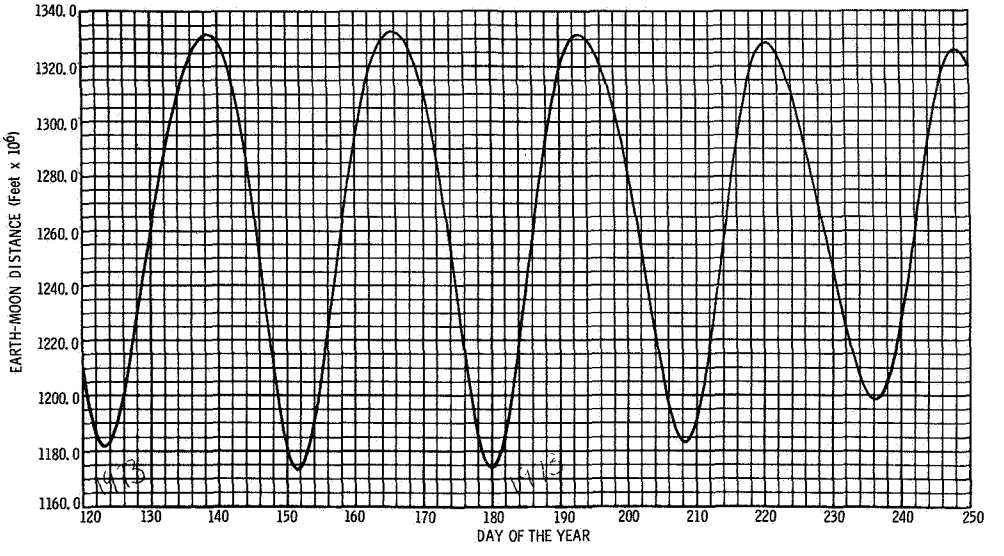


FIGURE B1972-16 TRANSLUNAR DIHEDRAL ANGLES

1973





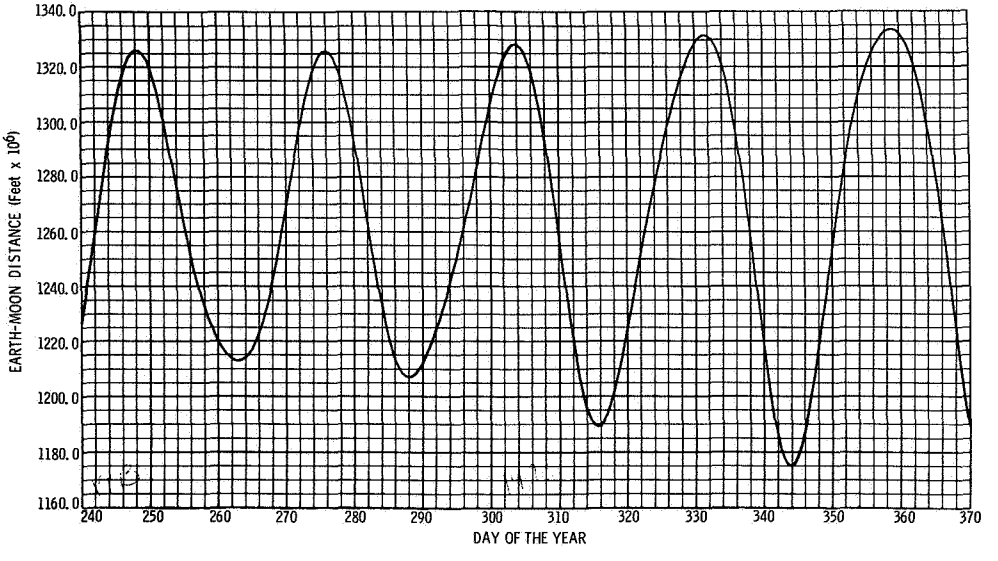
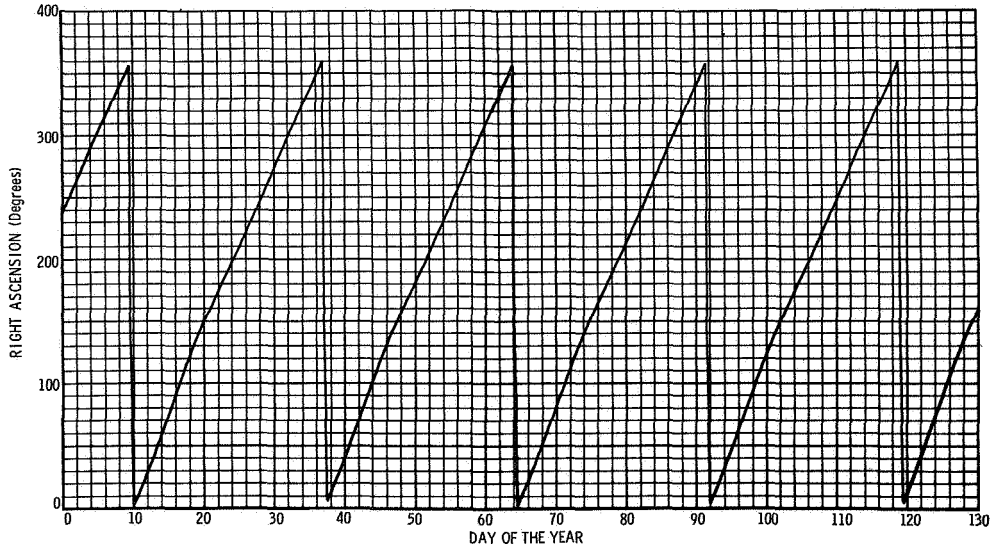
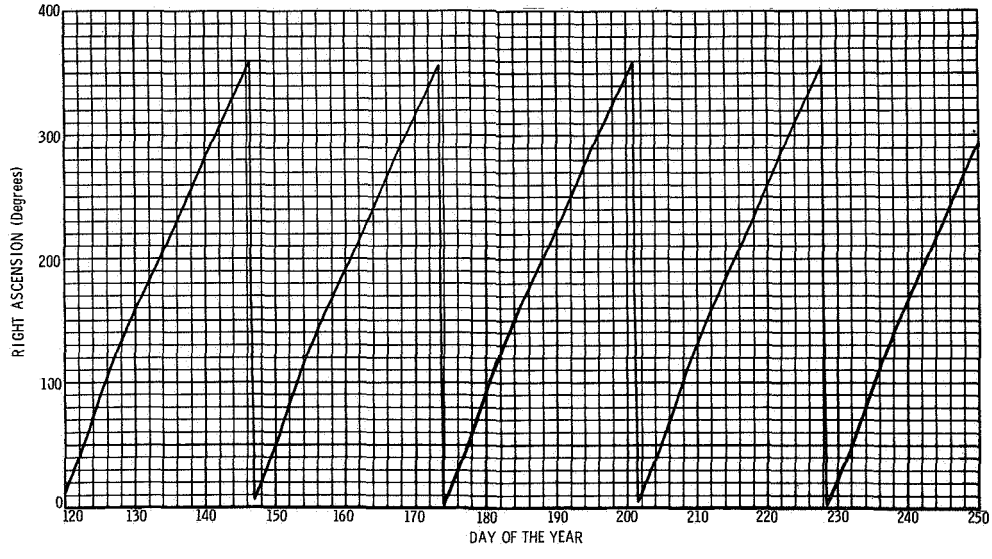


FIGURE B1973-1 EARTH-MOON DISTANCE





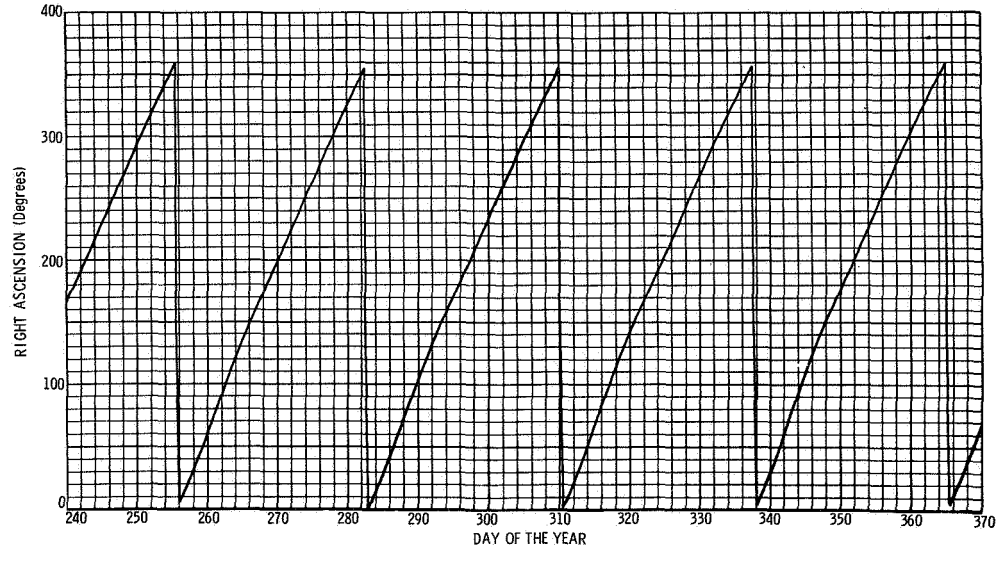
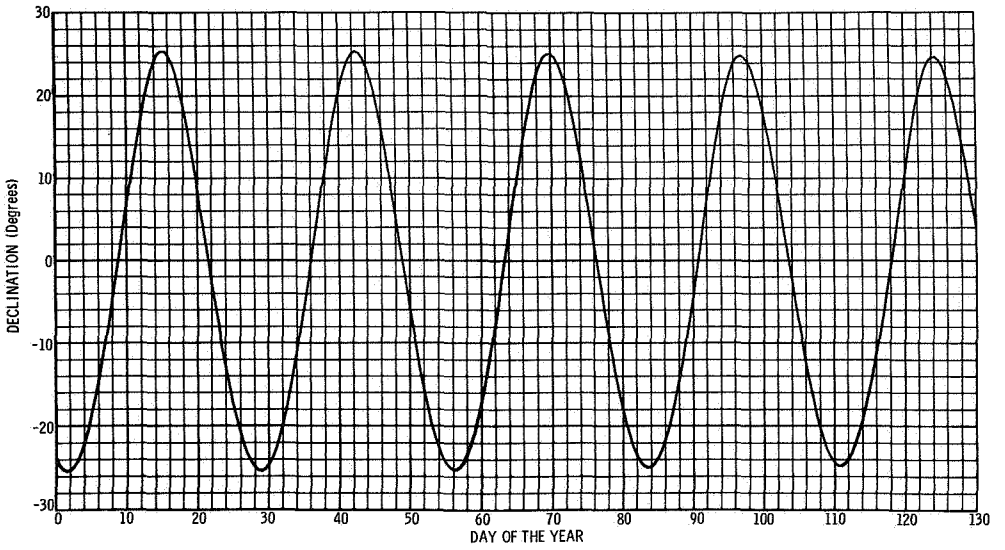
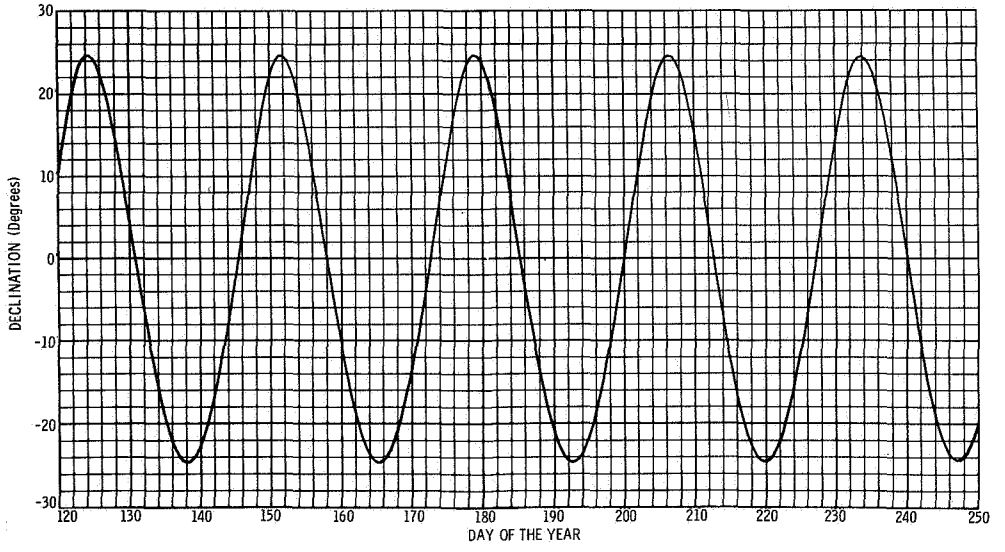


FIGURE B1973-2 RIGHT ASCENSION OF THE MOON





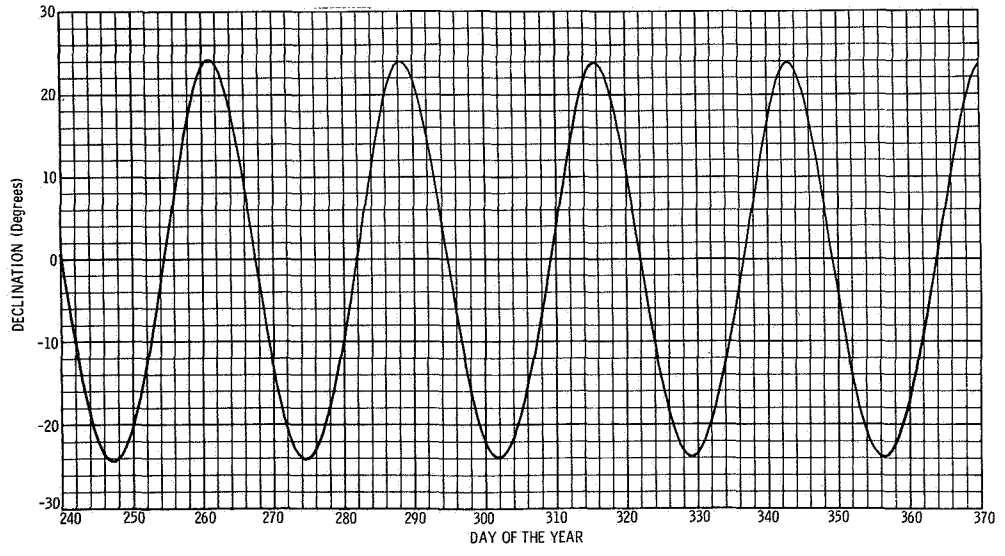
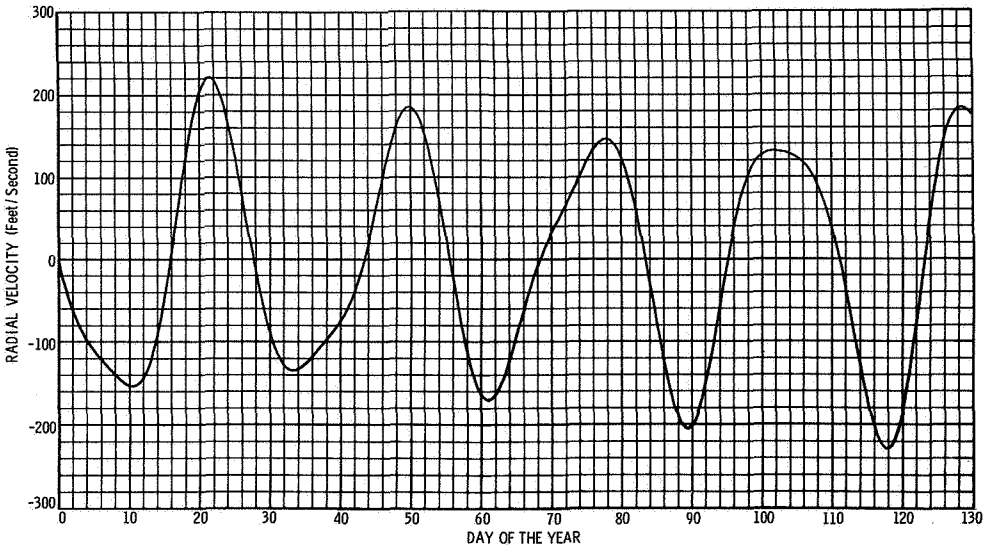
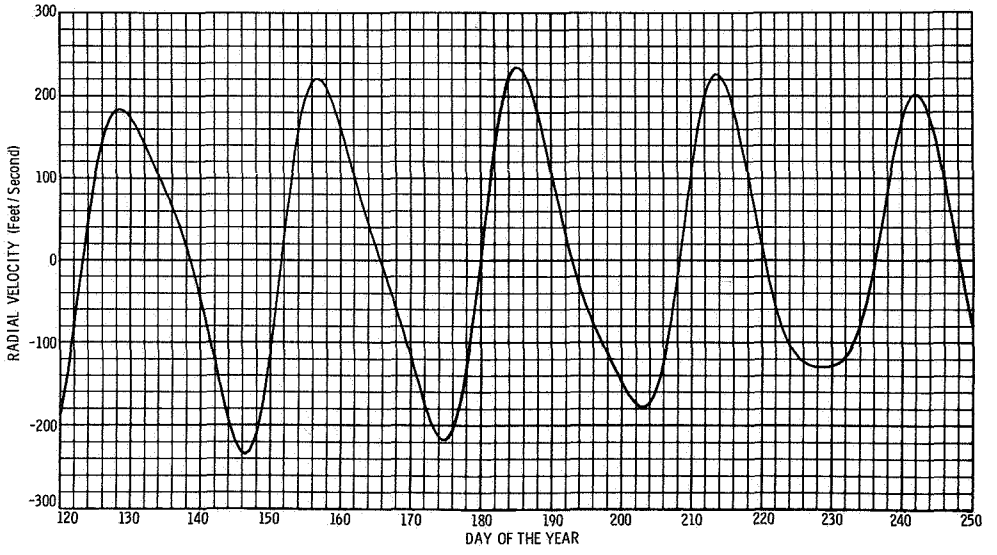


FIGURE B1973-3 DECLINATION OF THE MOON





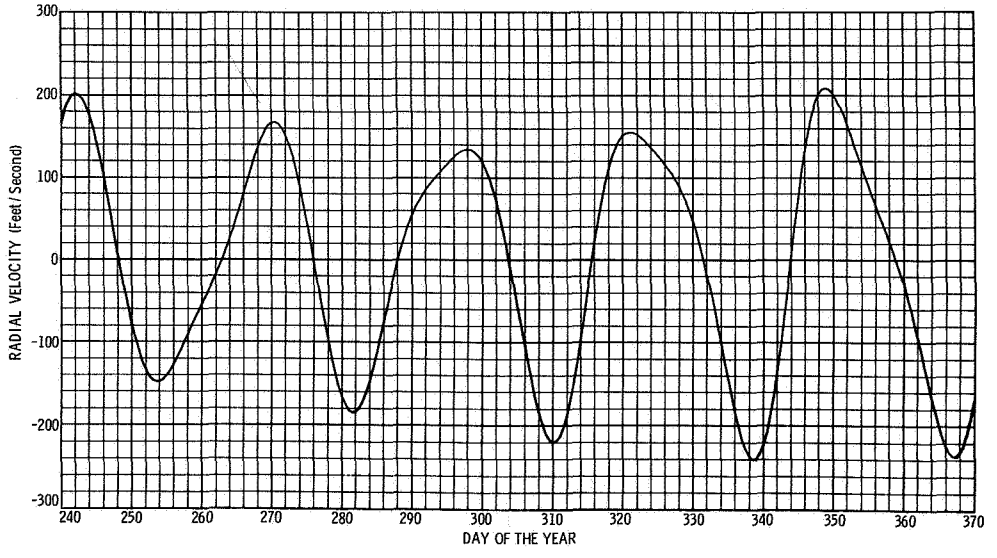
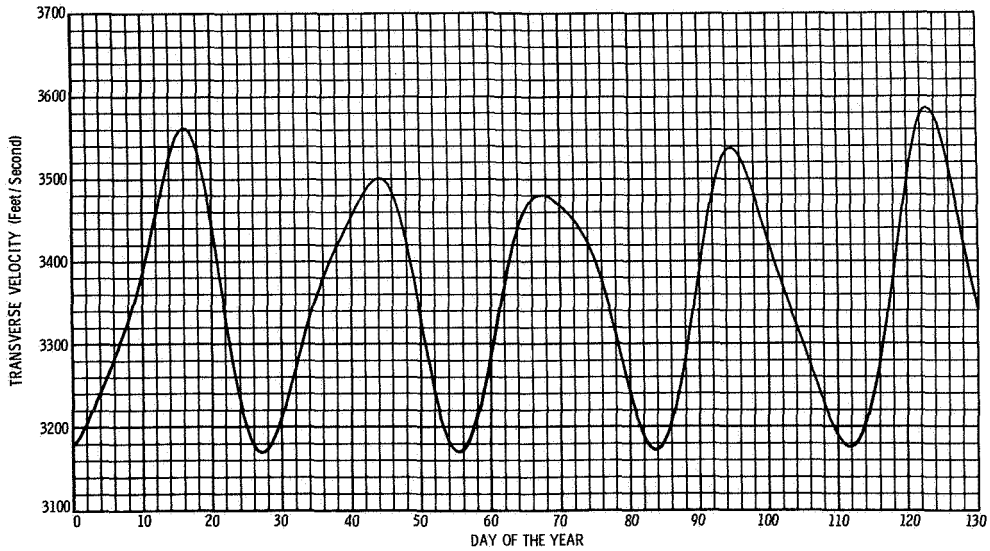
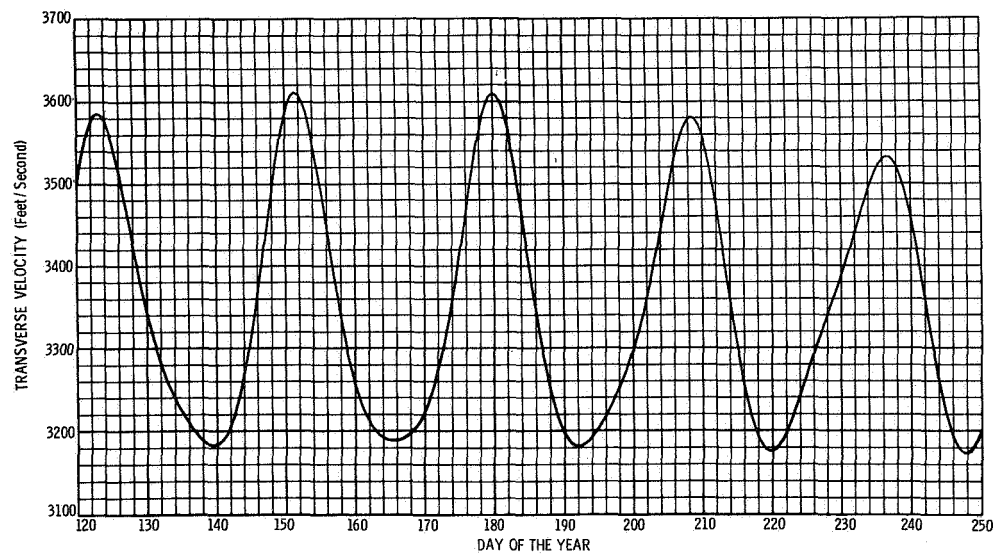


FIGURE B1973-4 RADIAL VELOCITY OF THE MOON





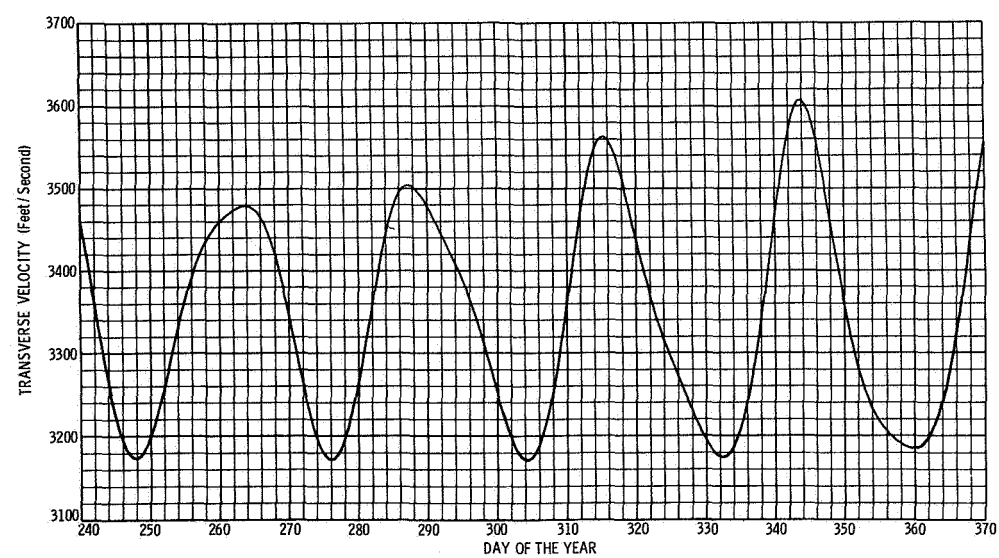


FIGURE B1973-5 TRANSVERSE VELOCITY OF THE MOON

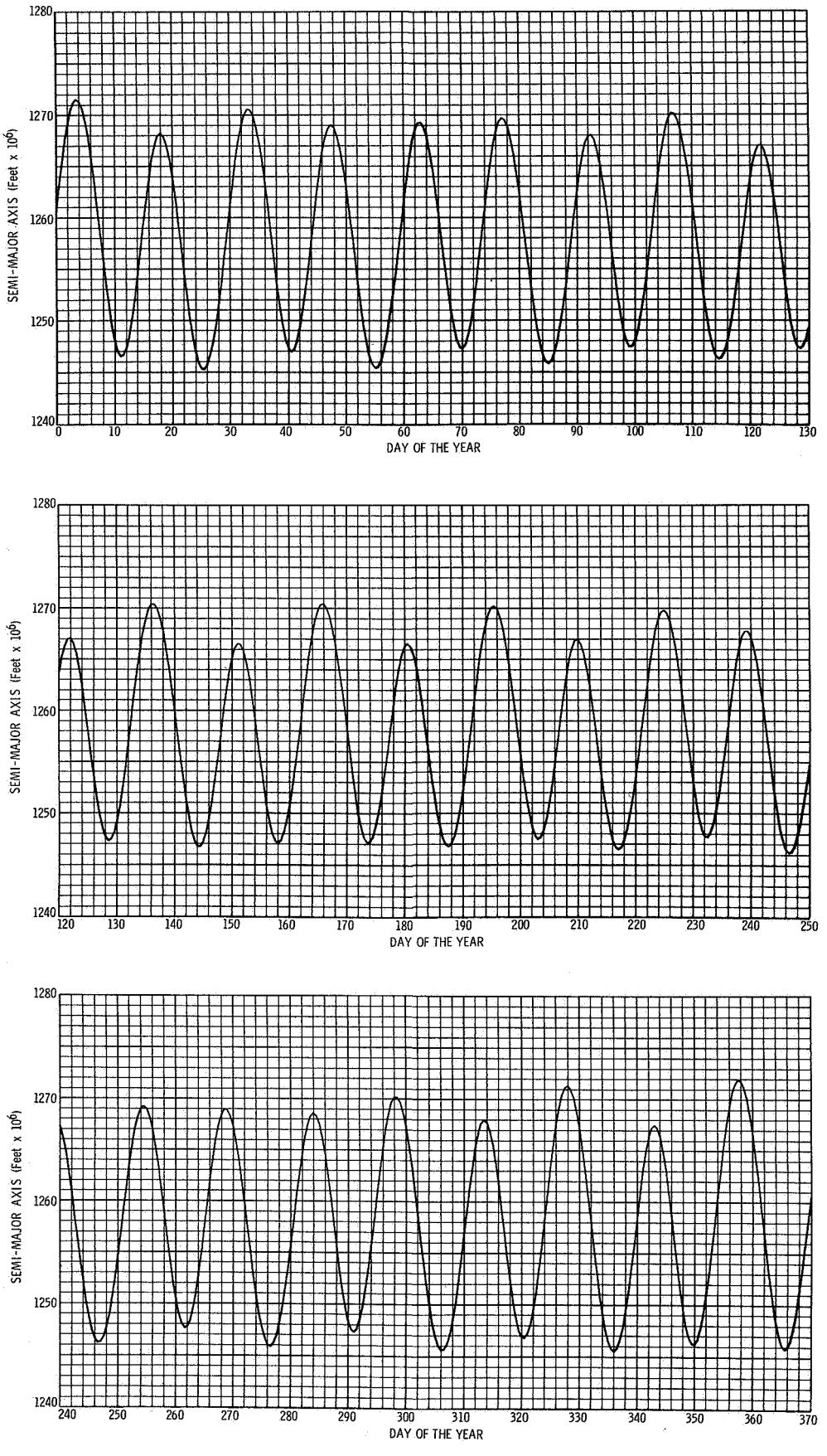
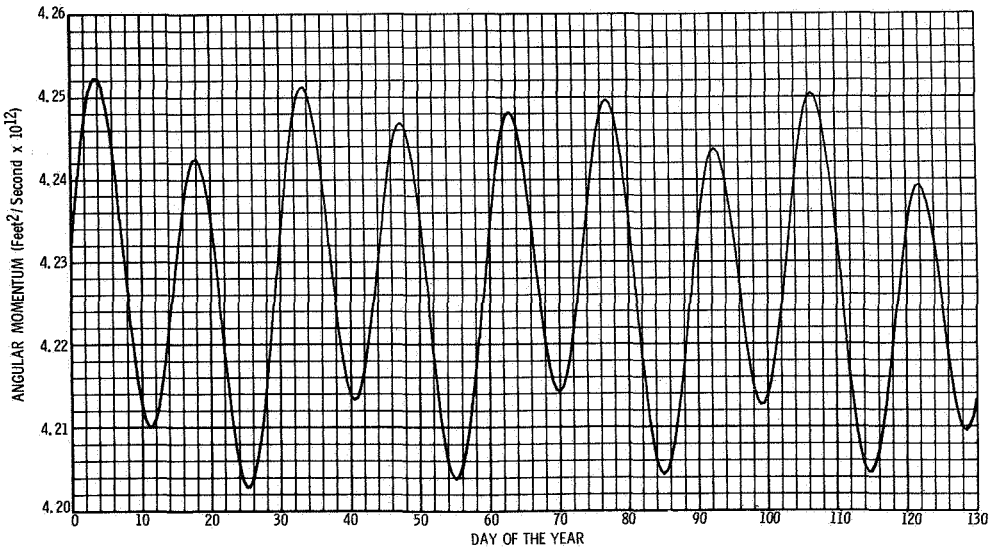
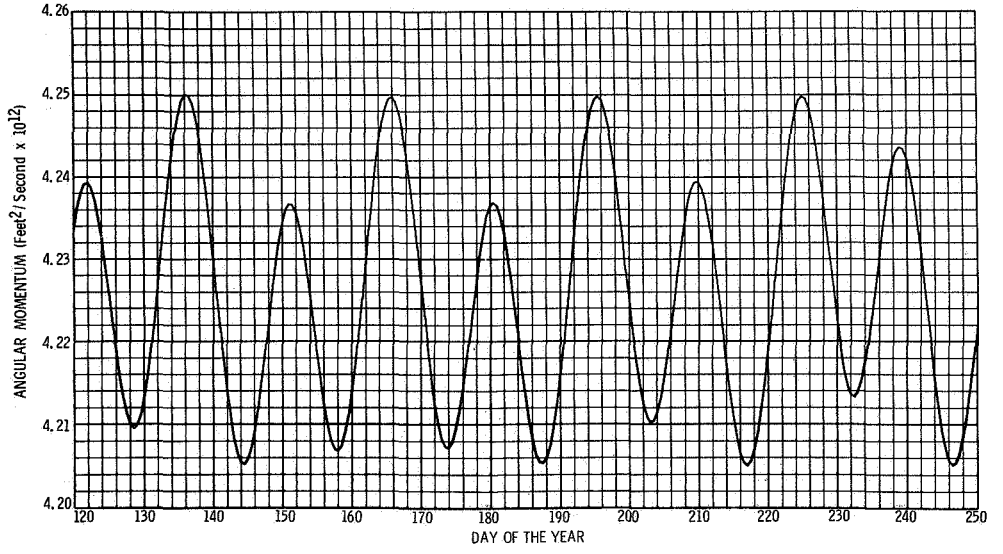


FIGURE B1973-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT





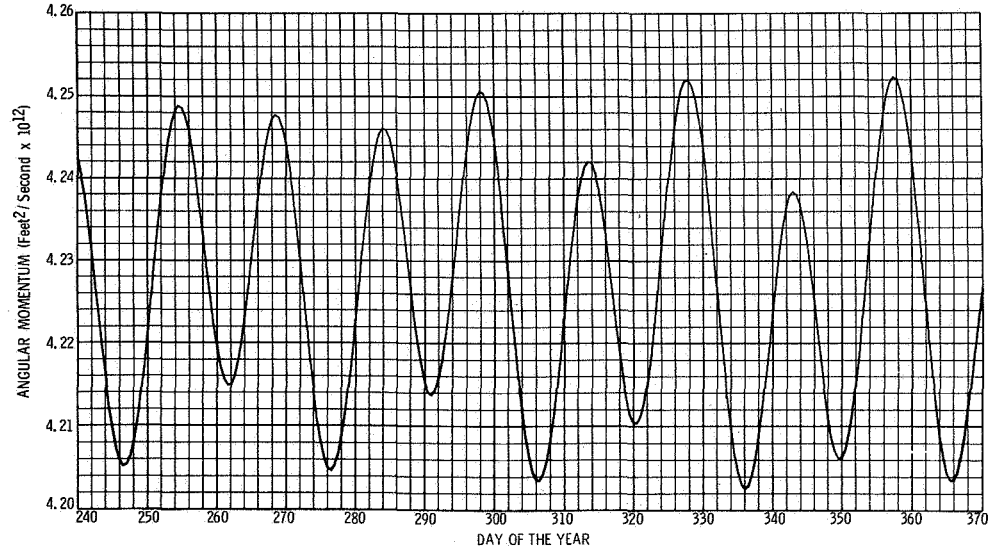
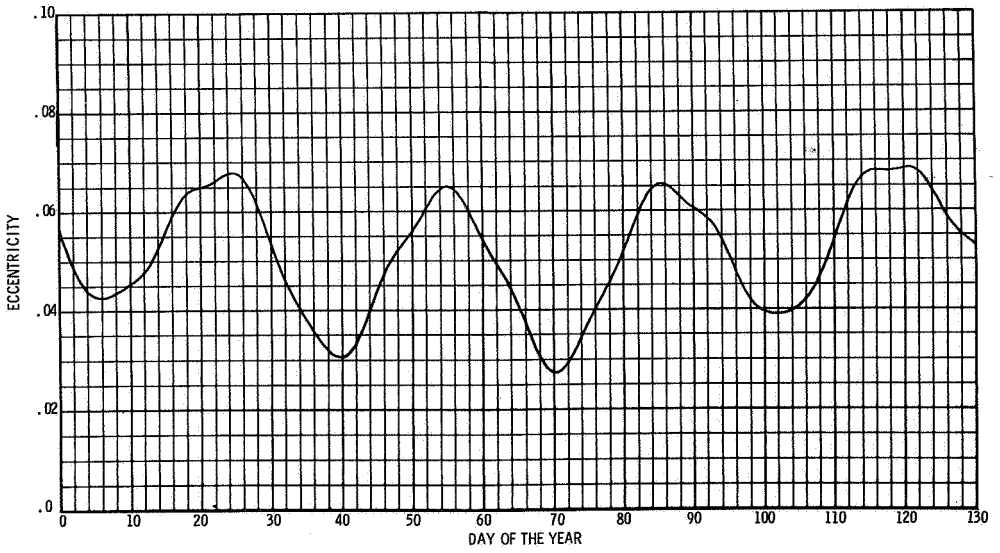
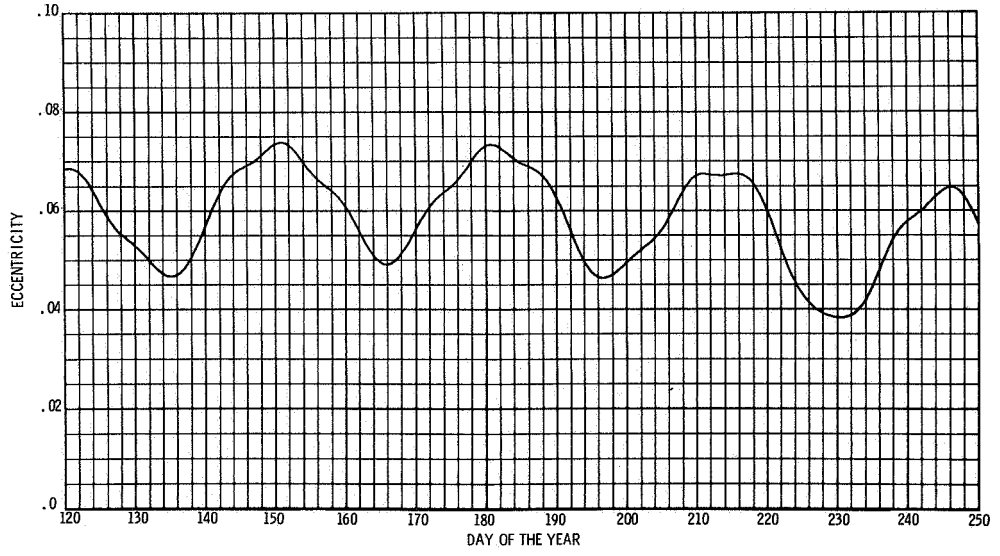


FIGURE B1973-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON





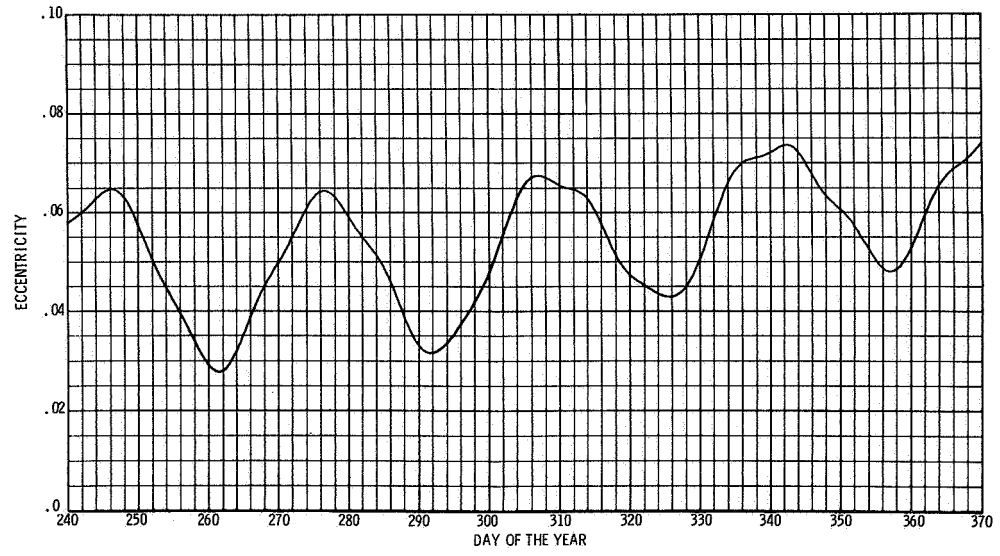
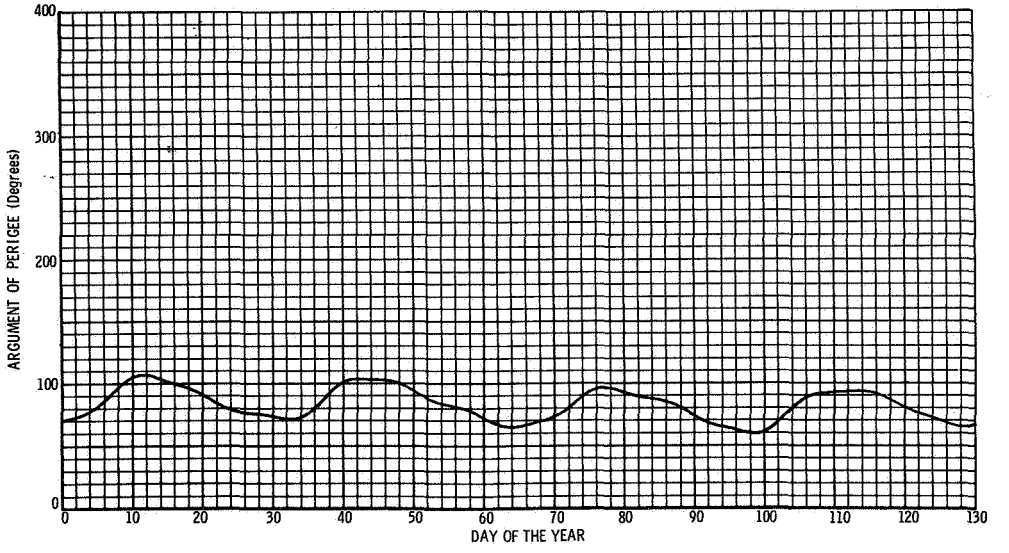
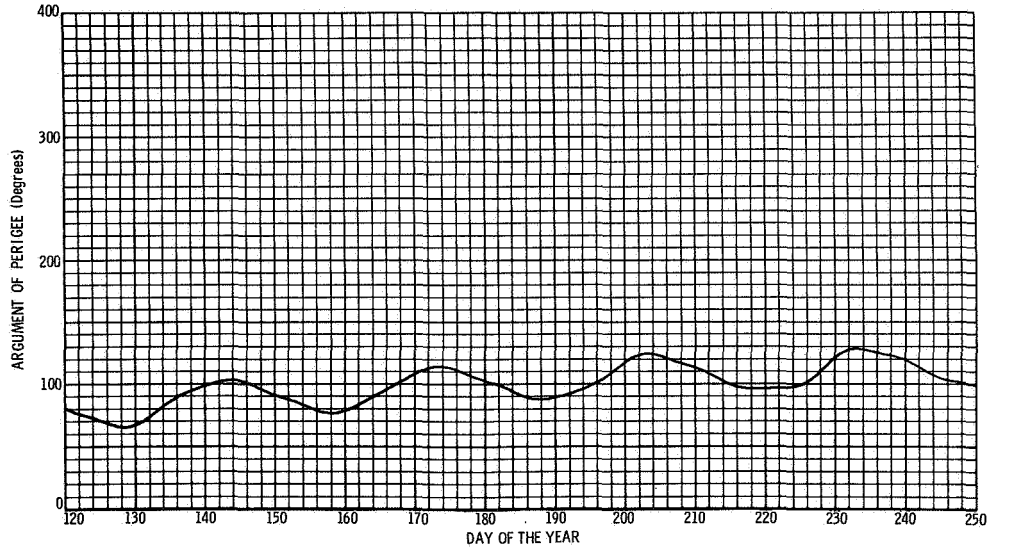


FIGURE B1973-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT





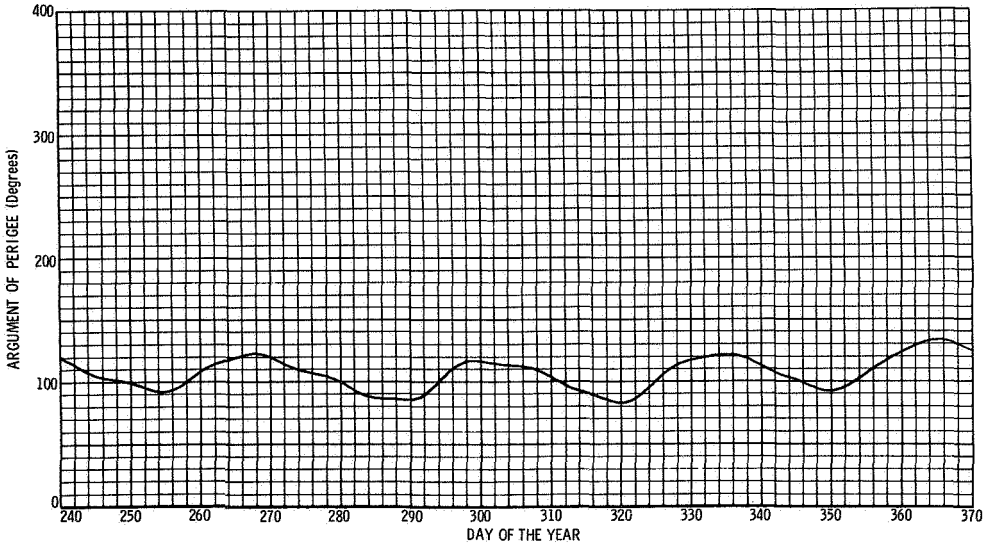
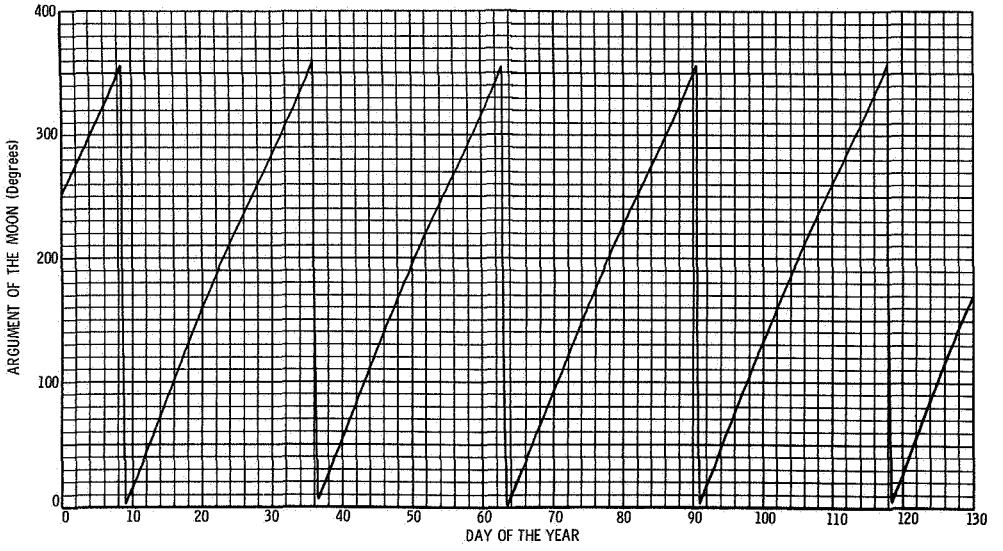
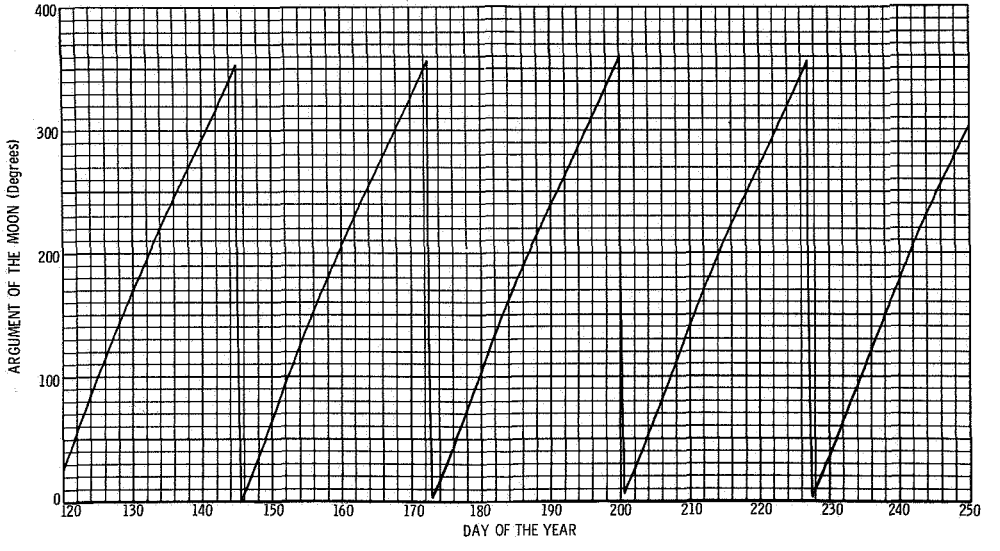


FIGURE B1973-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE





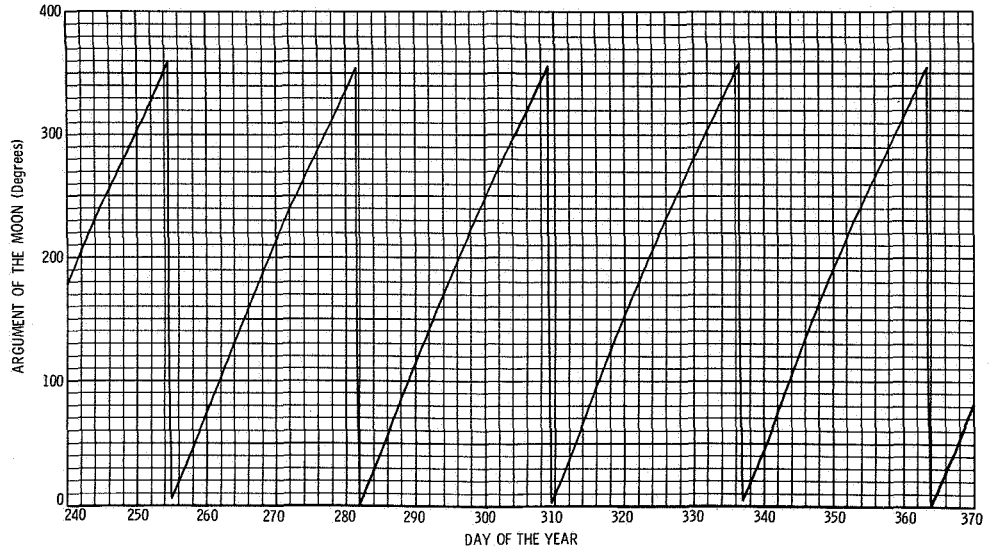


FIGURE B1973-10 ARGUMENT OF THE MOON'S POSITION

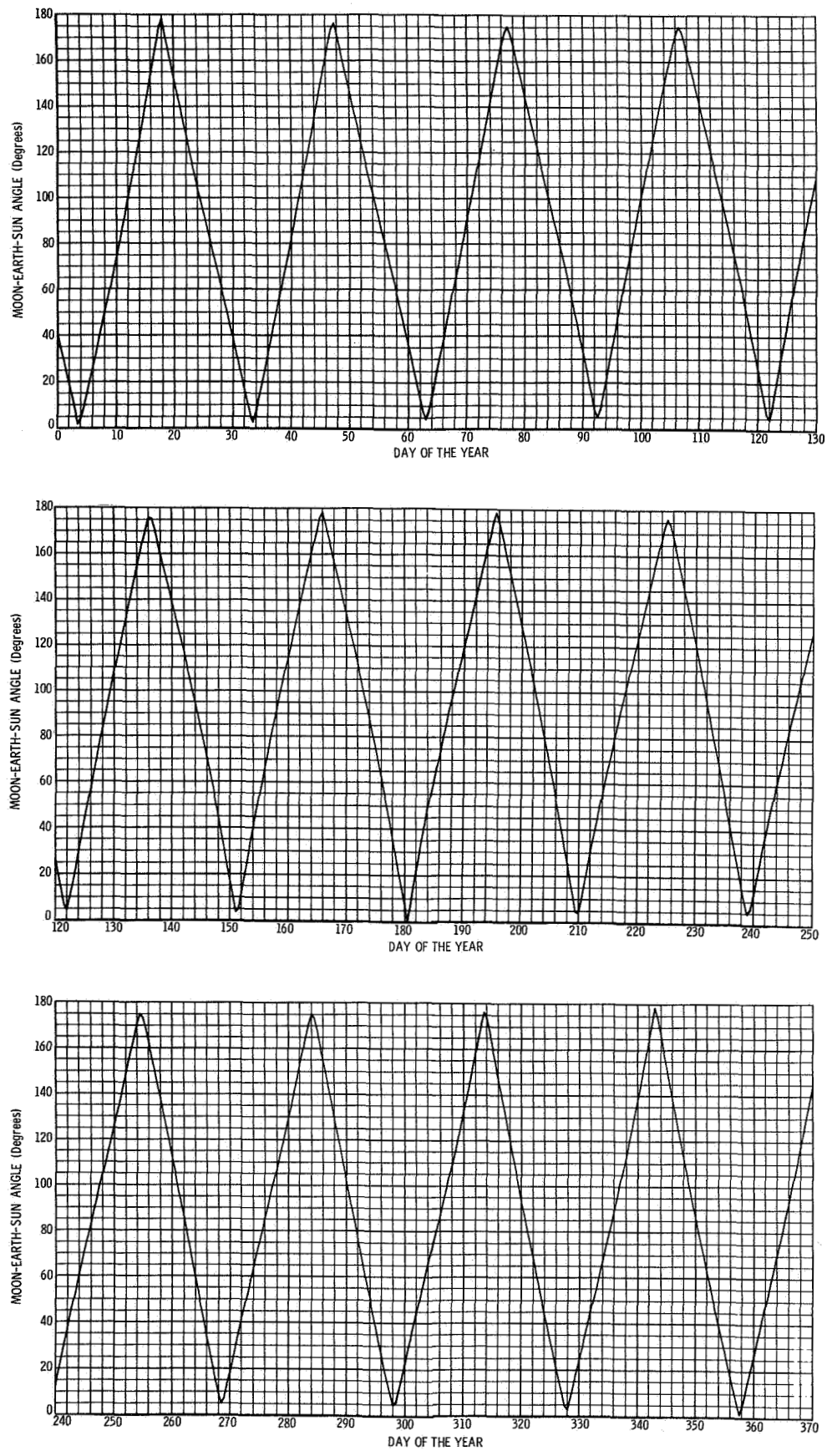
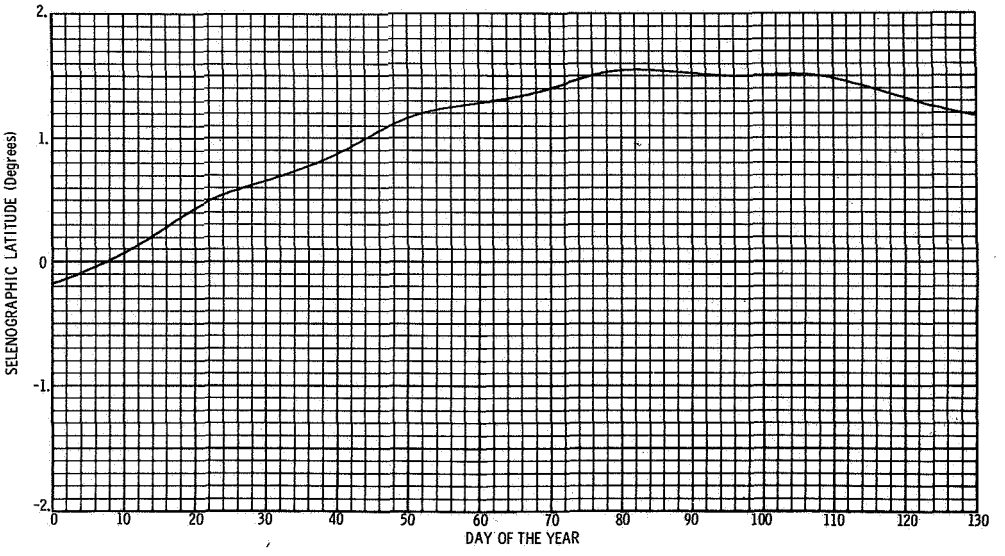
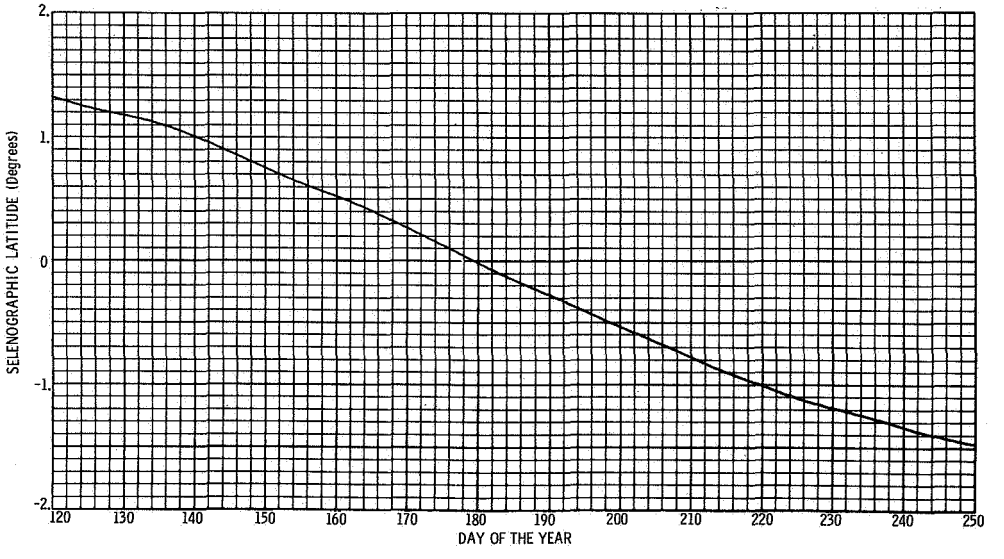


FIGURE B1973-11 MOON-EARTH-SUN ANGLE





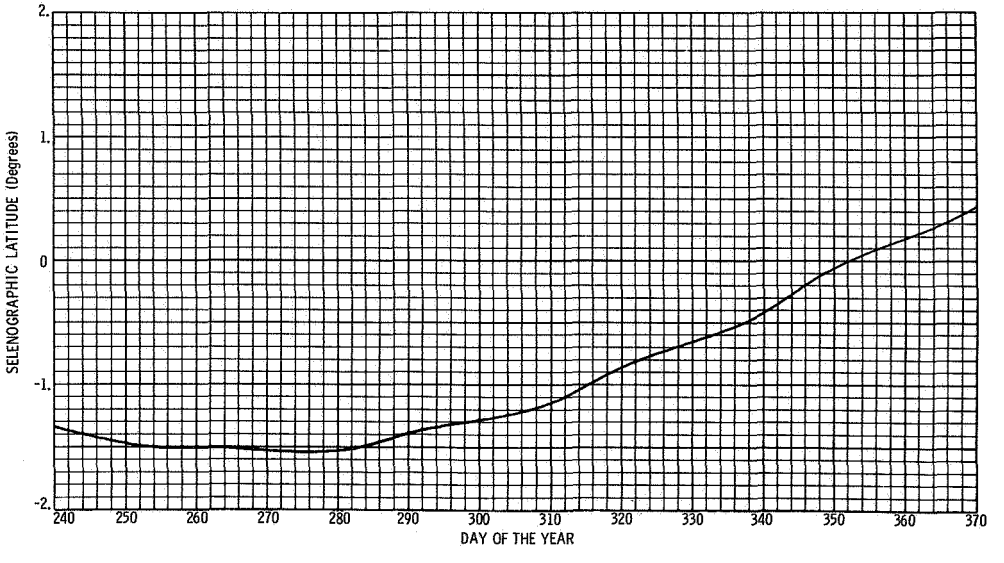
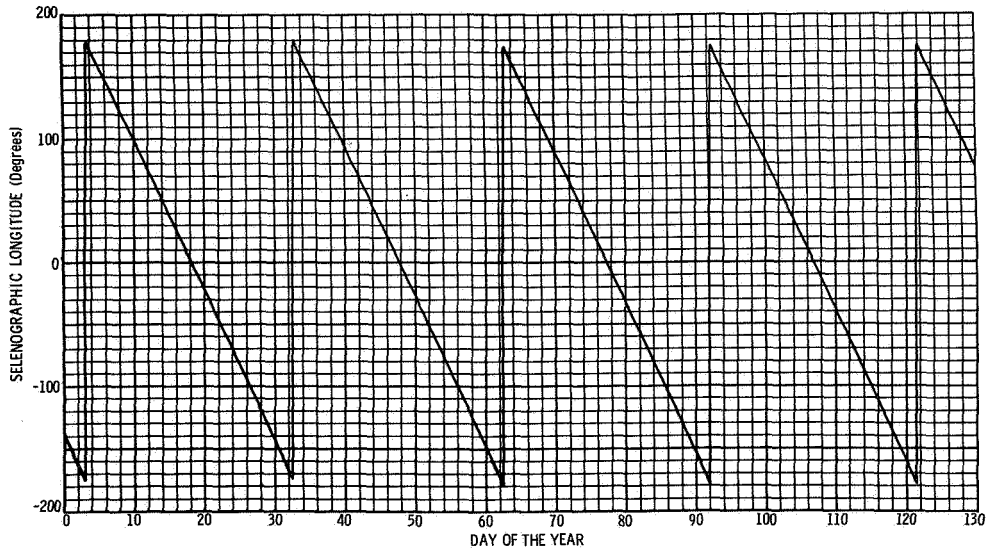
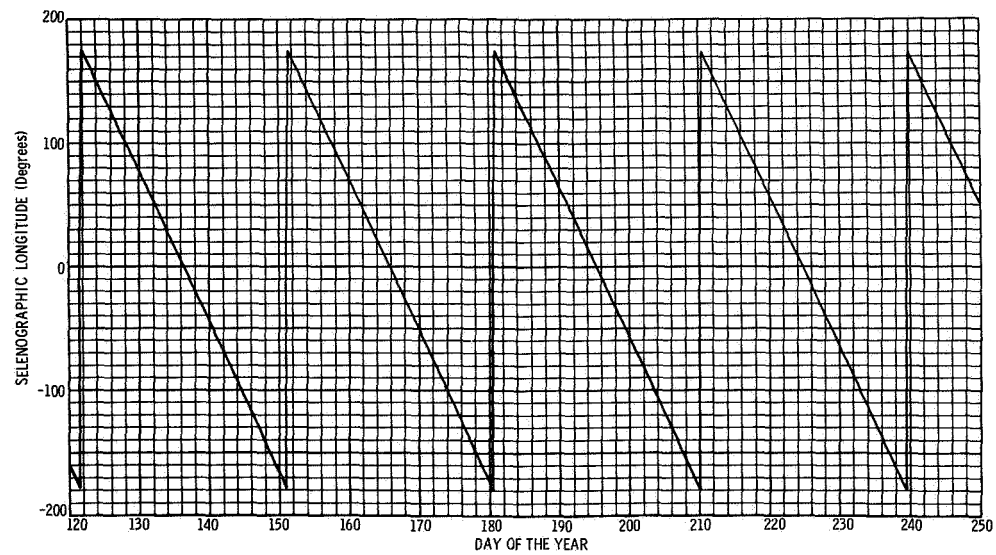


FIGURE B1973-12 SELENOGRAPHIC LATITUDE OF THE SUN





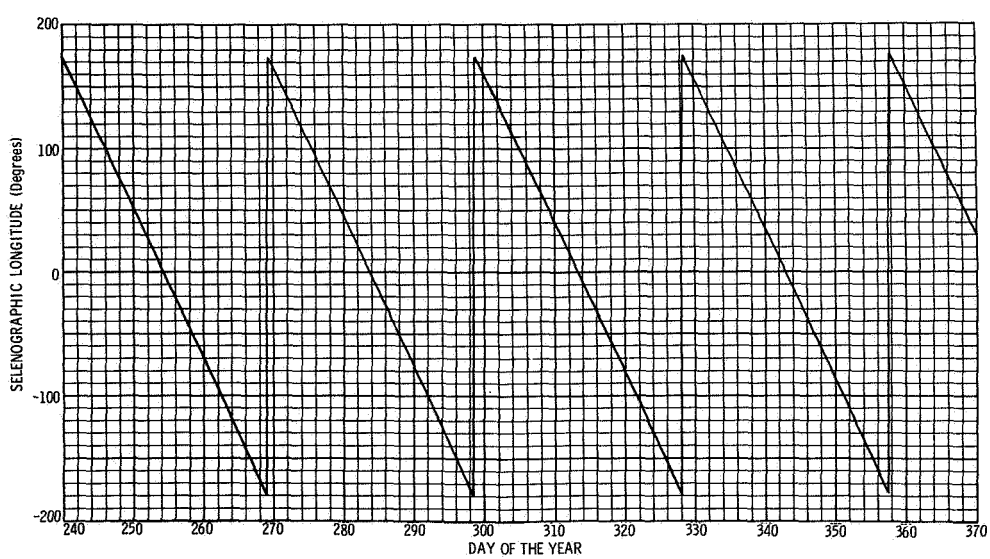
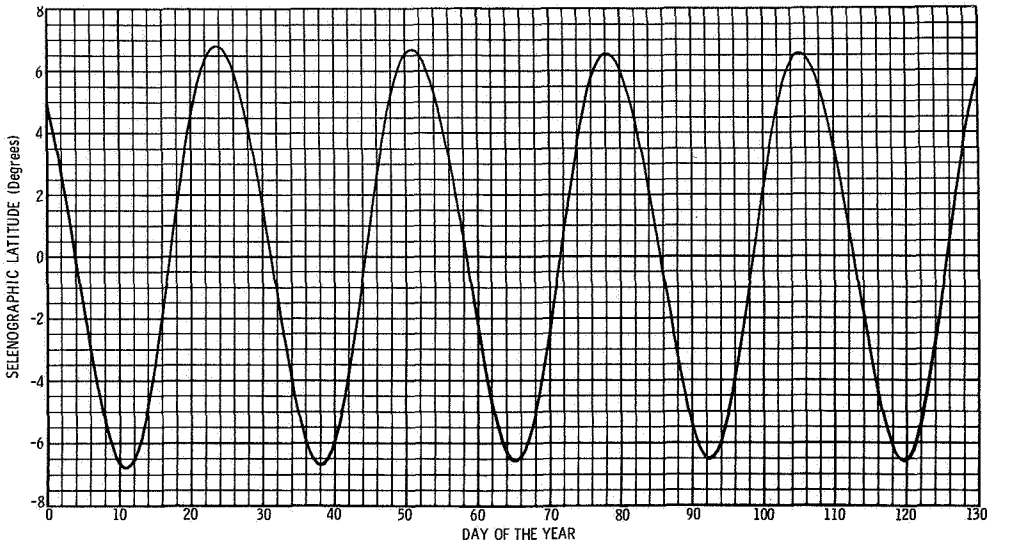
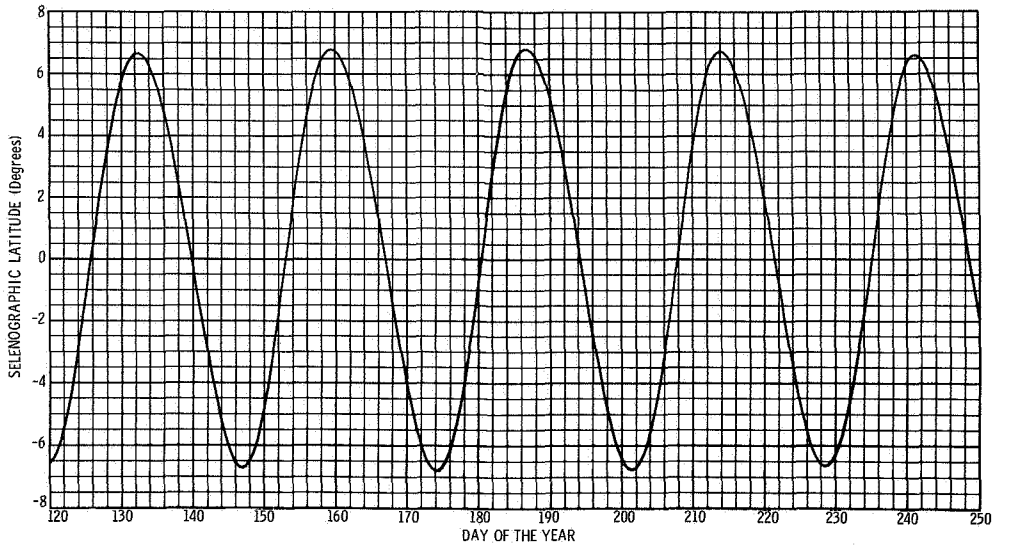


FIGURE B1973-13 SELENOGRAPHIC LONGITUDE OF THE SUN





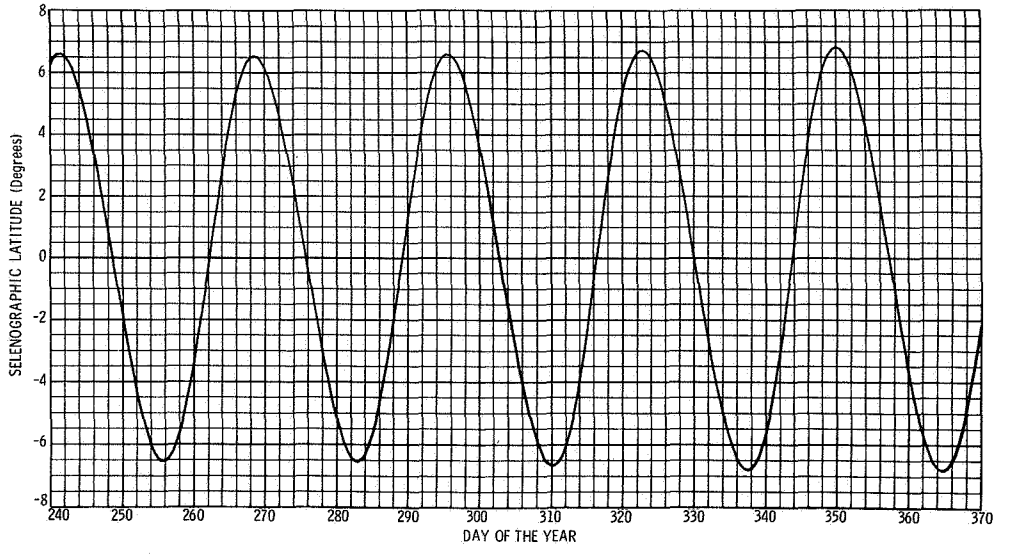


FIGURE B1973-14 SELENOGRAPHIC LATITUDE OF THE EARTH

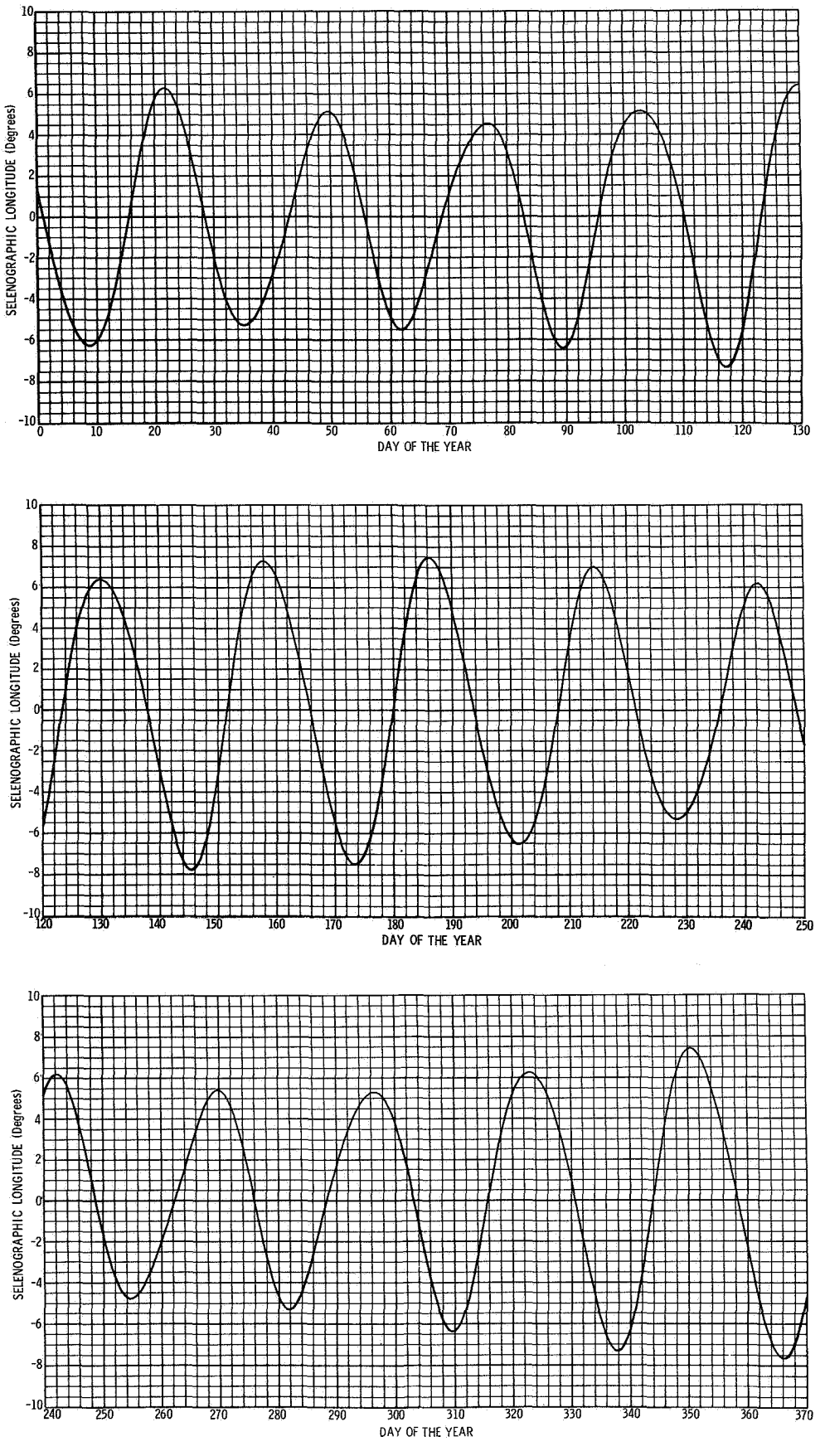
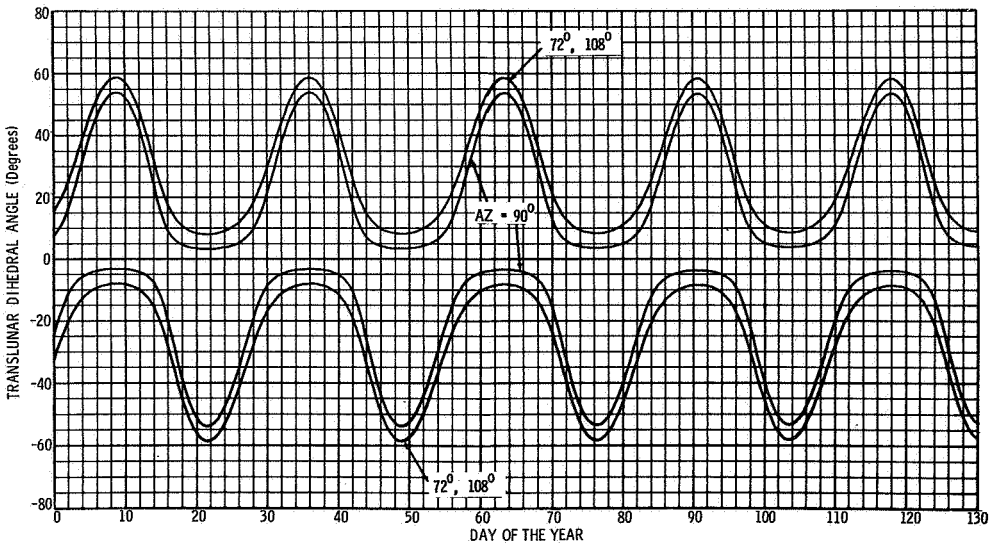
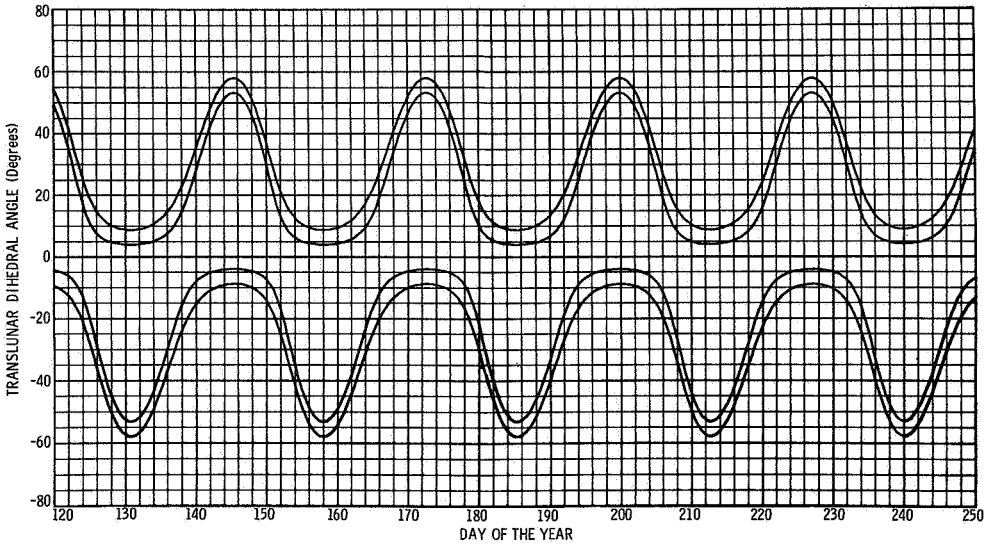


FIGURE B1973-15 SELENOGRAPHIC LONGITUDE OF THE EARTH





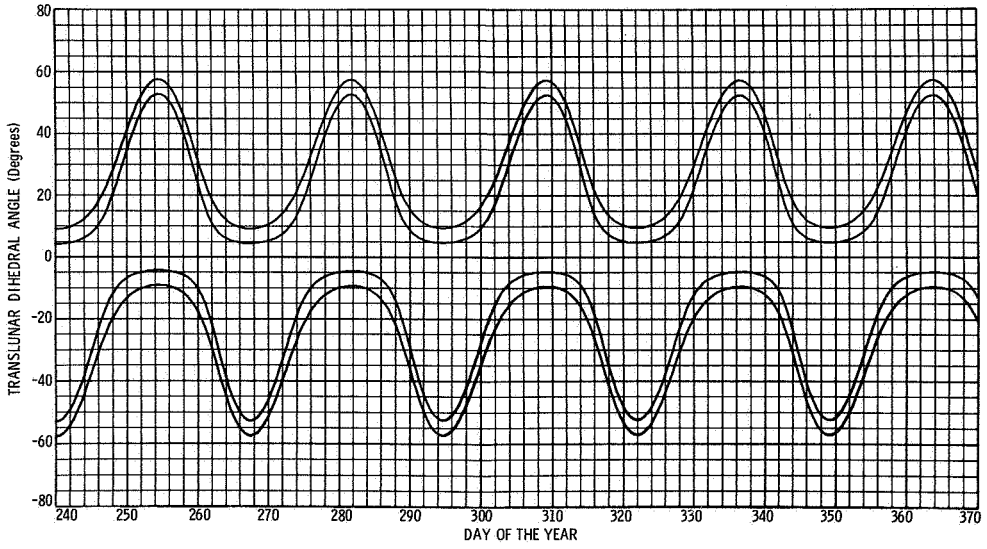
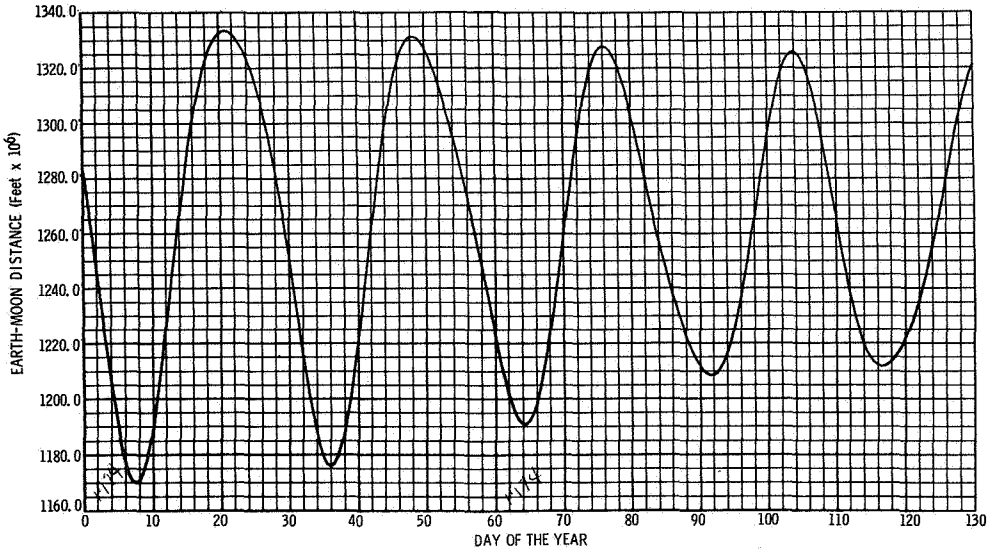
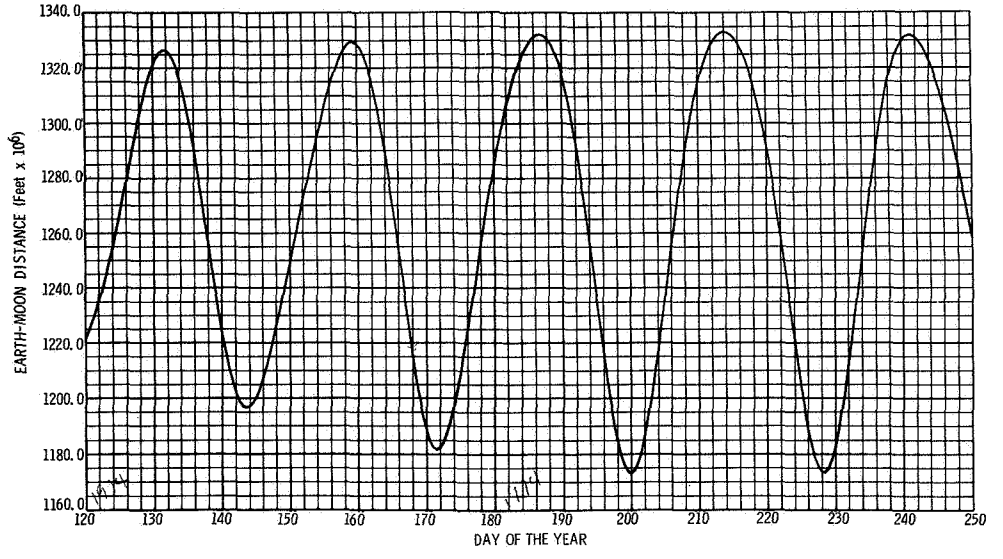


FIGURE B1973-16 TRANSLUNAR DIHEDRAL ANGLES

1974





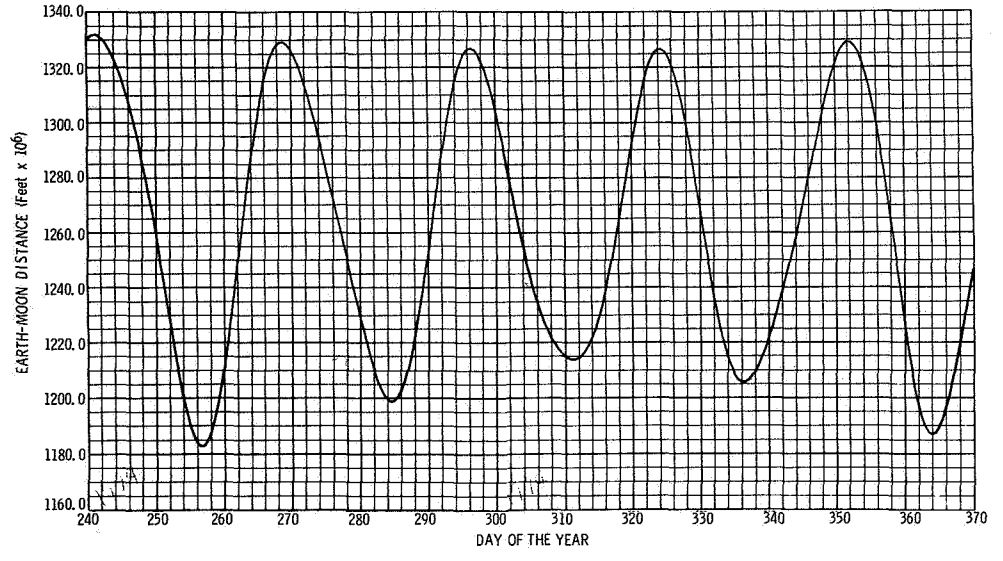
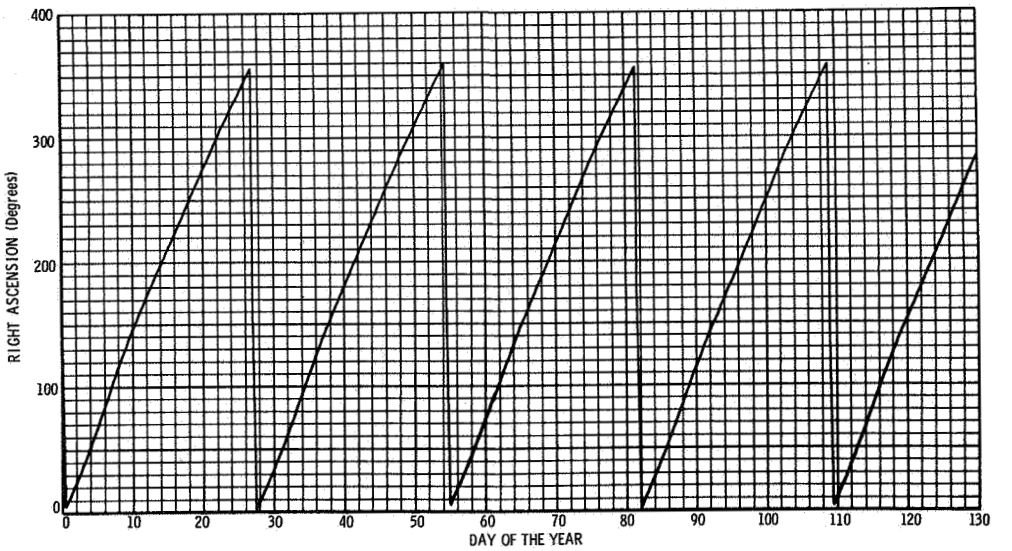
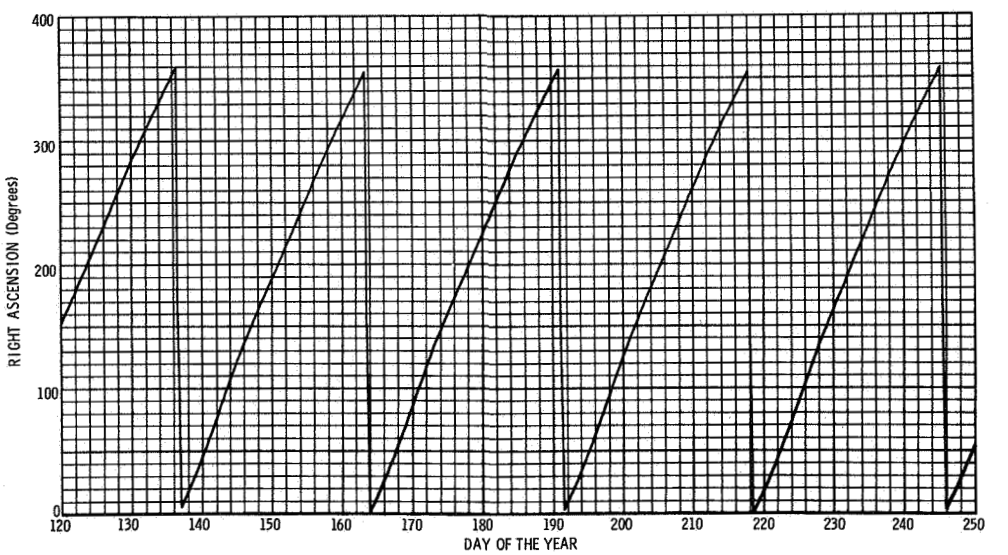


FIGURE B1974-1 EARTH-MOON DISTANCE





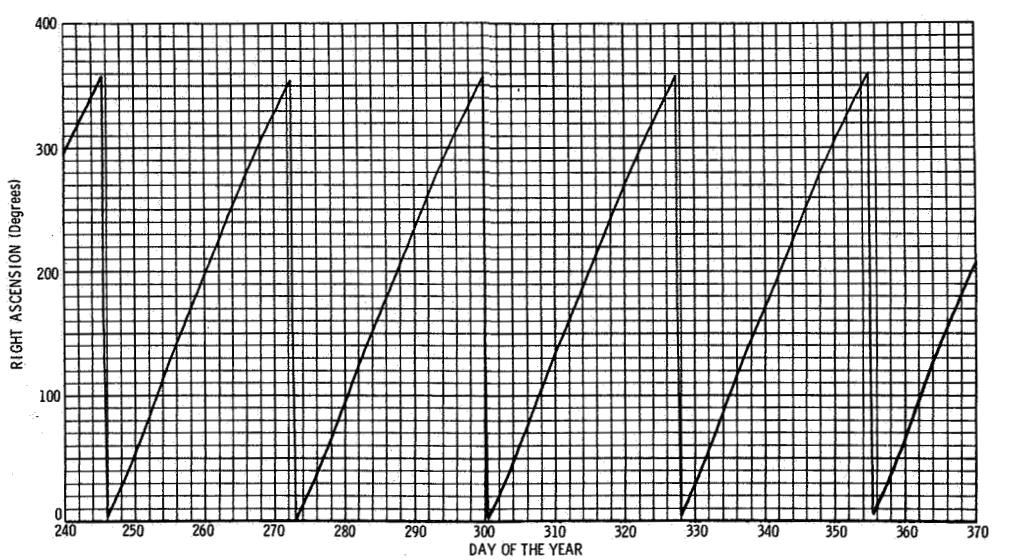
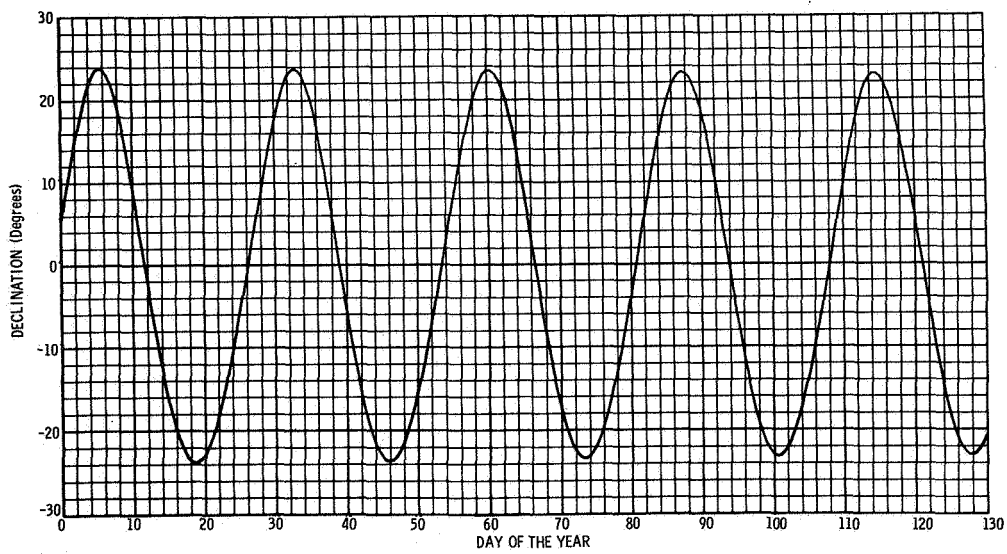
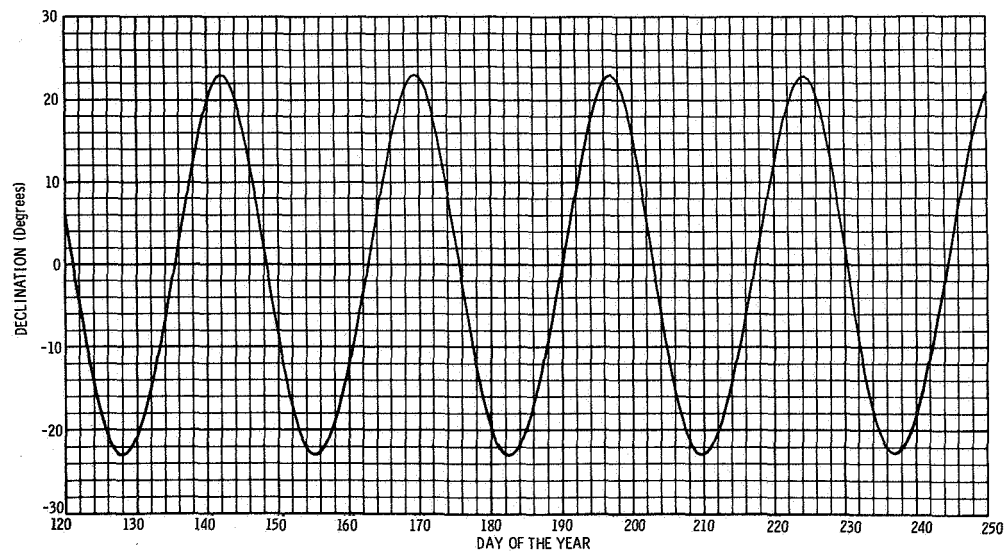


FIGURE B1974-2 RIGHT ASCENSION OF THE MOON





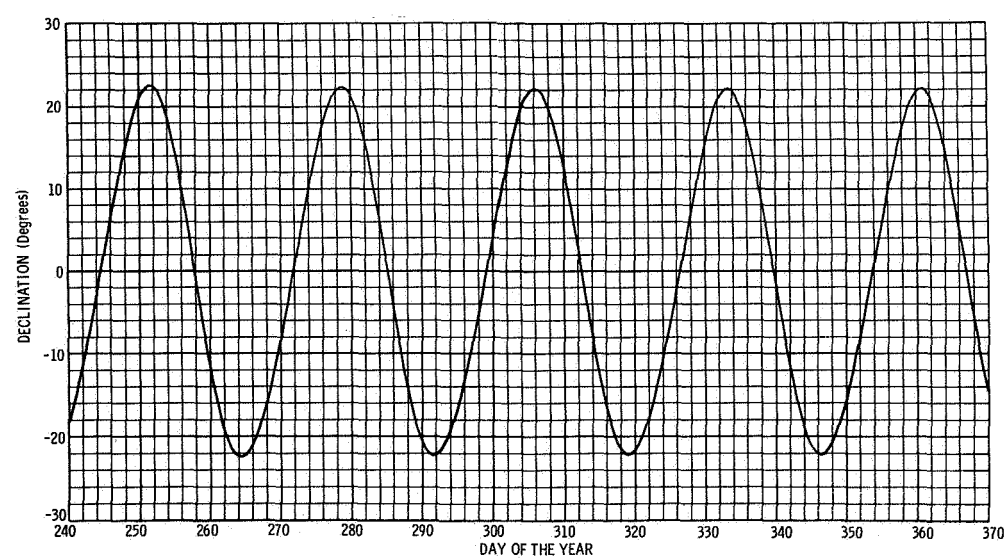
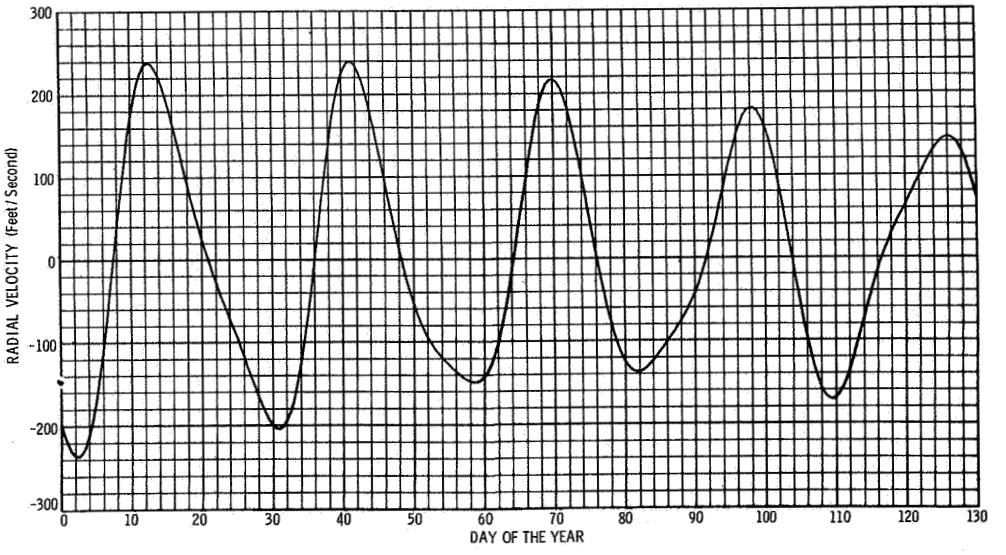
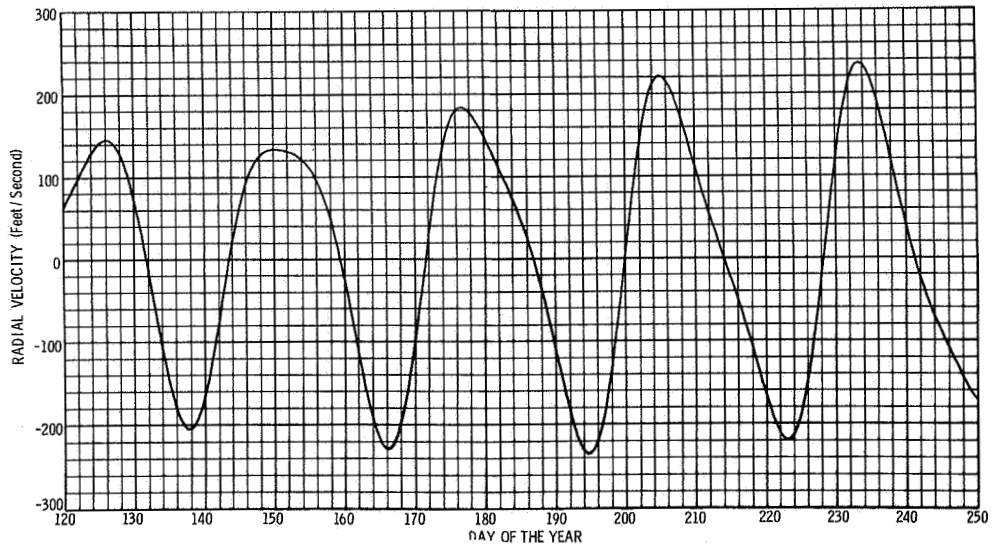


FIGURE B1974-3 DECLINATION OF THE MOON





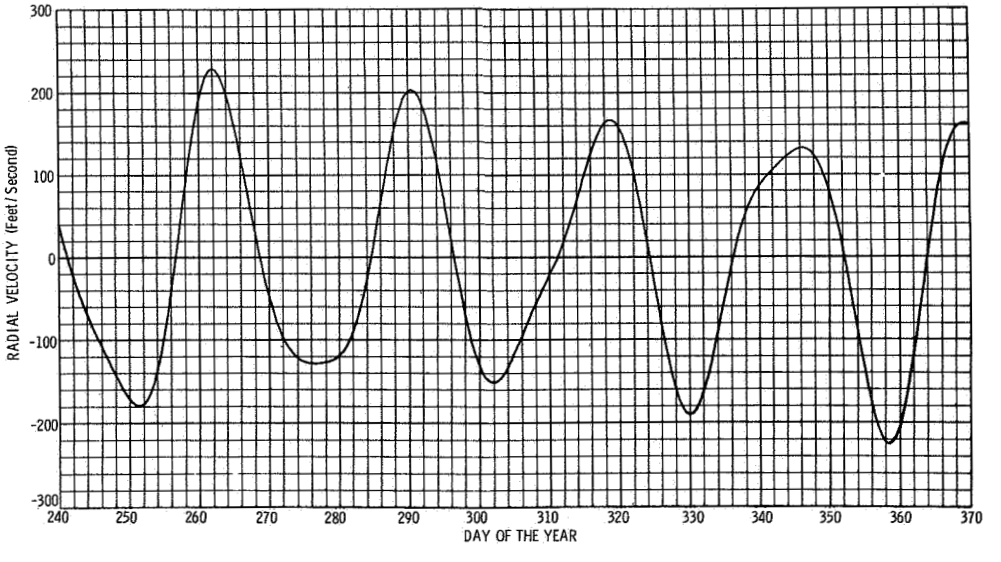
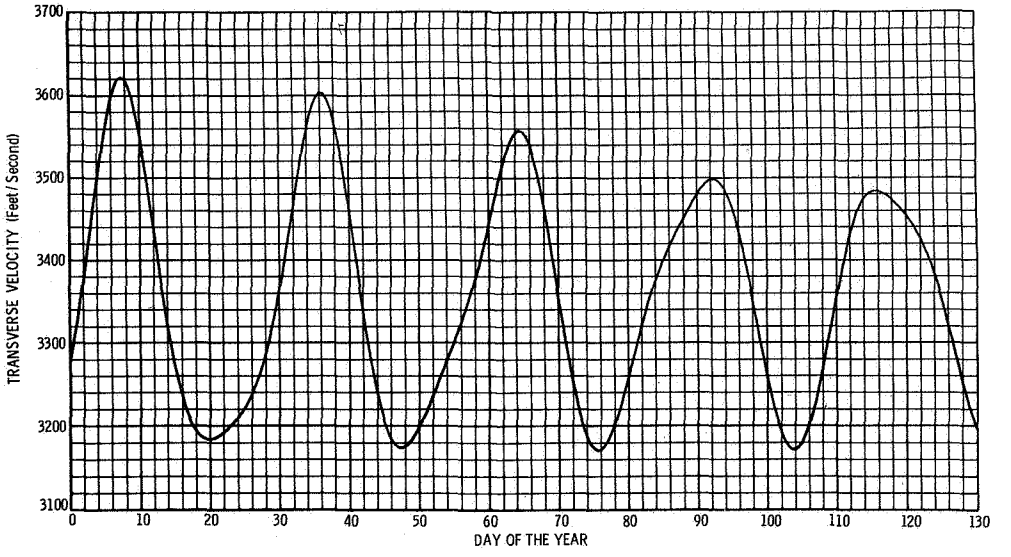
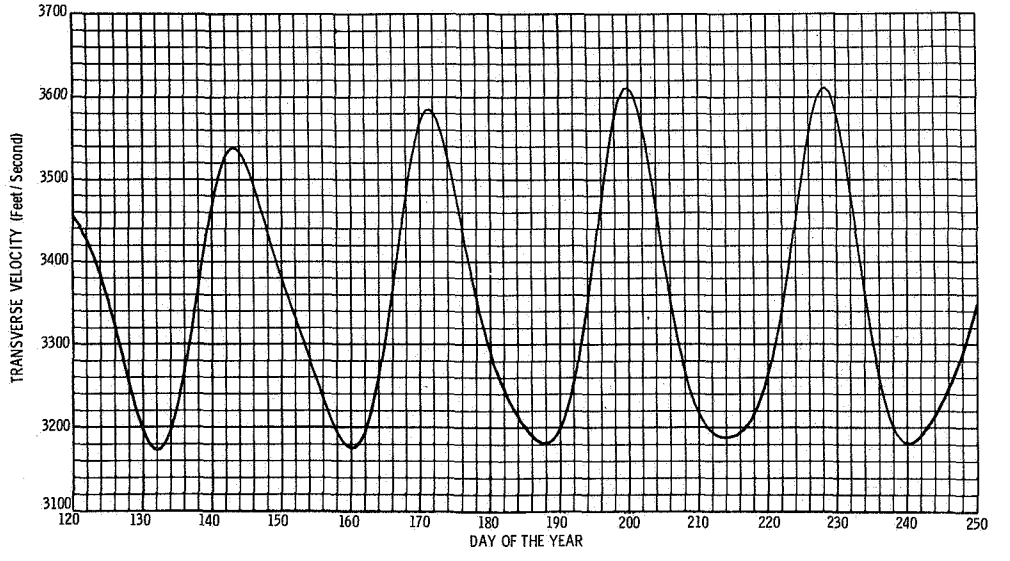


FIGURE B1974-4 RADIAL VELOCITY OF THE MOON





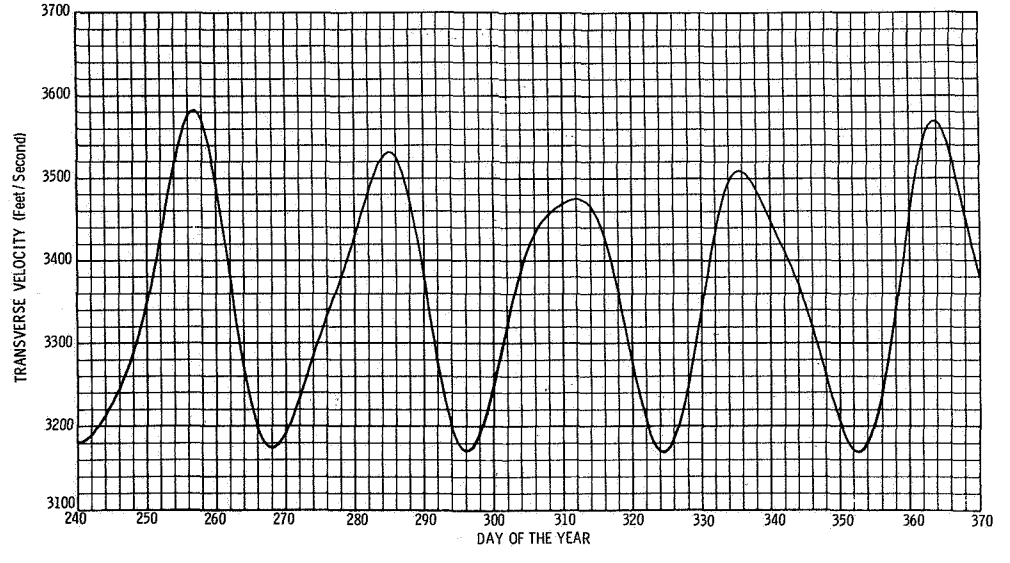


FIGURE B1974-5 TRANSVERSE VELOCITY OF THE MOON

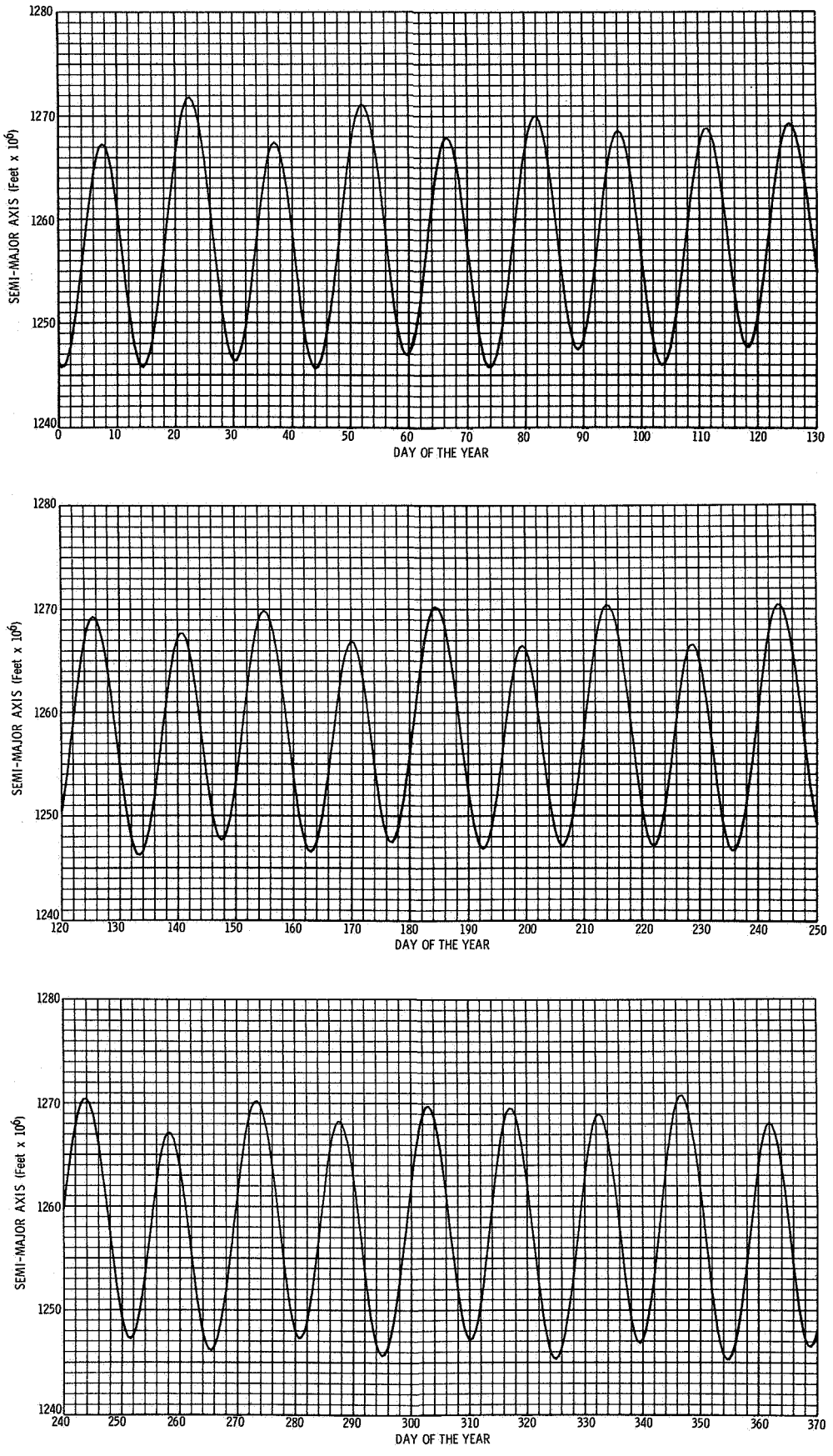


FIGURE B1974-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT

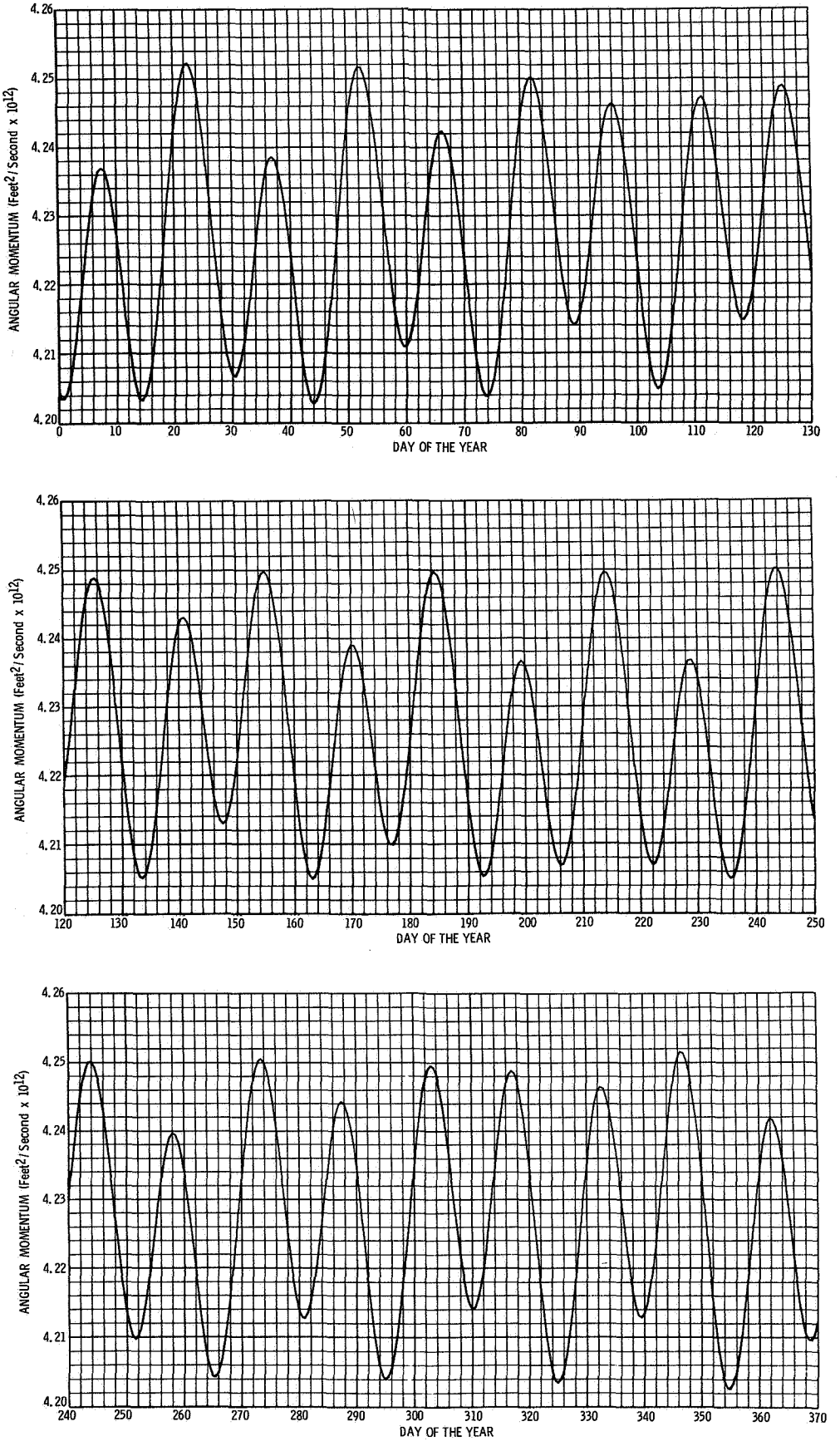
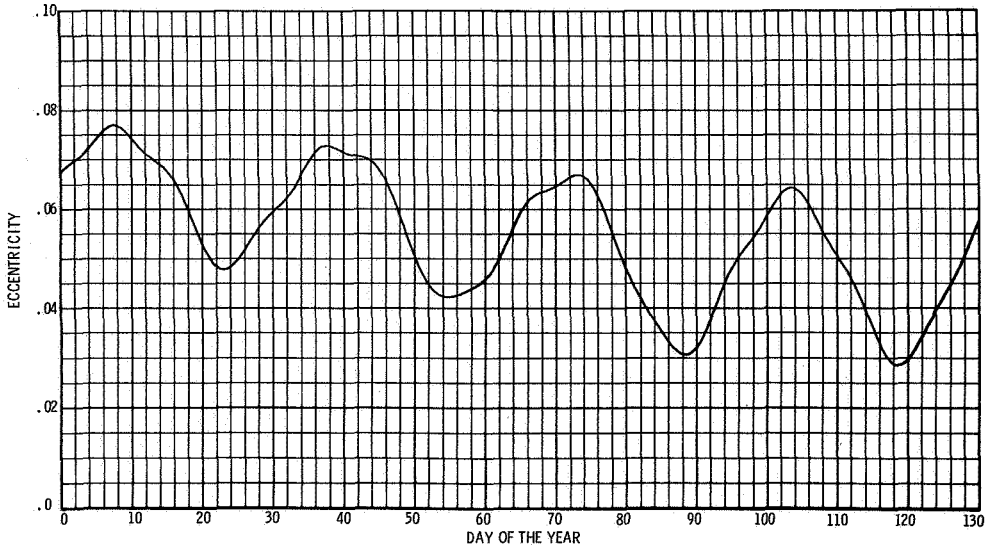
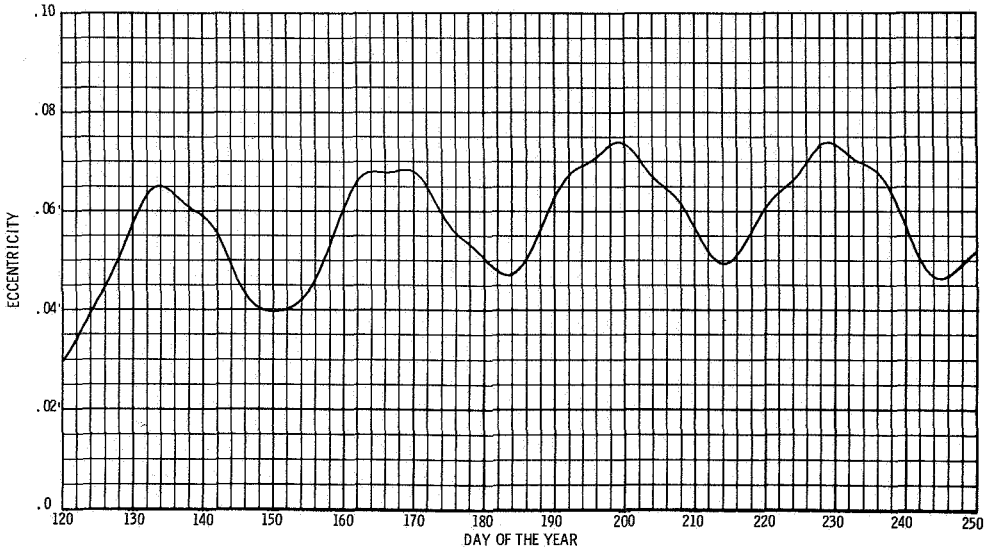


FIGURE B1974-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON





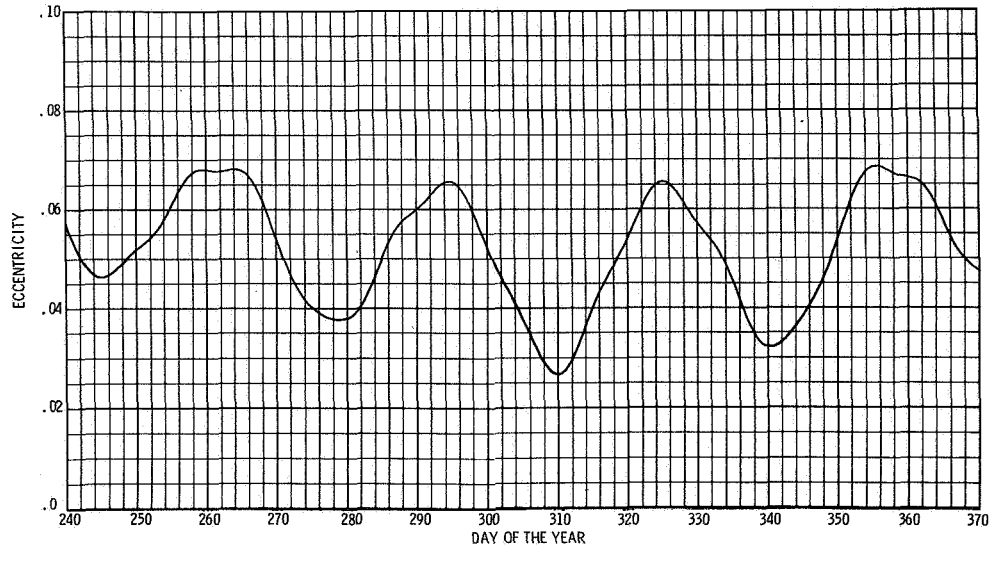


FIGURE B1974-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT

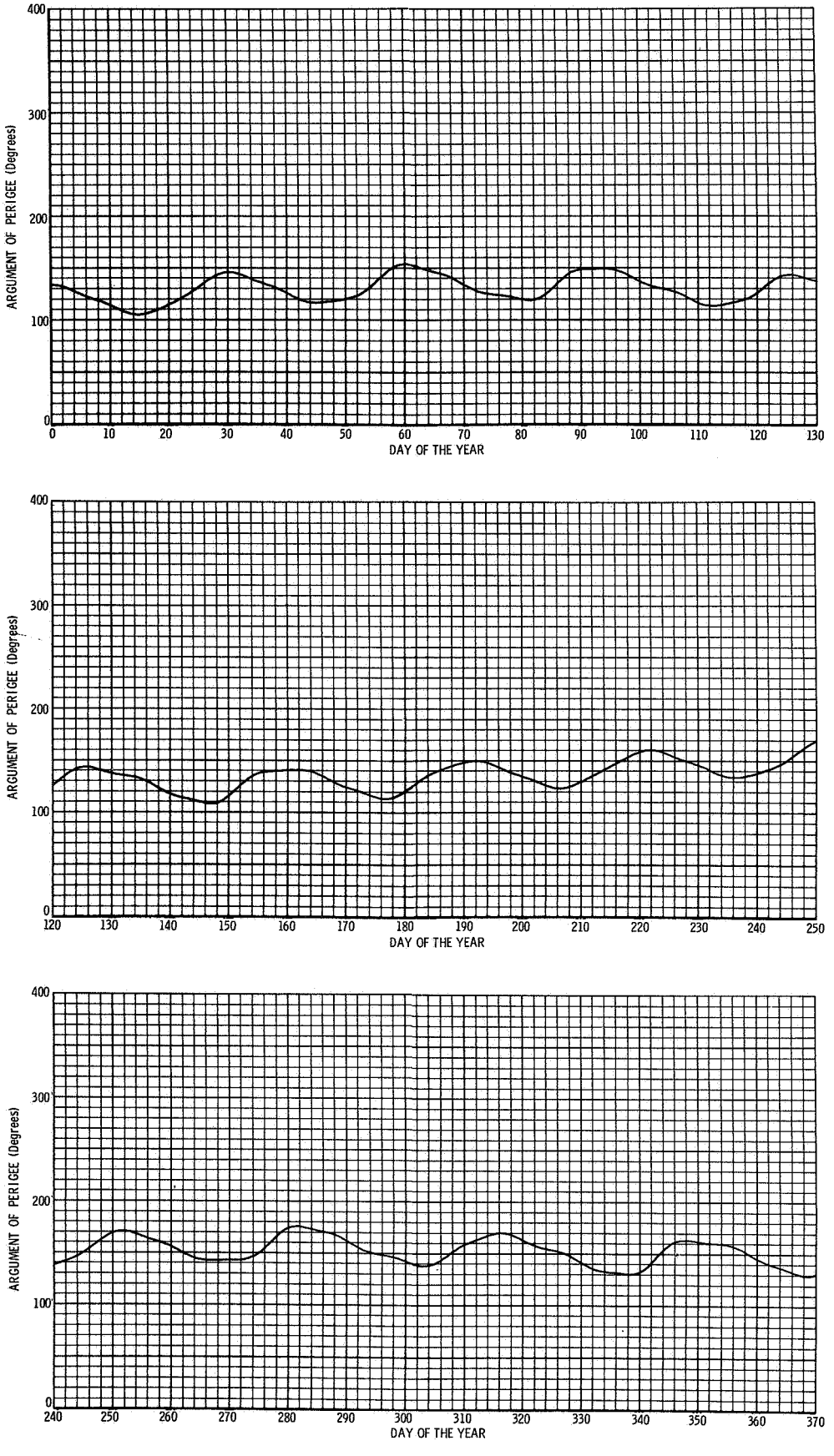
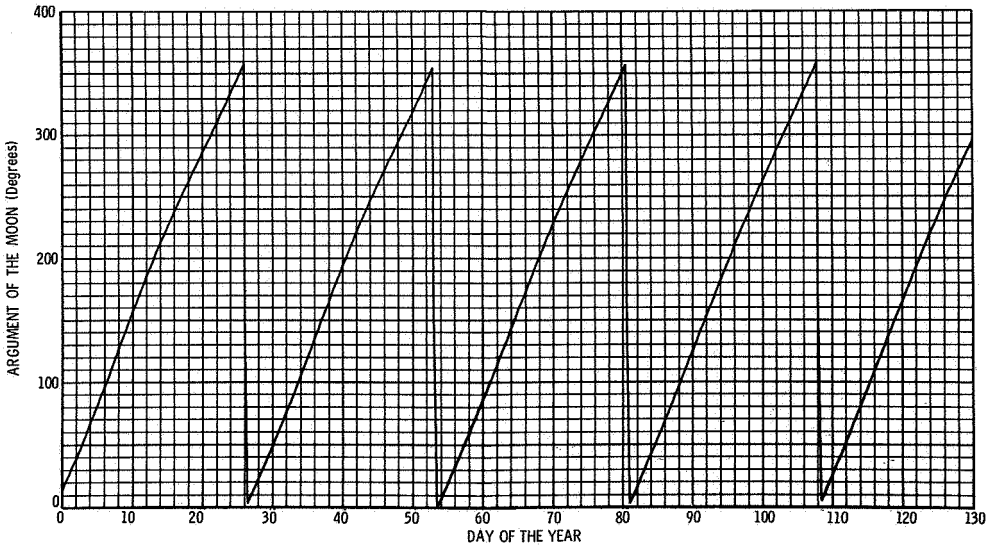
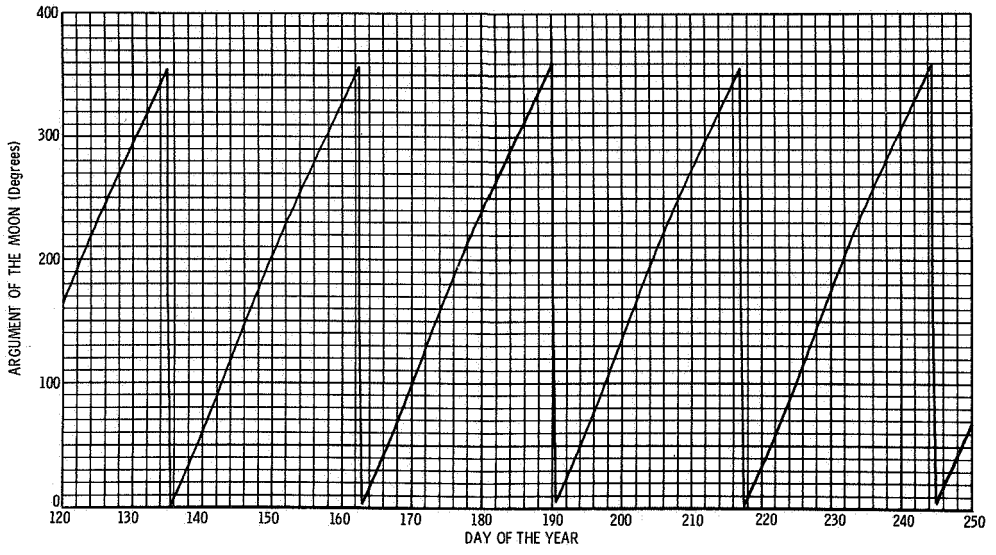


FIGURE B1974-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE





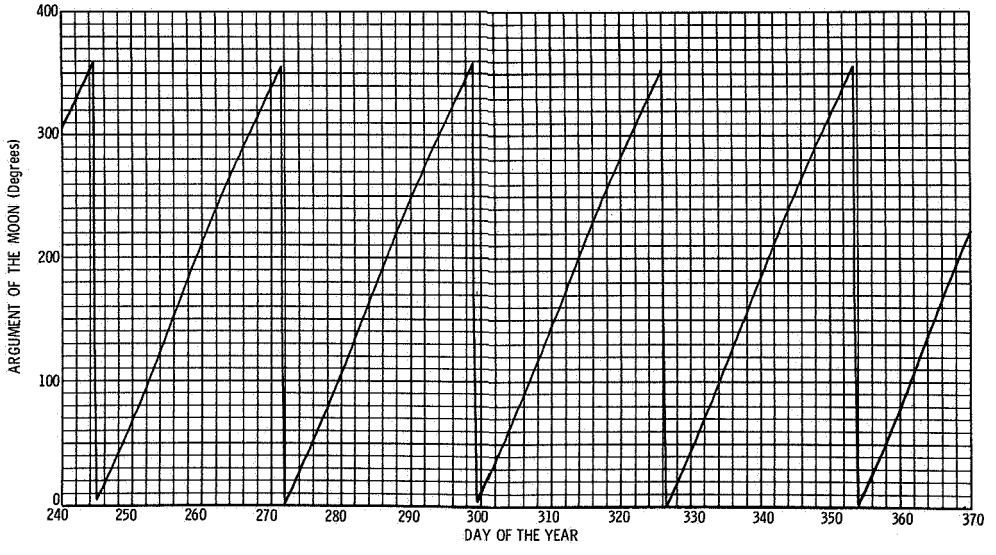
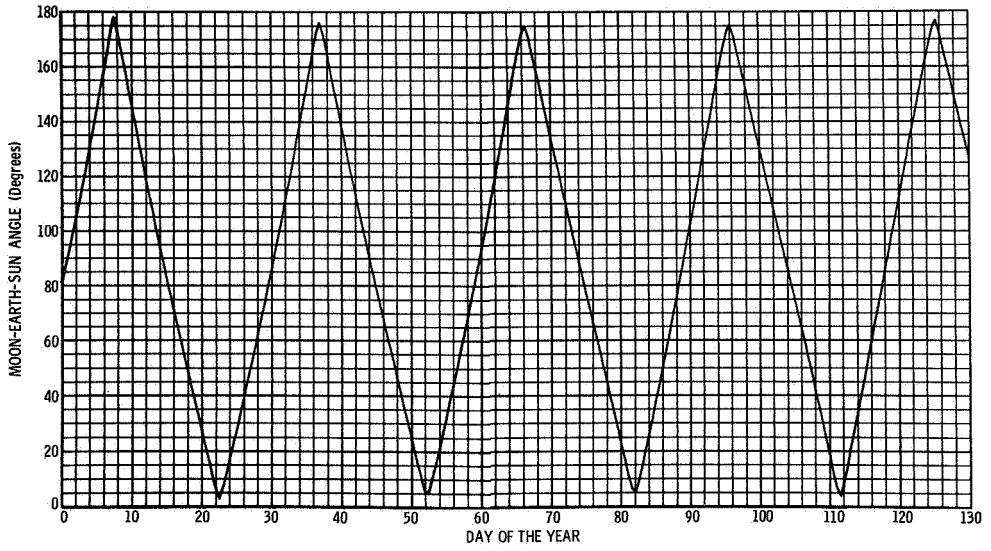
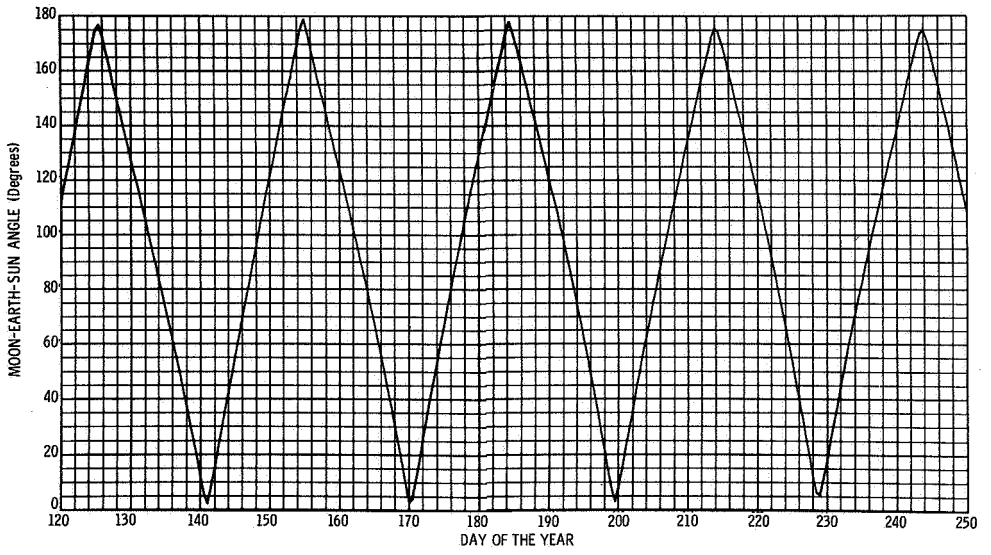


FIGURE B1974-10 ARGUMENT OF THE MOON'S POSITION





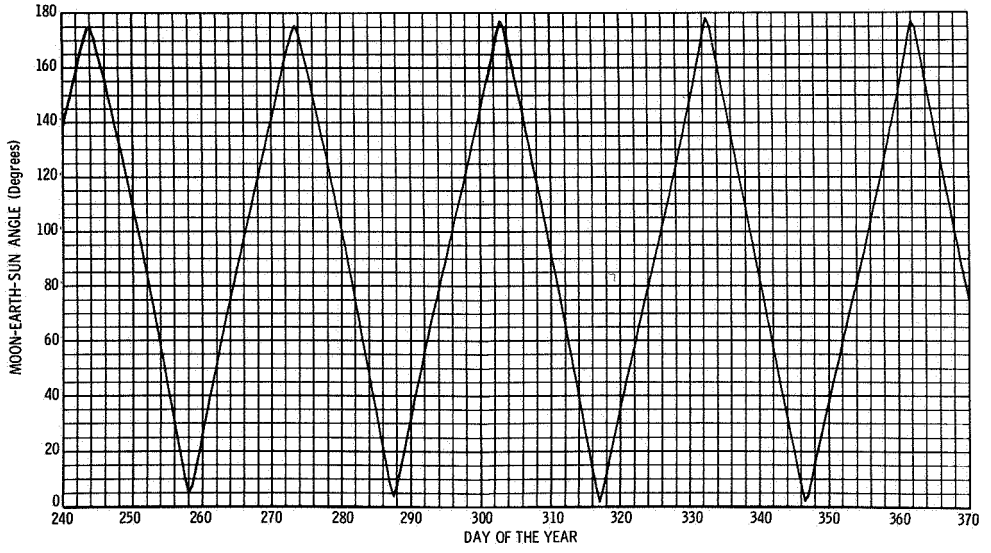


FIGURE B1974-11 MOON-EARTH-SUN ANGLE

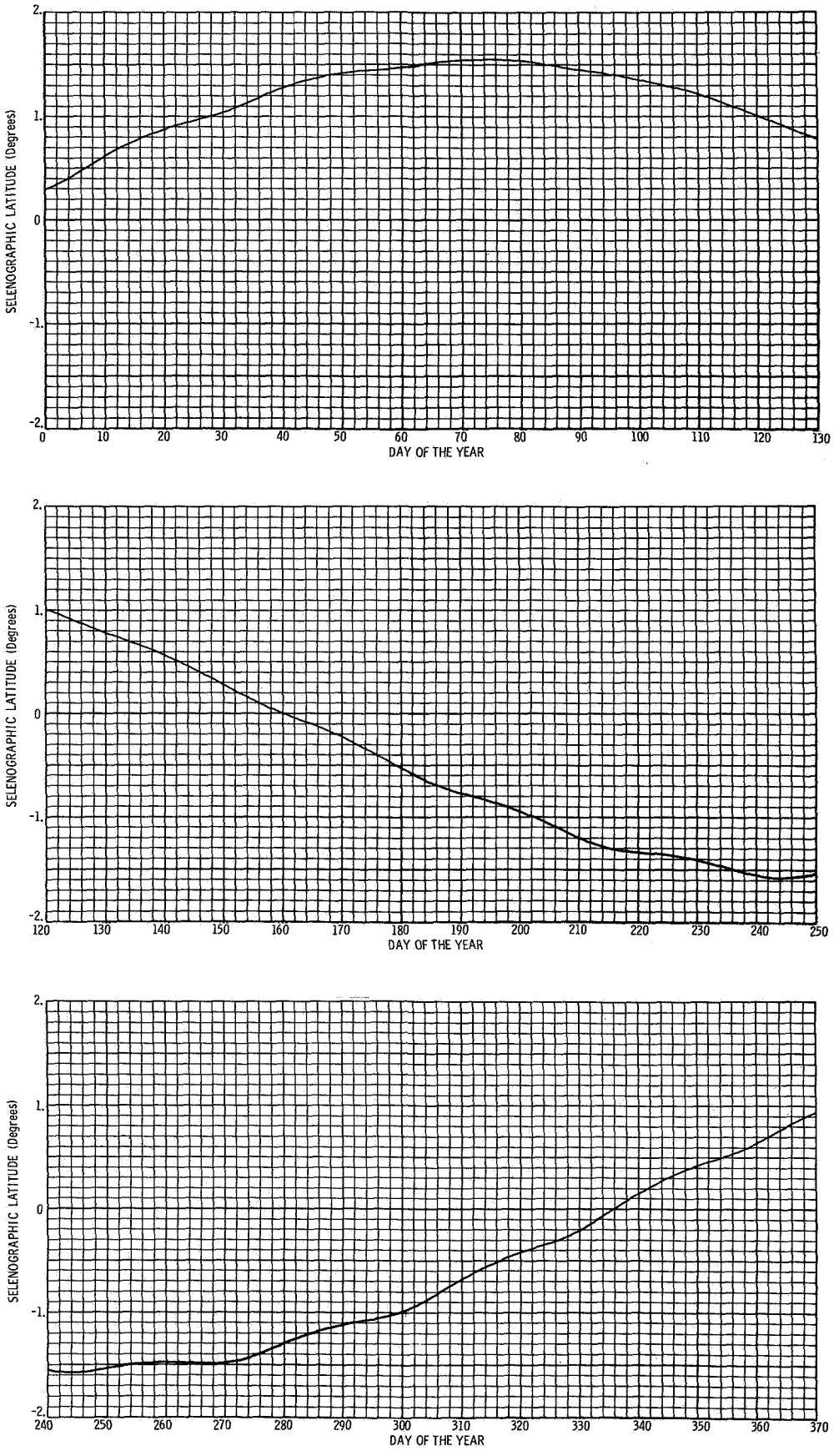


FIGURE B1974-12 SELENOGRAPHIC LATITUDE OF THE SUN

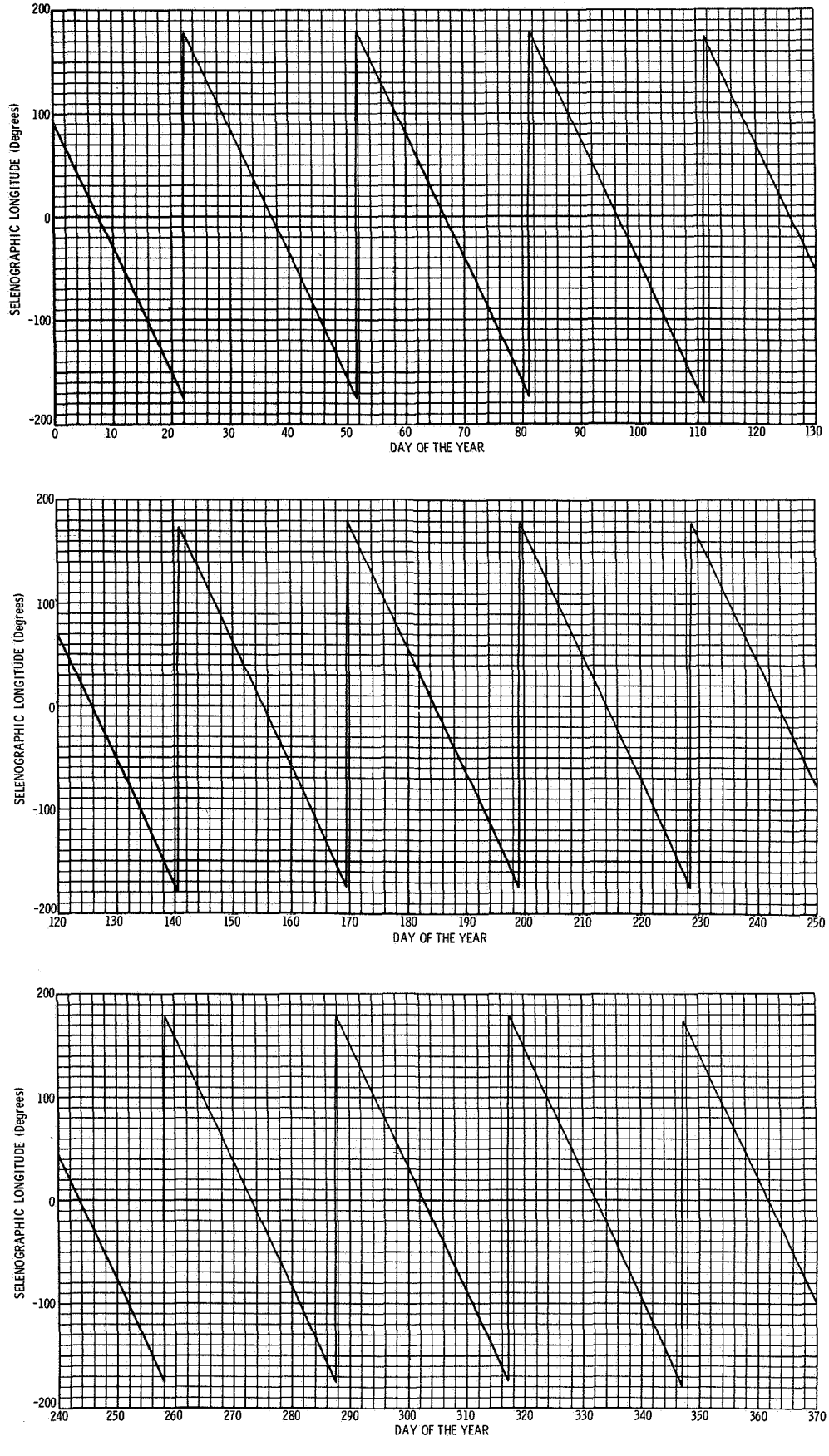
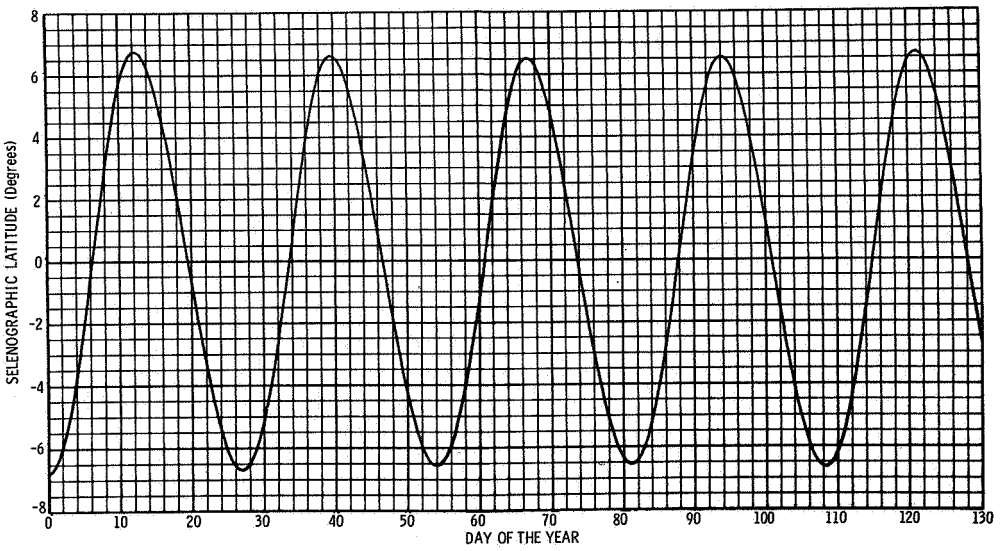
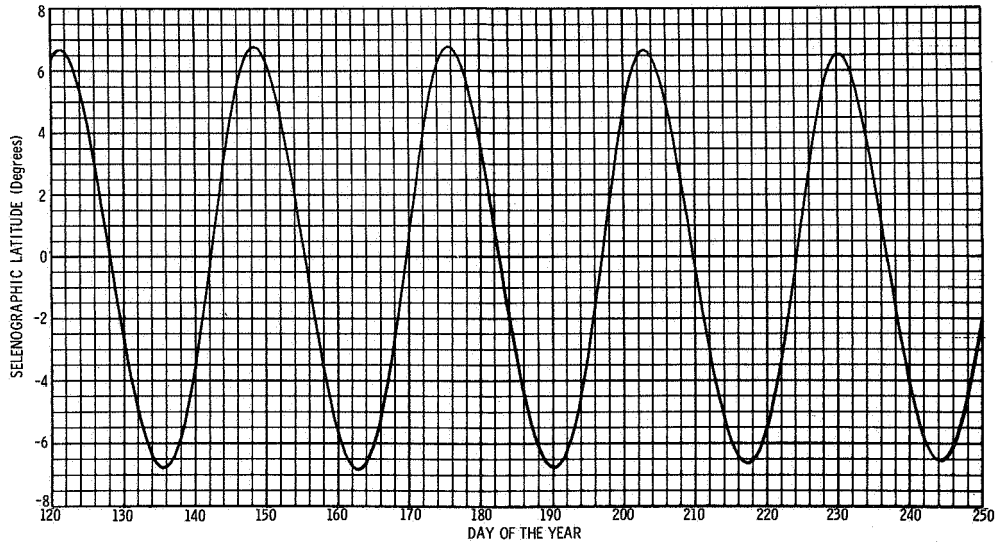


FIGURE B1974-13 SELENOGRAPHIC LONGITUDE OF THE SUN





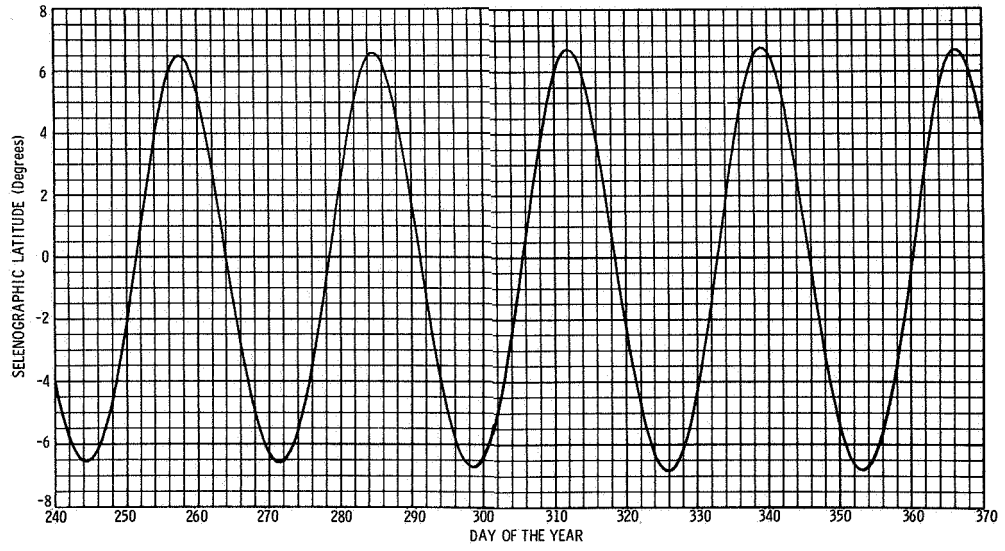
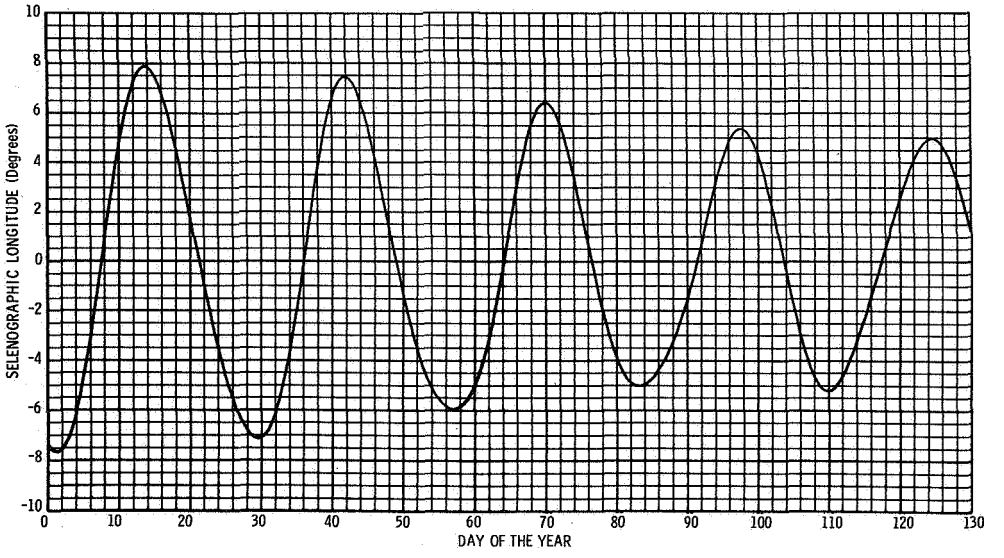
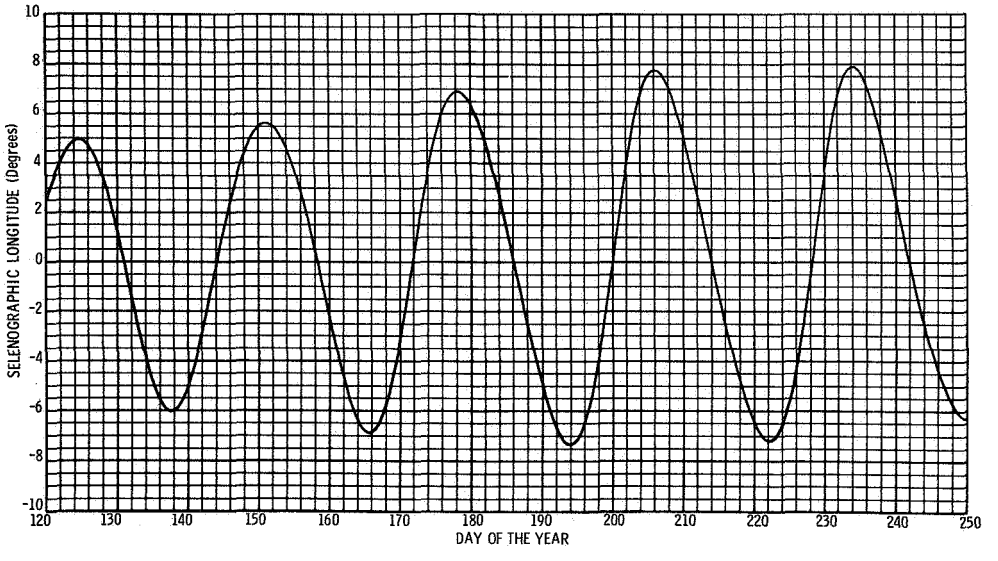


FIGURE B1974-14 SELENOGRAPHIC LATITUDE OF THE EARTH





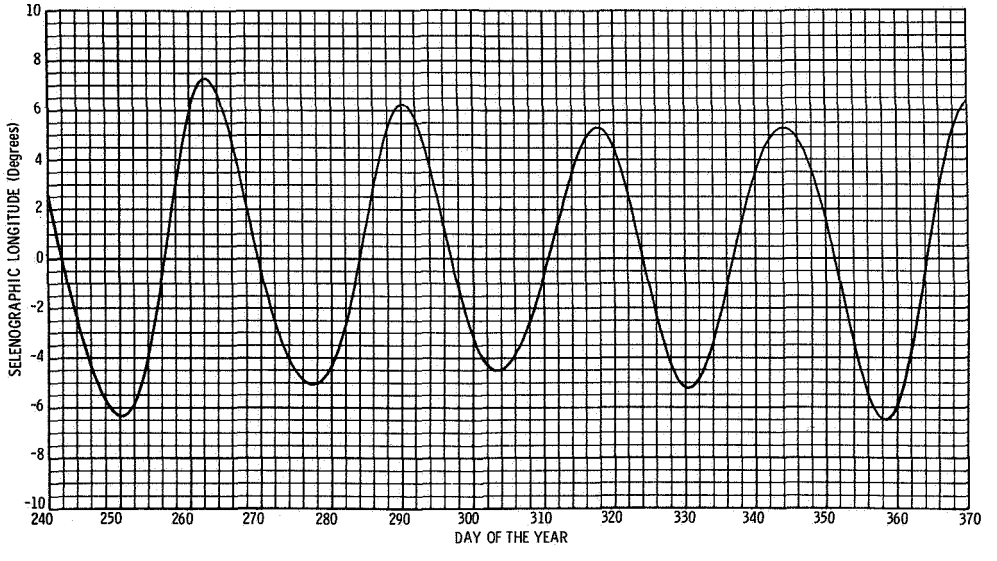


FIGURE B1974-15 SELENOGRAPHIC LONGITUDE OF THE EARTH

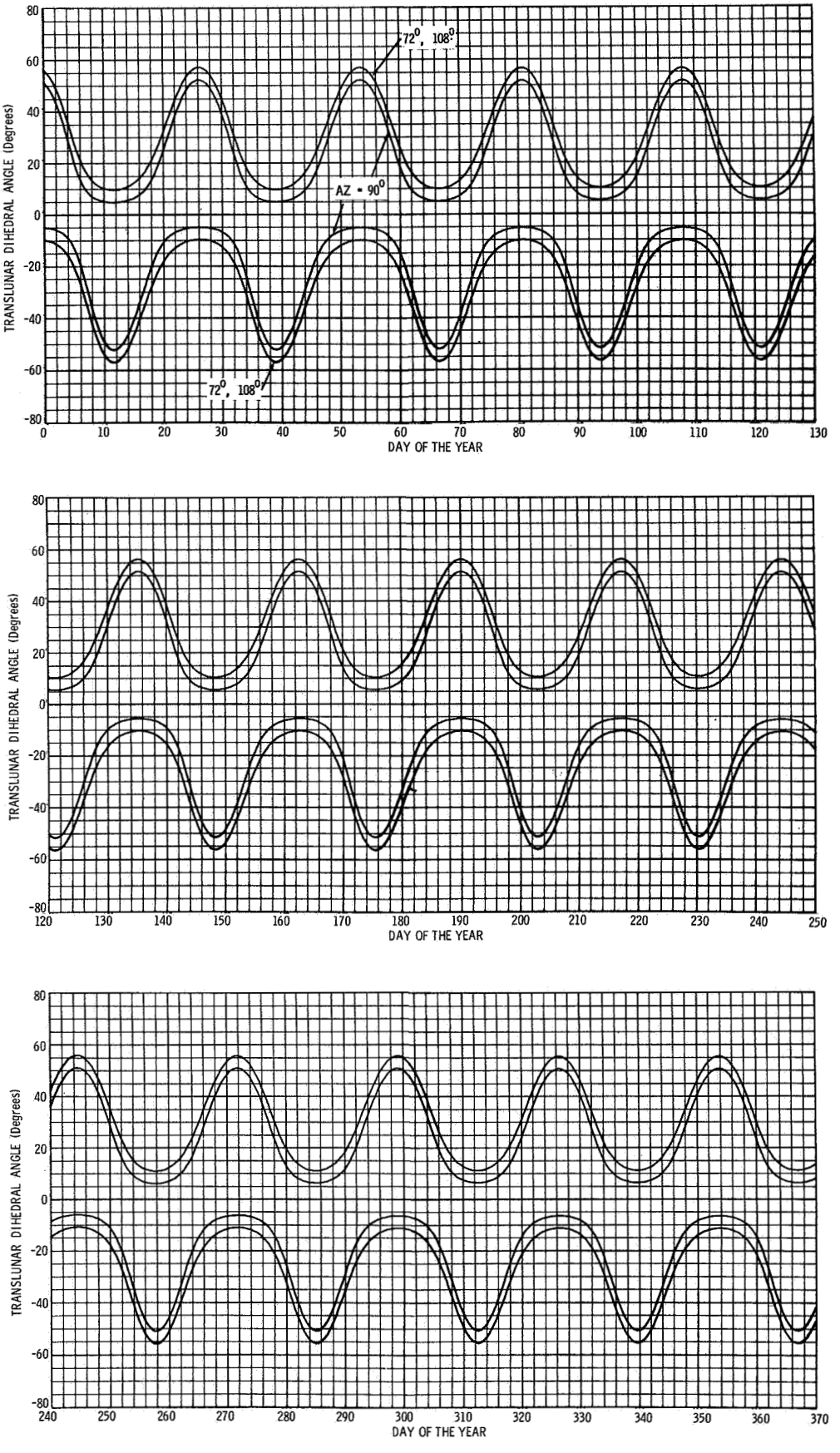


FIGURE B1974-16 TRANSLUNAR DIHEDRAL ANGLES

1975

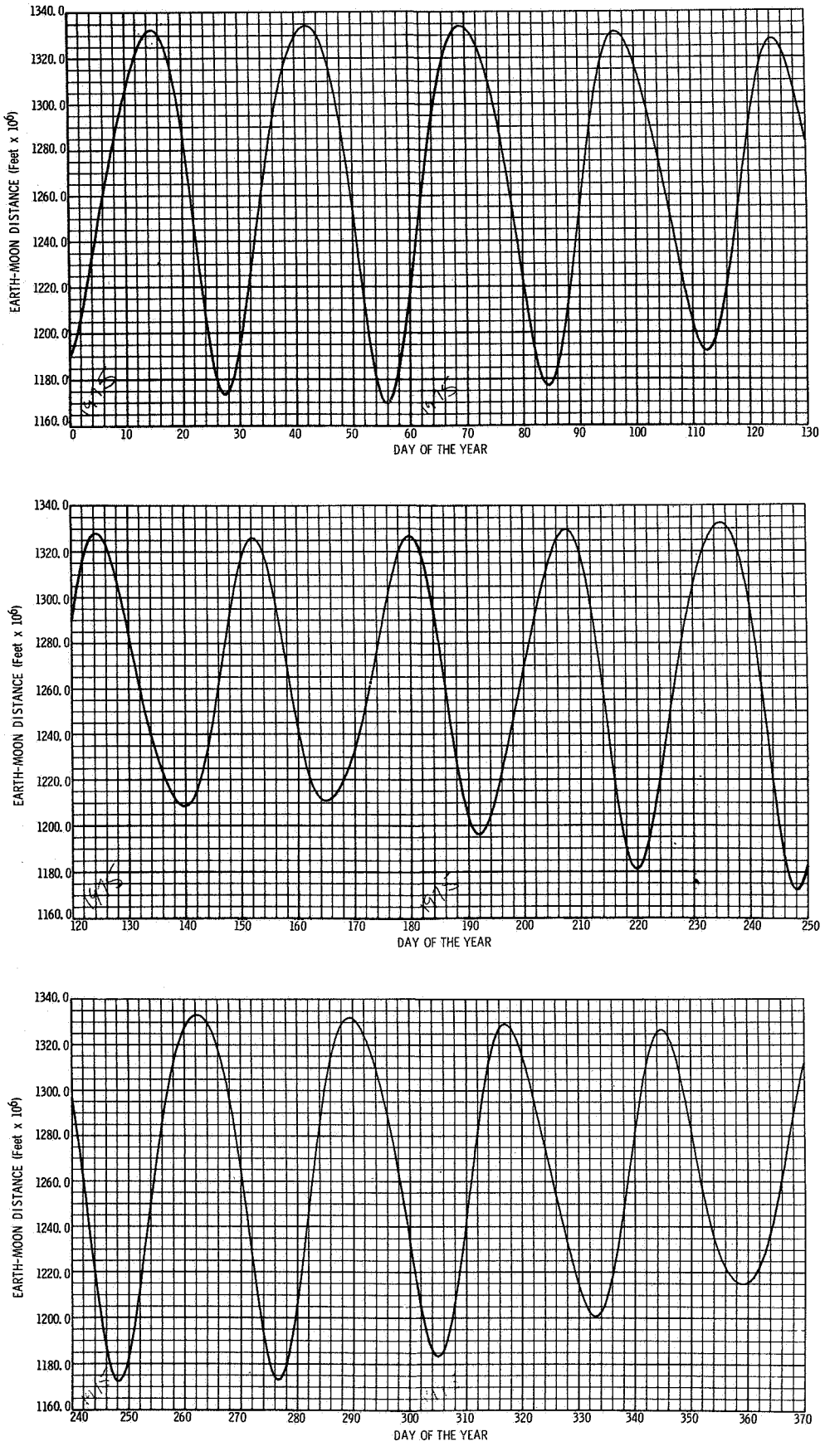
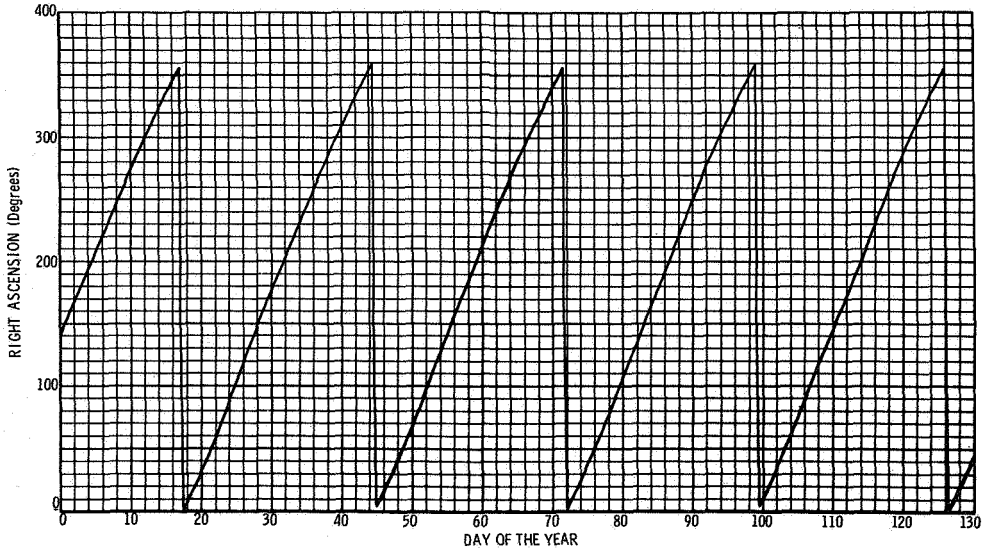
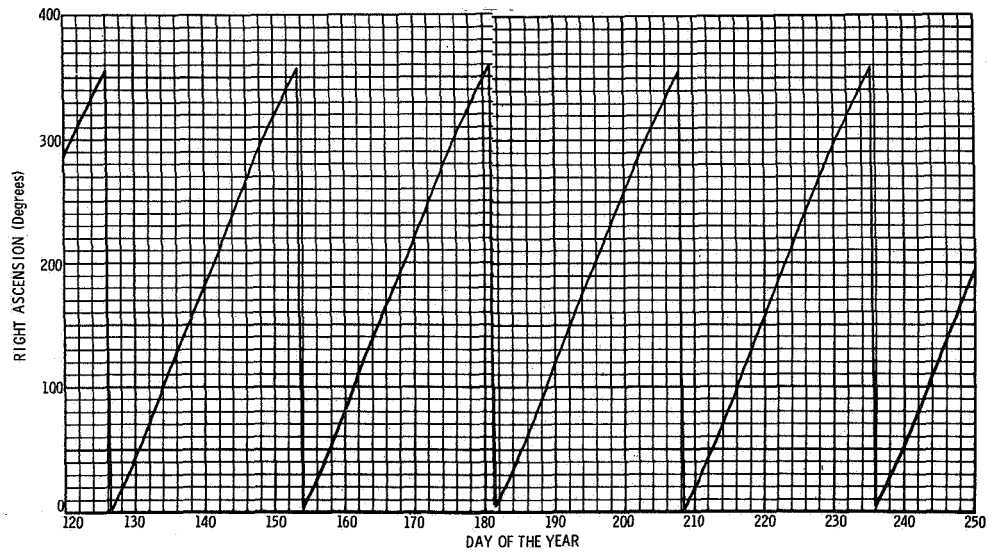


FIGURE B1975-1 EARTH-MOON DISTANCE





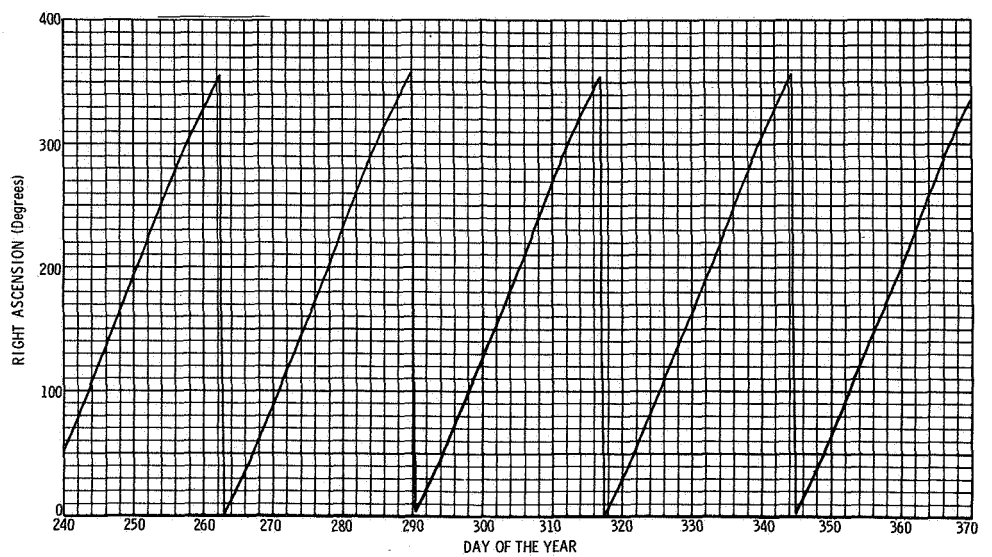
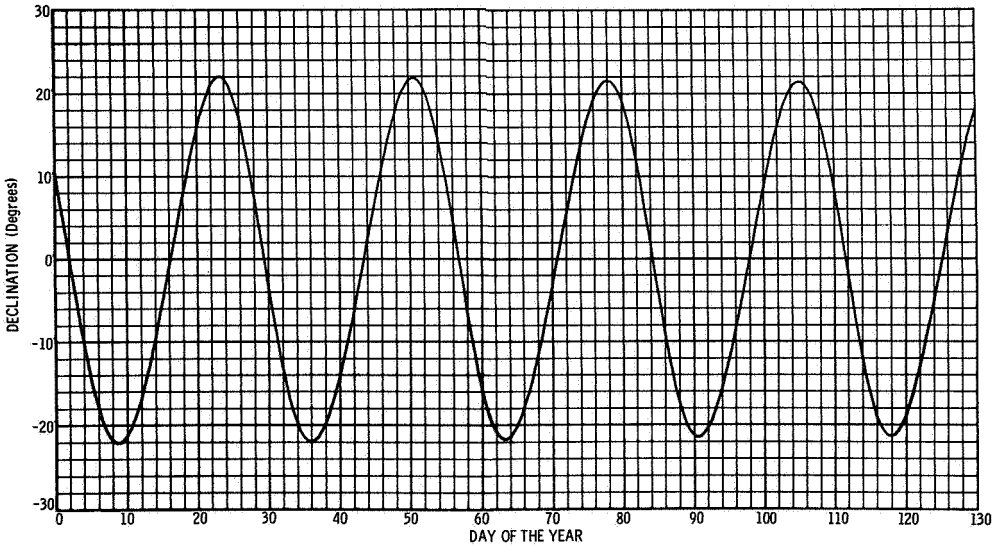
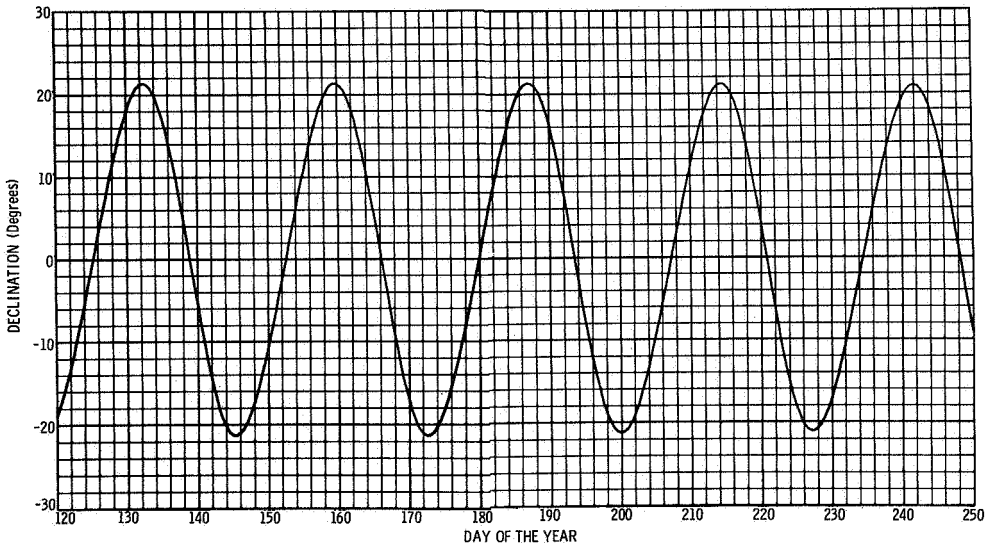


FIGURE B1975-2 RIGHT ASCENSION OF THE MOON





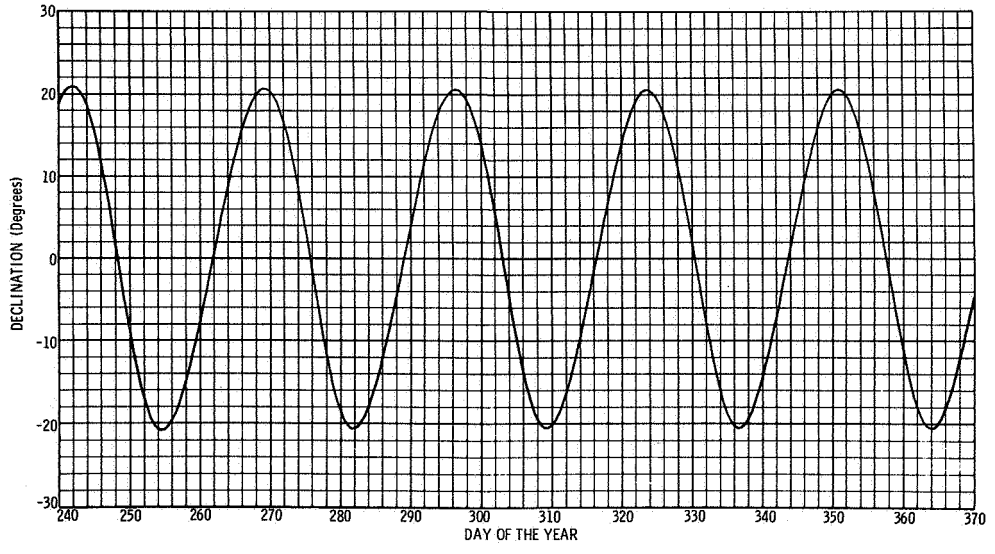


FIGURE B1975-3 DECLINATION OF THE MOON

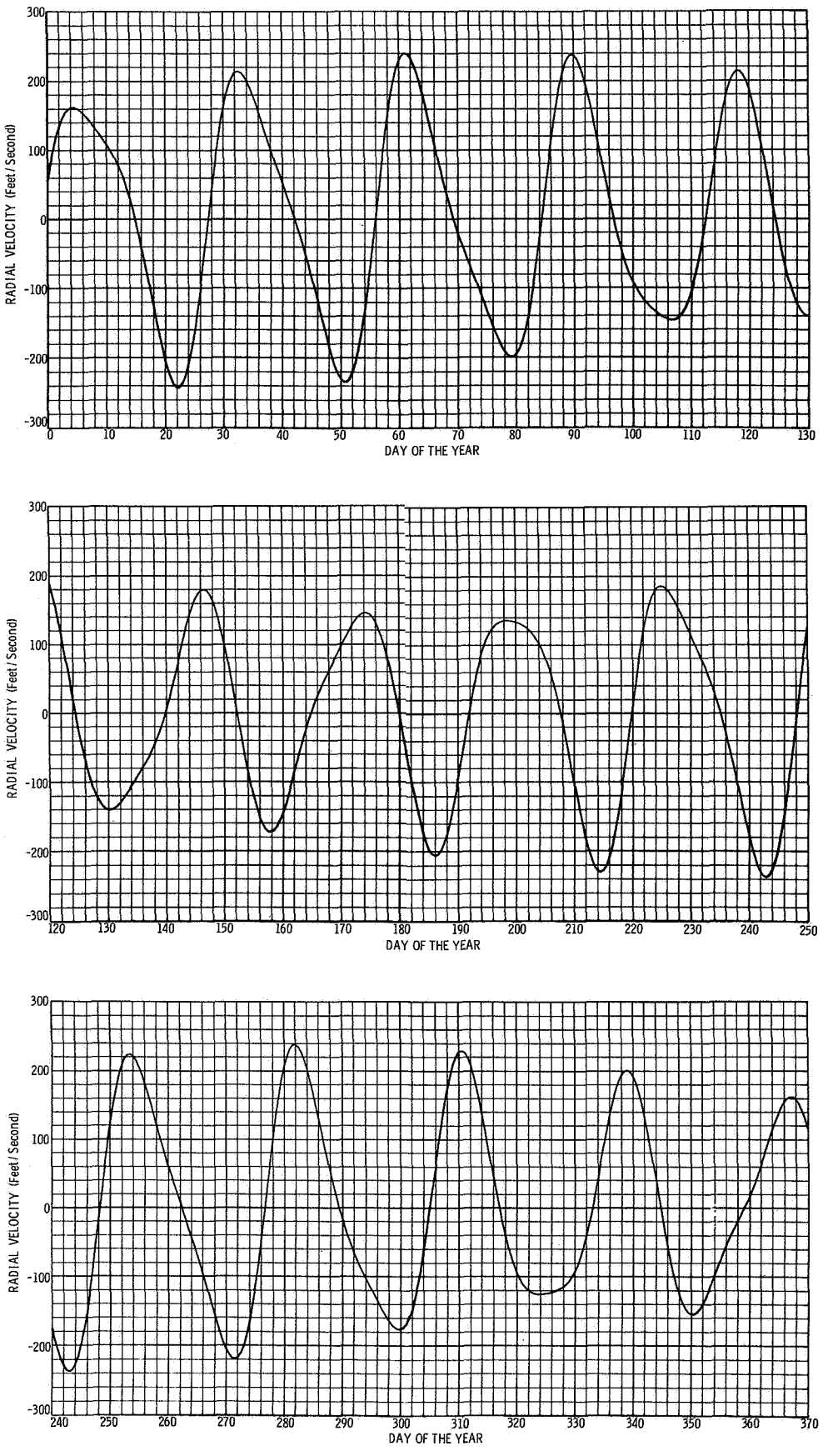


FIGURE B1975-4 RADIAL VELOCITY OF THE MOON

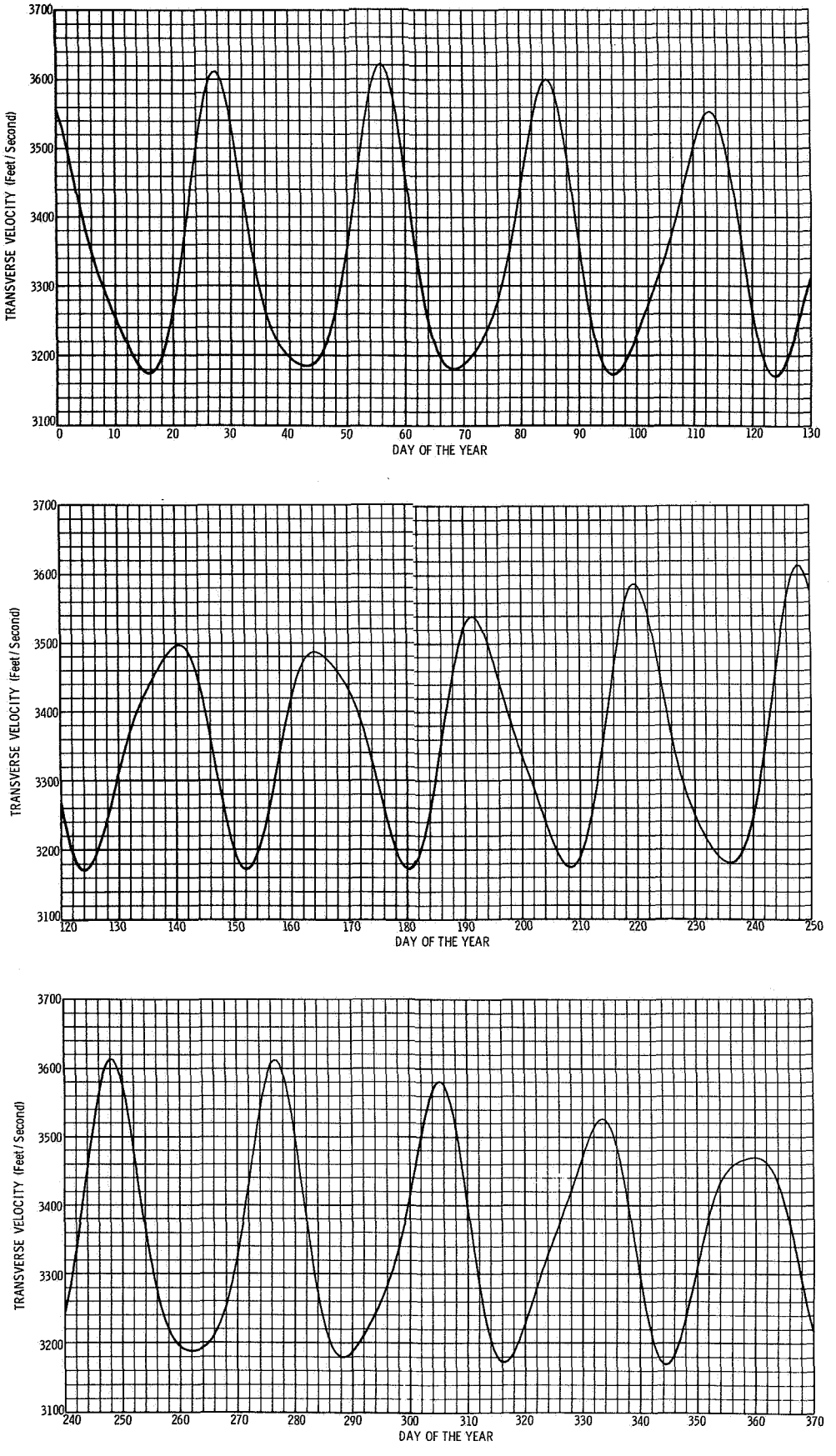


FIGURE B1975-5 TRANSVERSE VELOCITY OF THE MOON

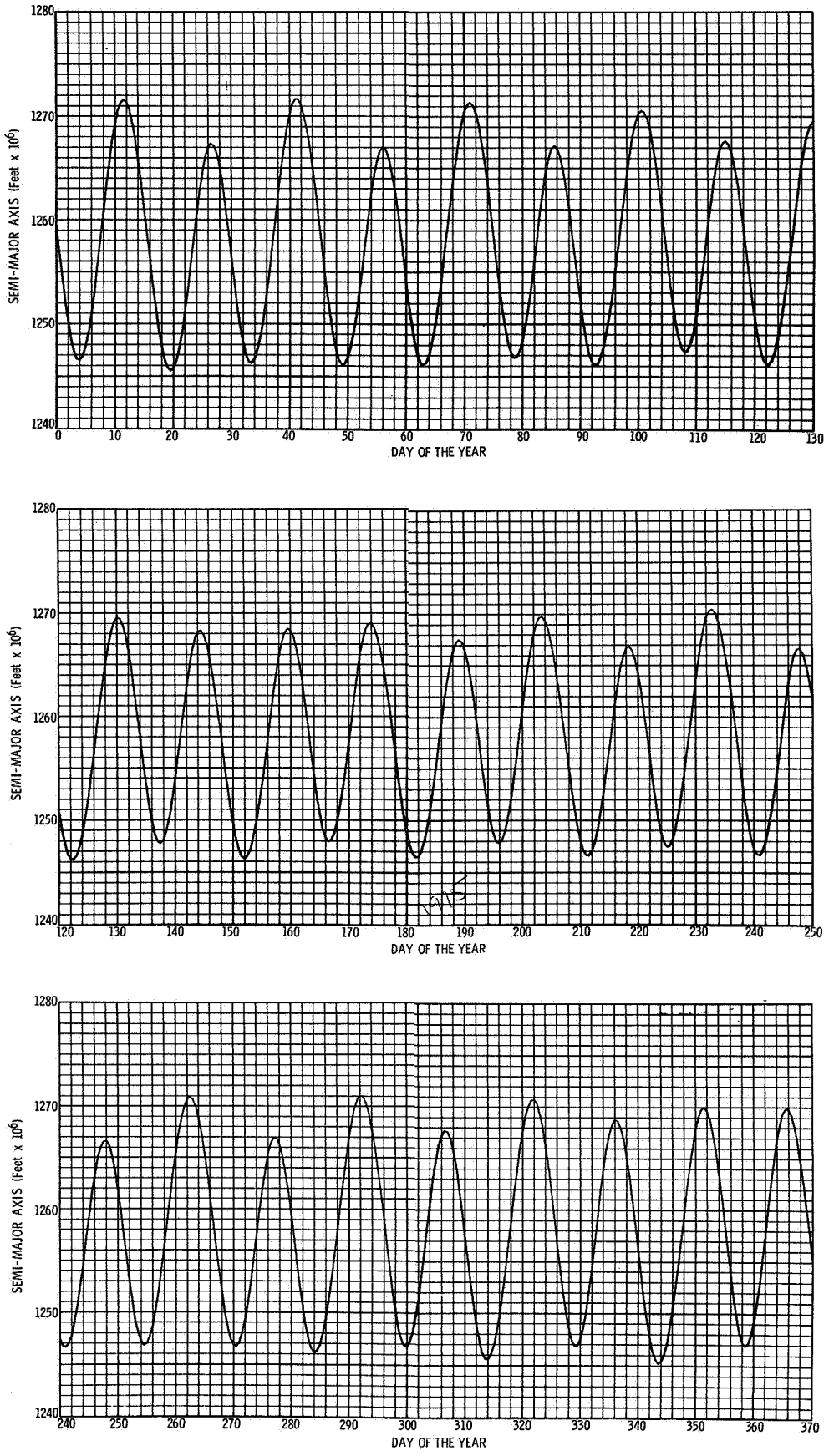


FIGURE B1975-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT

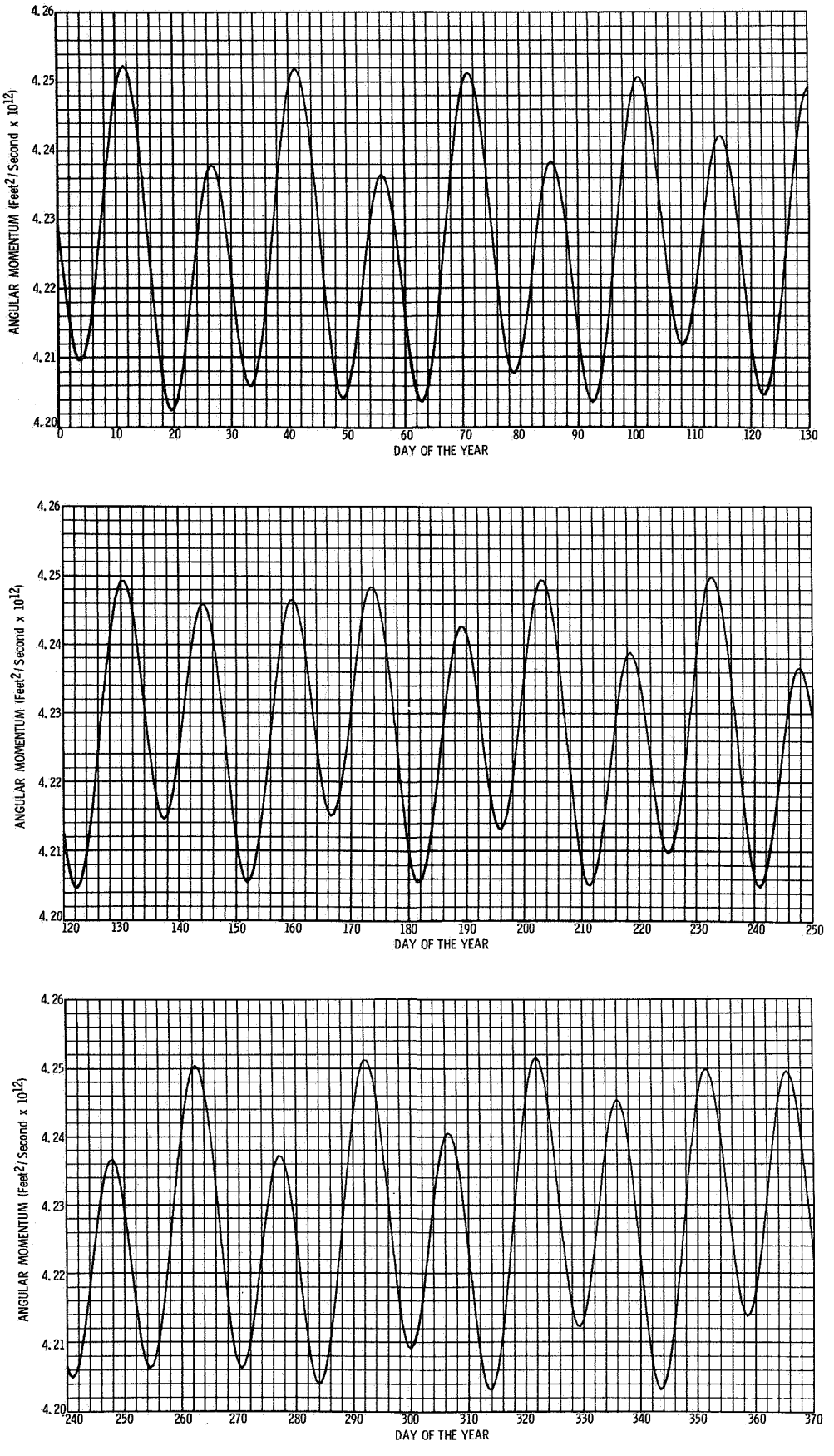


FIGURE B1975-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON

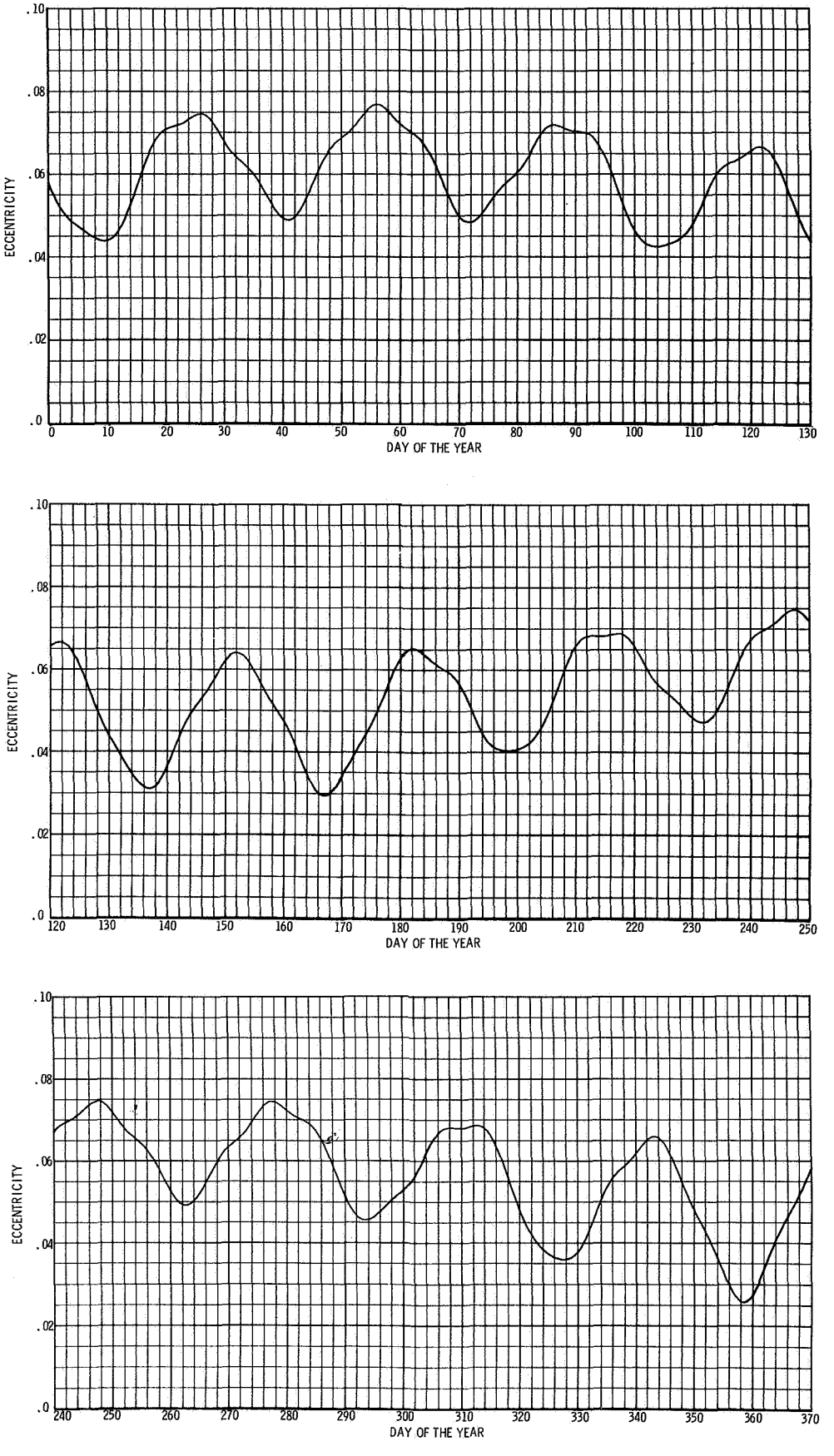
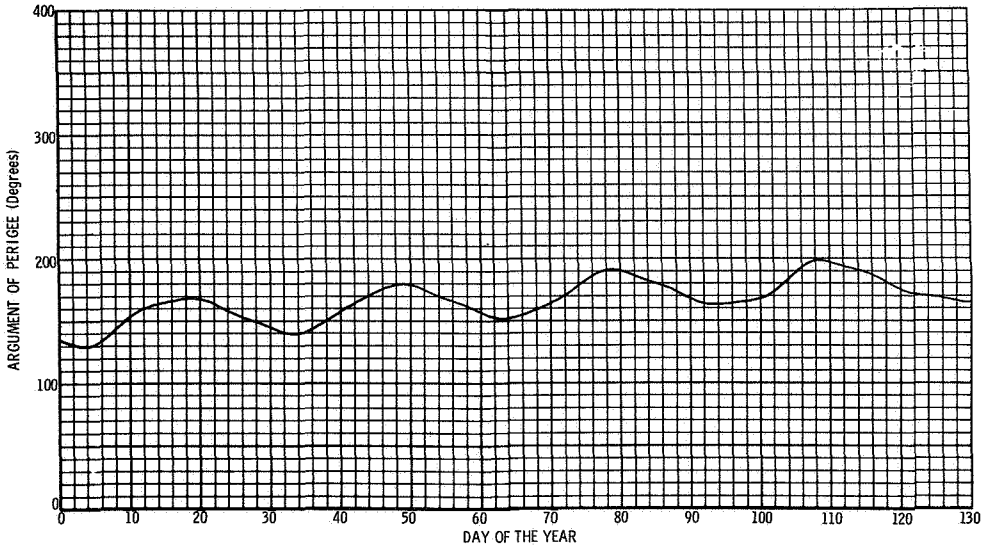
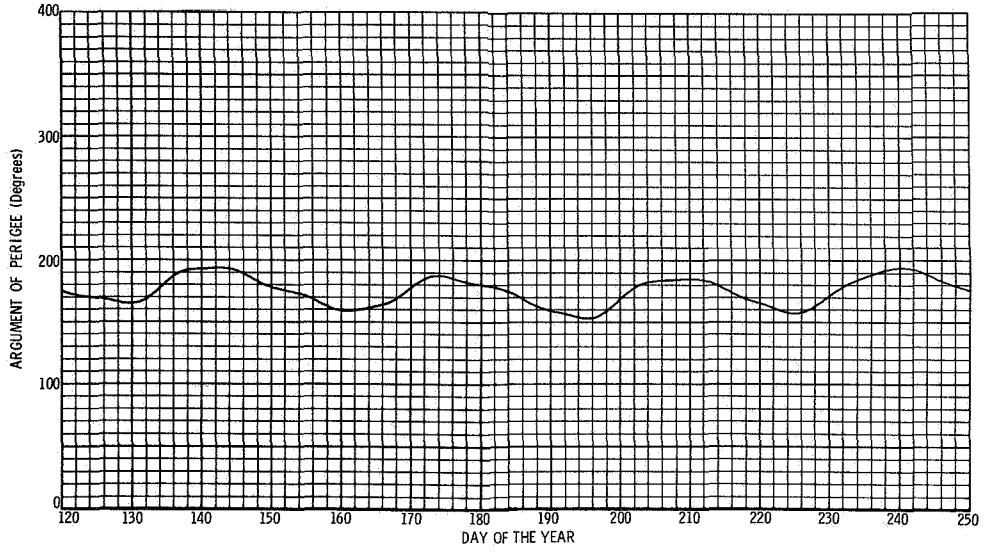


FIGURE B1975-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT





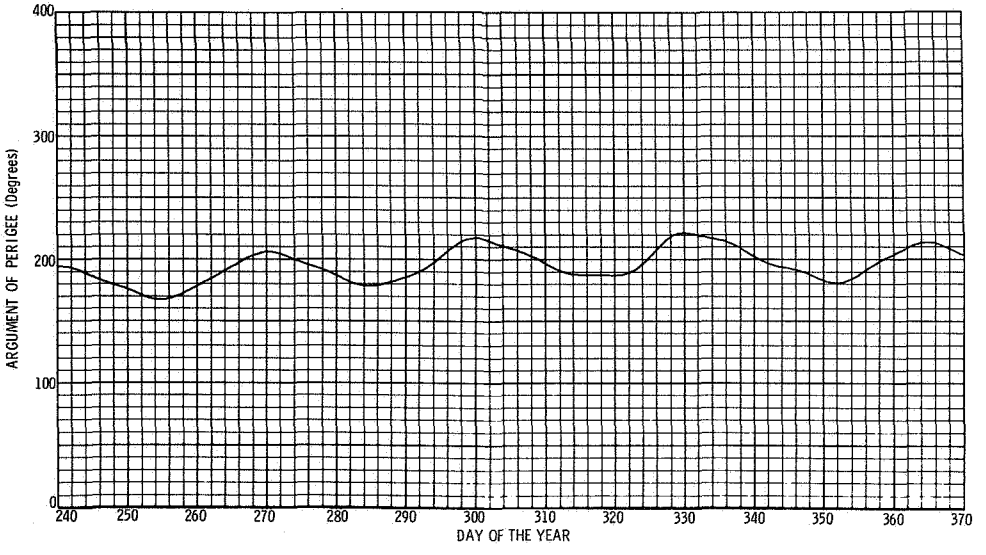


FIGURE B1975-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE

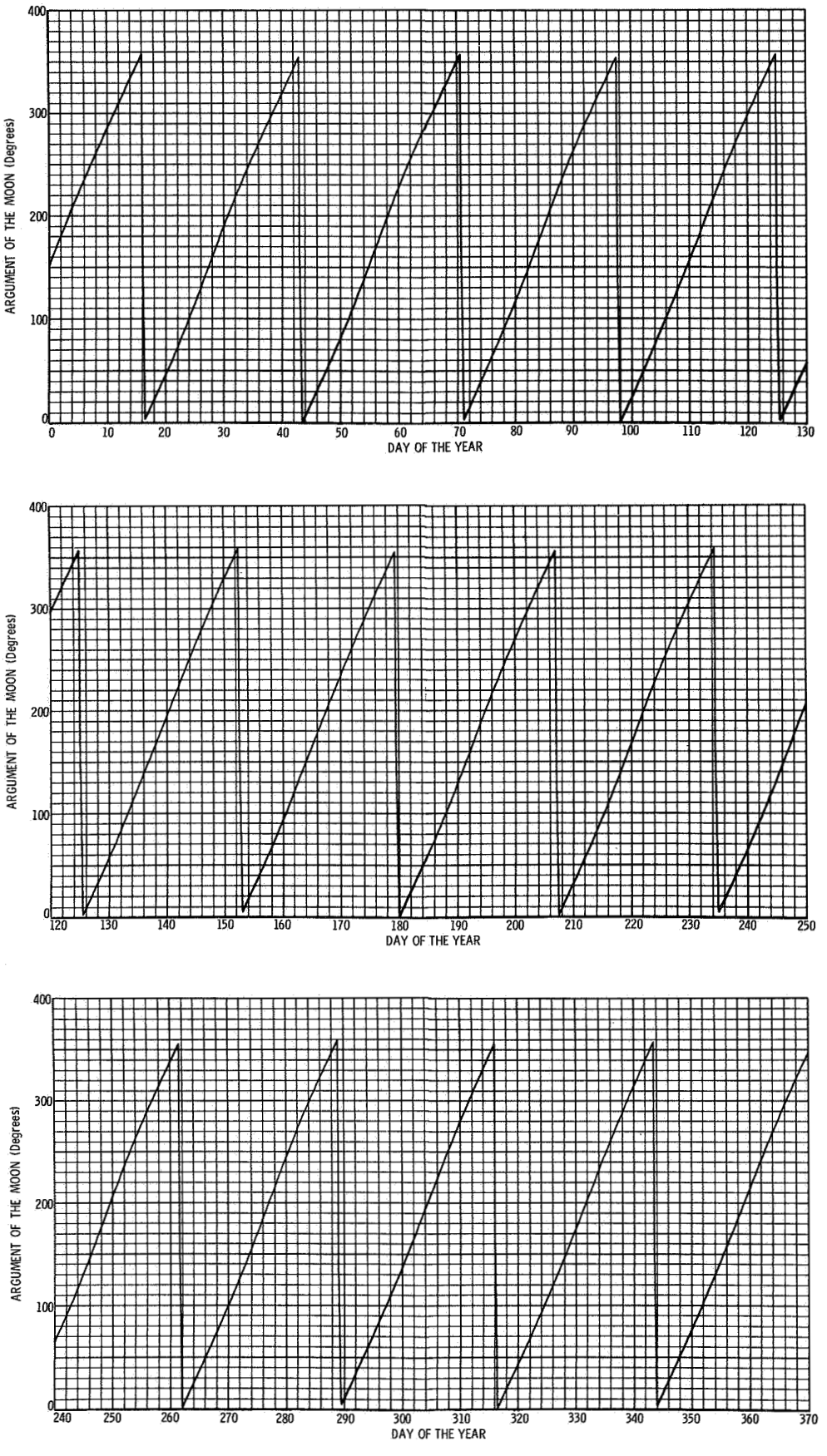


FIGURE B1975-10 ARGUMENT OF THE MOON'S POSITION

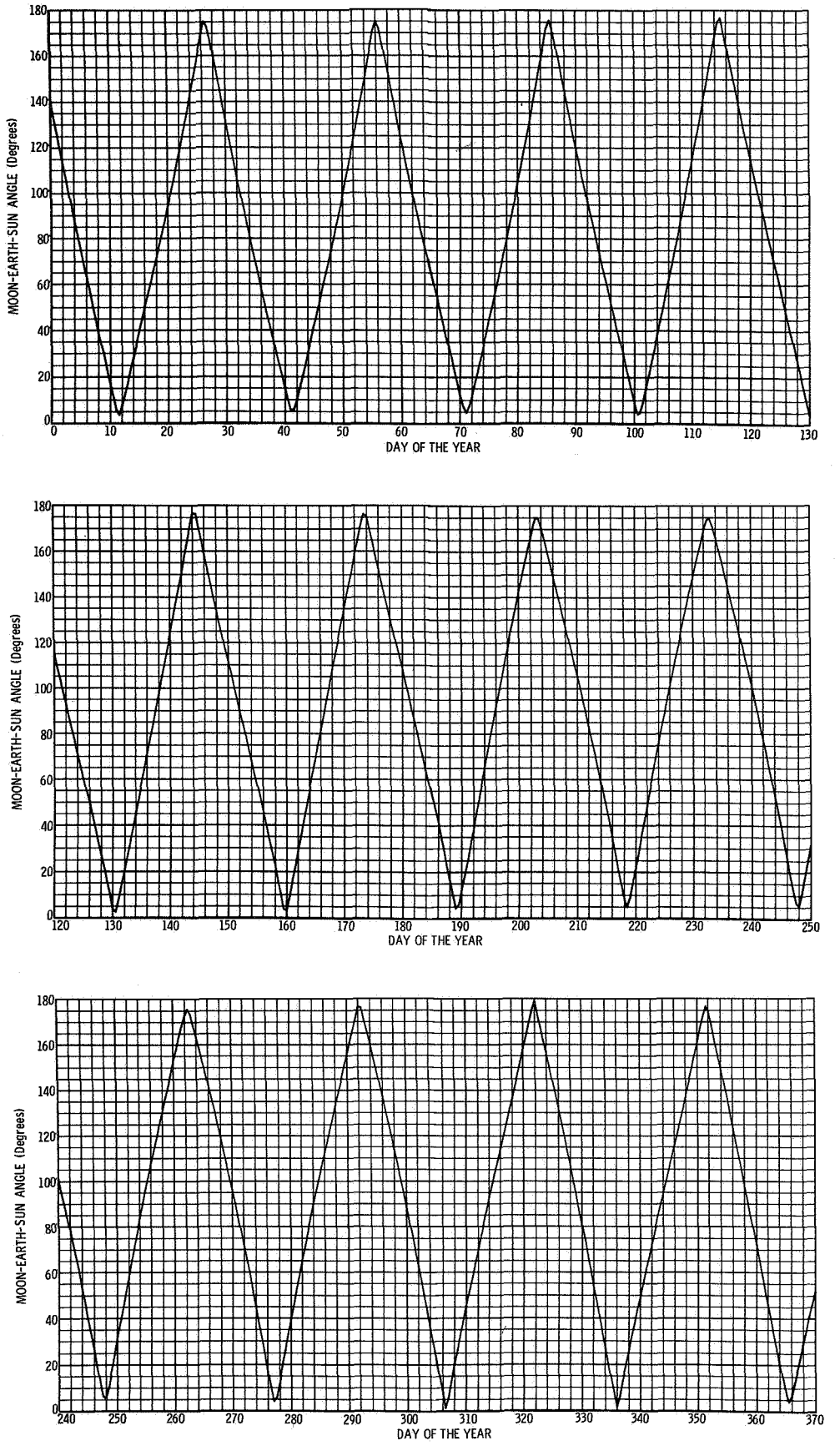
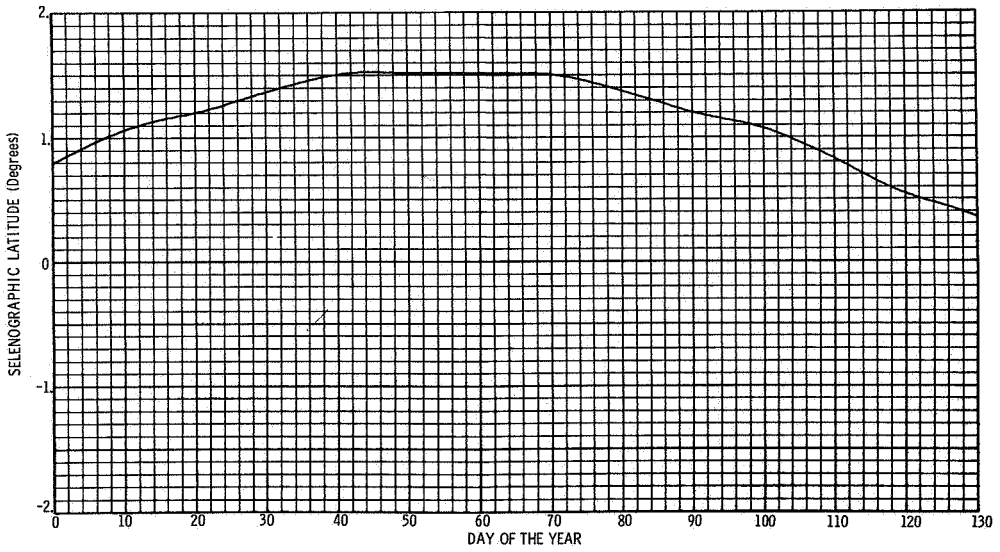
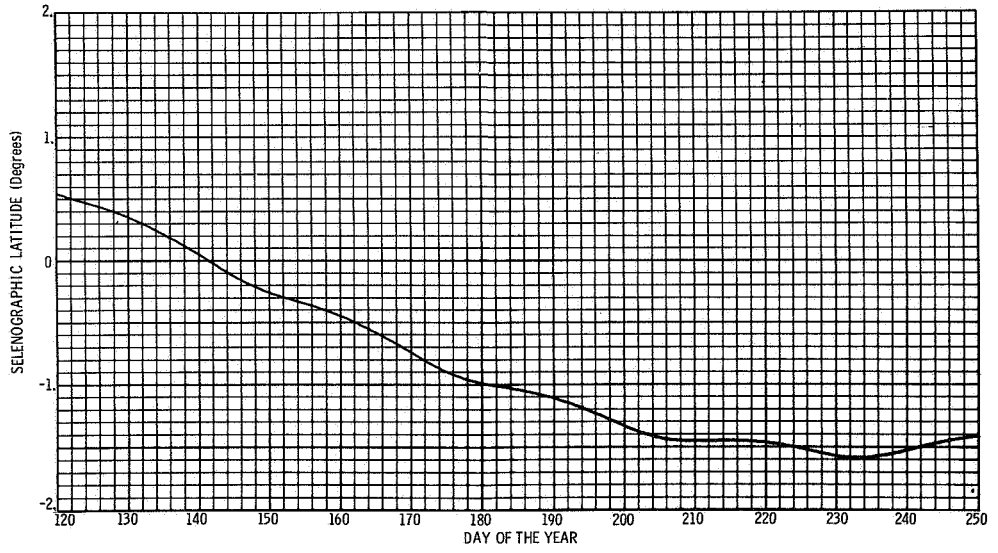


FIGURE B1975-11 MOON-EARTH-SUN ANGLE





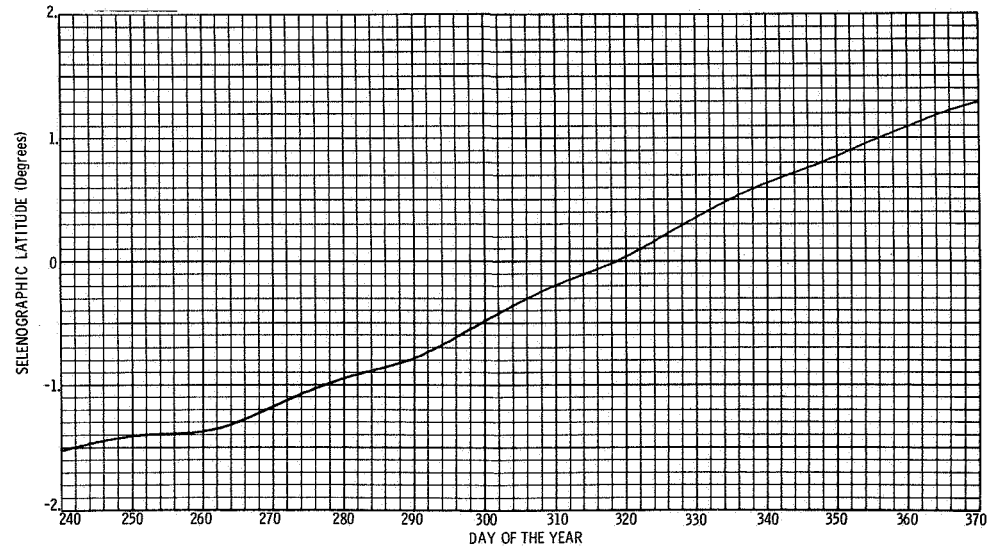
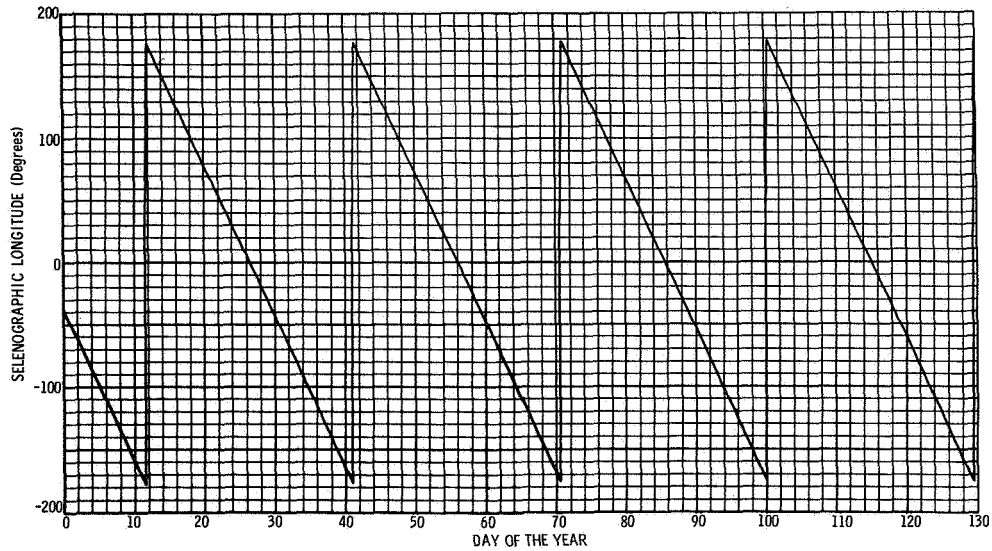
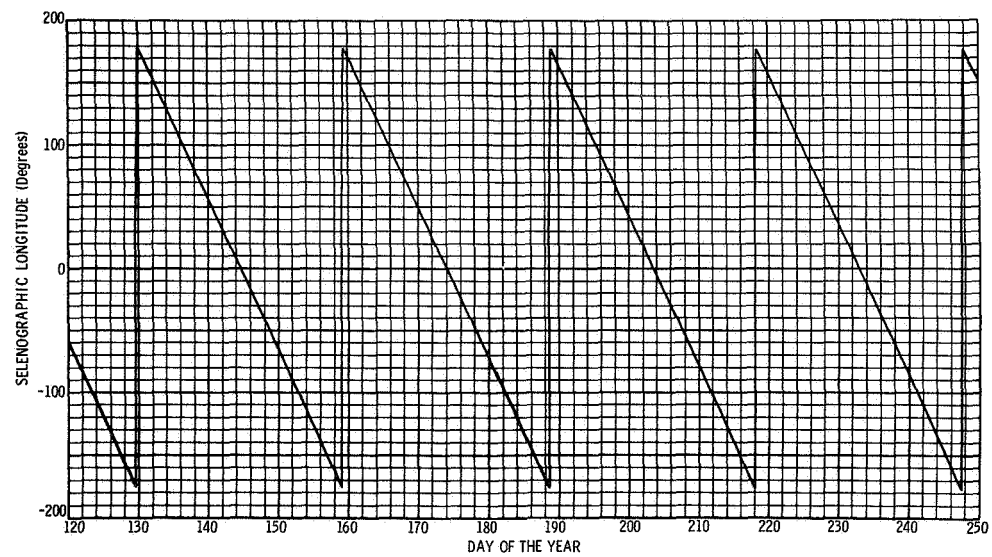


FIGURE B1975-12 SELENOGRAPHIC LATITUDE OF THE SUN





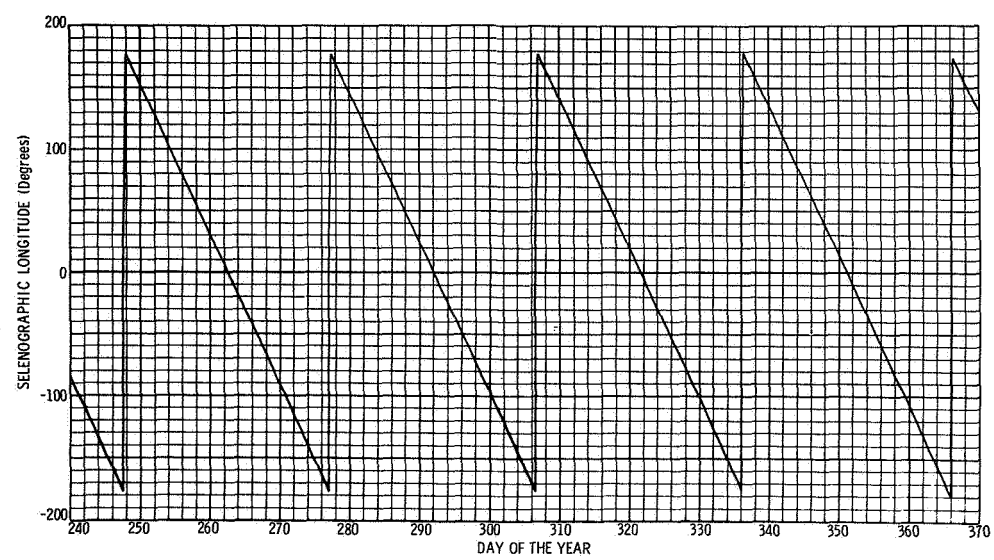
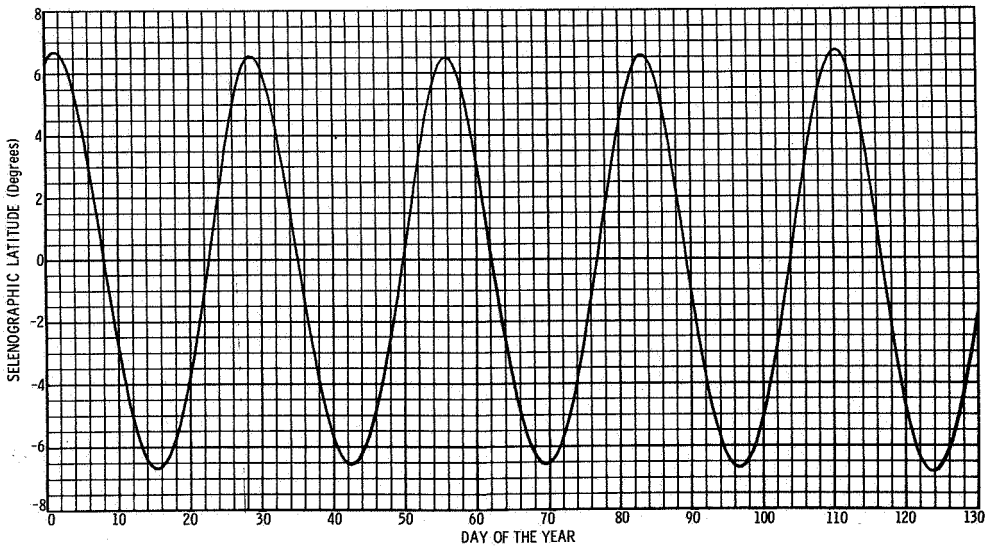
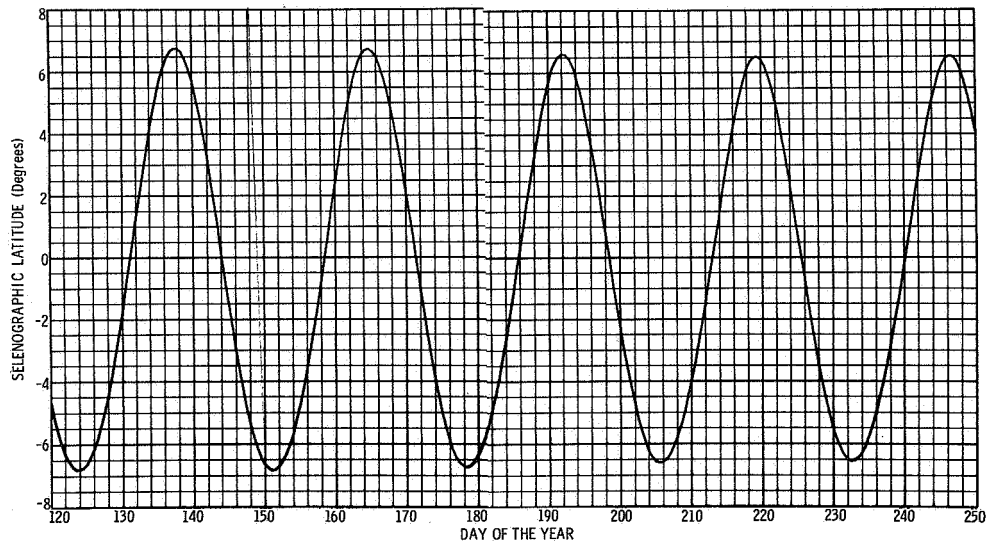


FIGURE B1975-13 SELENOGRAPHIC LONGITUDE OF THE SUN





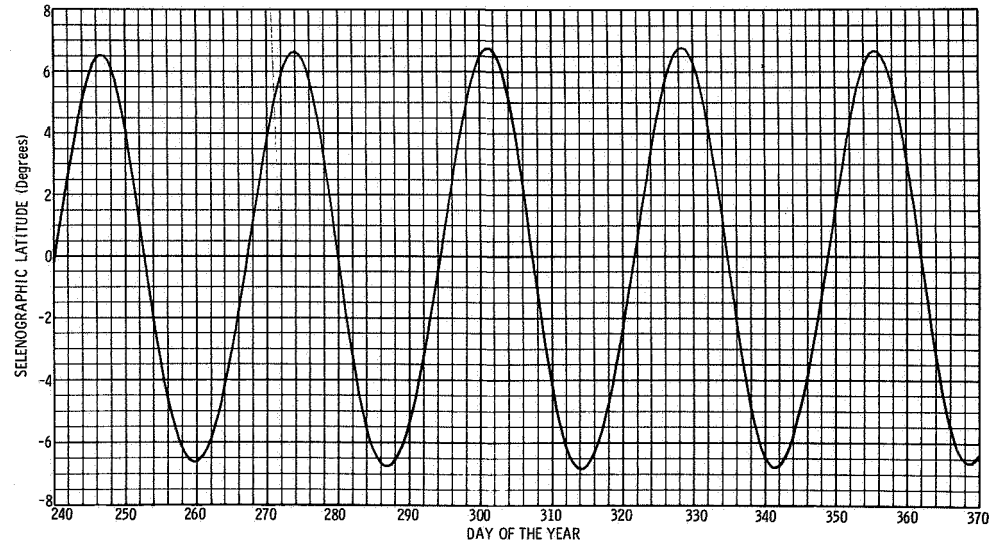


FIGURE B1975-14 SELENOGRAPHIC LATITUDE OF THE EARTH

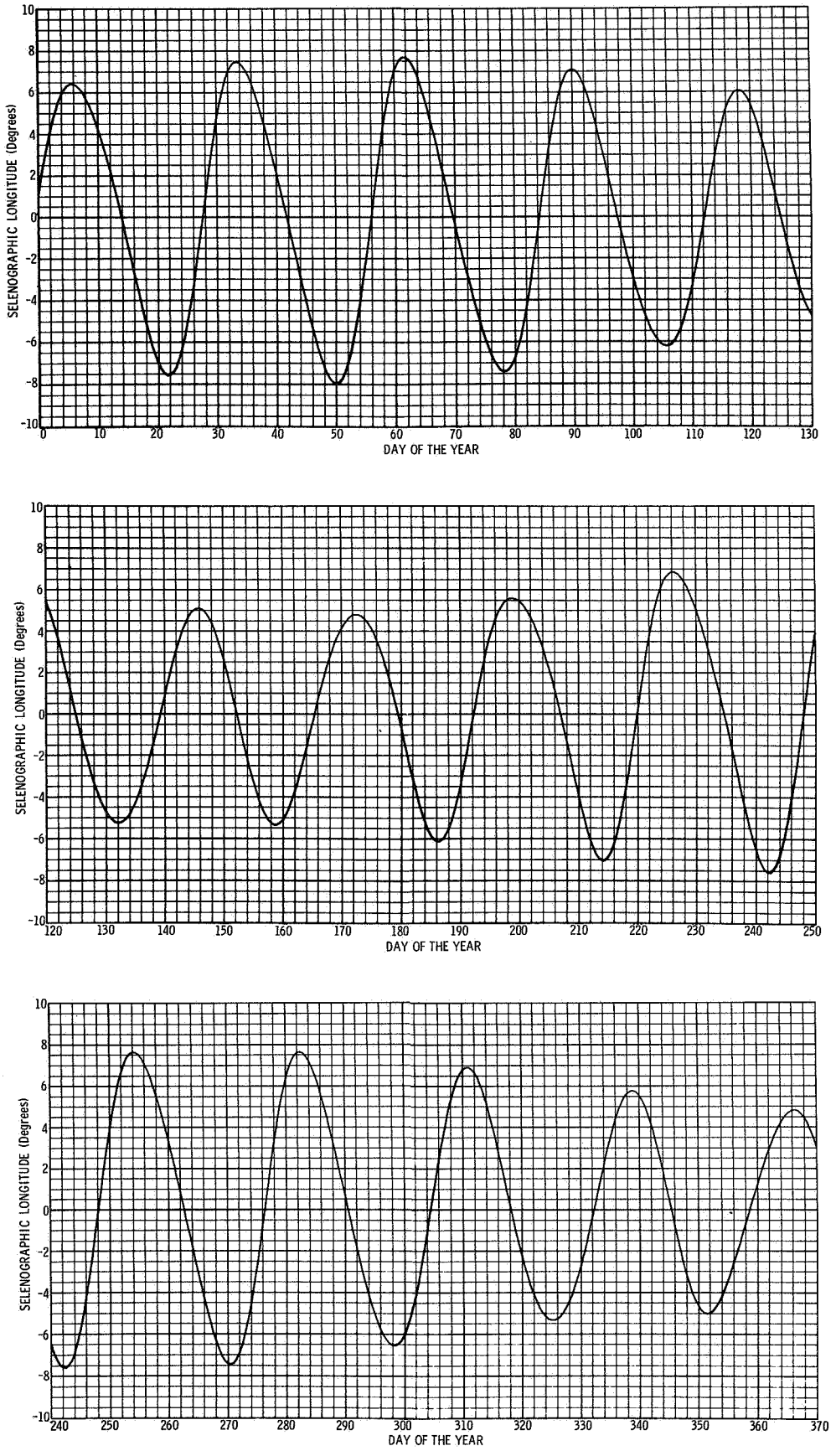


FIGURE B1975-15 SELENOGRAPHIC LONGITUDE OF THE EARTH

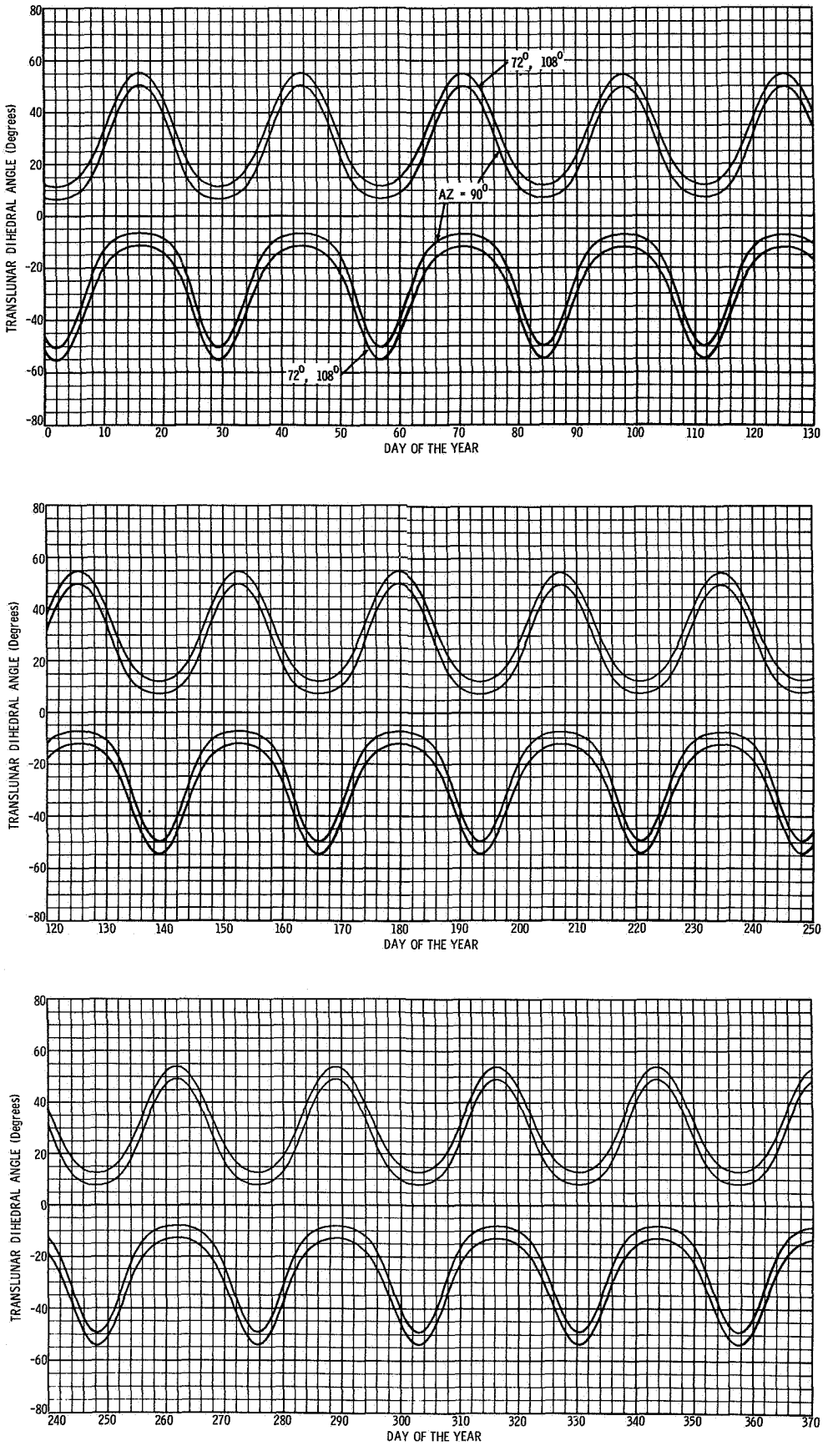


FIGURE B1975-16 TRANSLUNAR DIHEDRAL ANGLES

1976

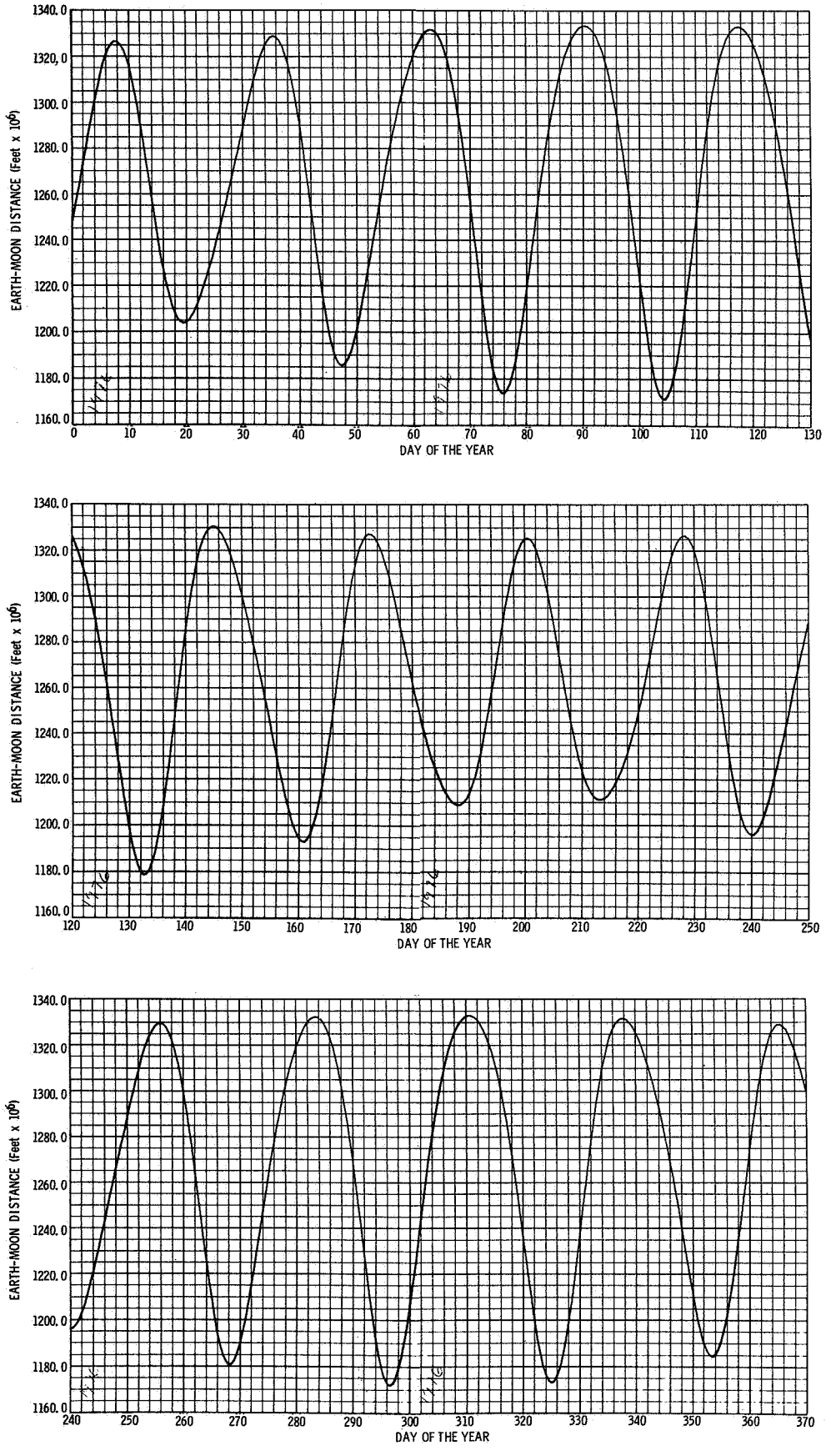
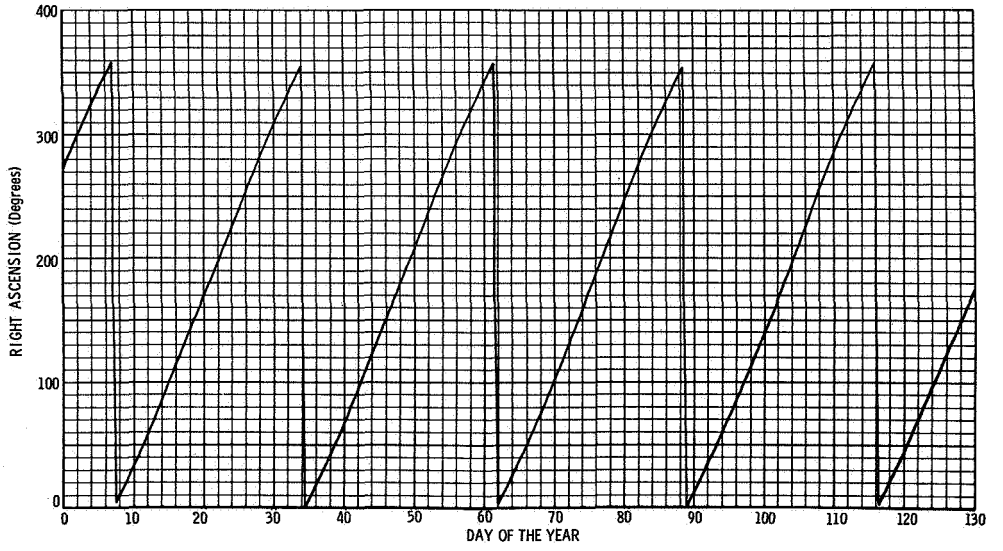
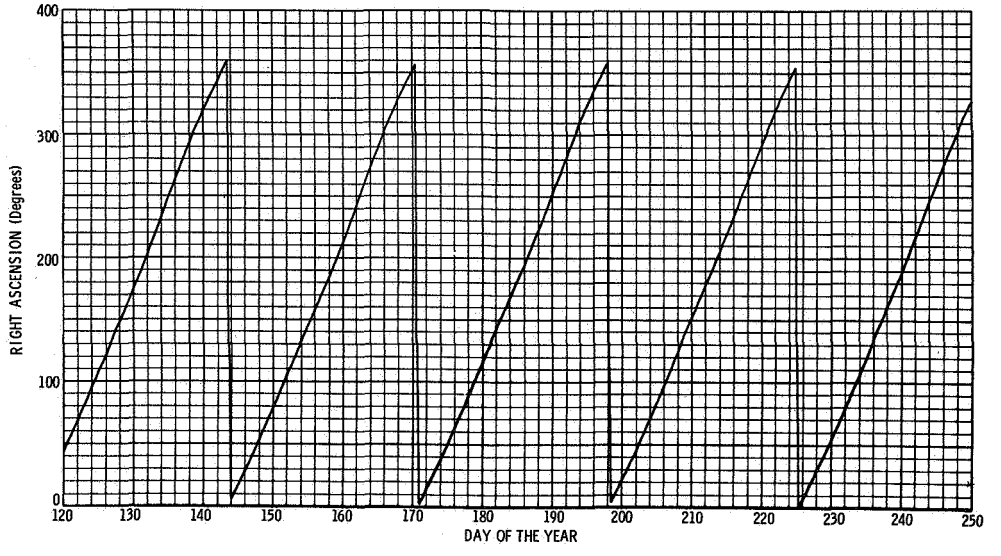


FIGURE B1976-1 EARTH-MOON DISTANCE





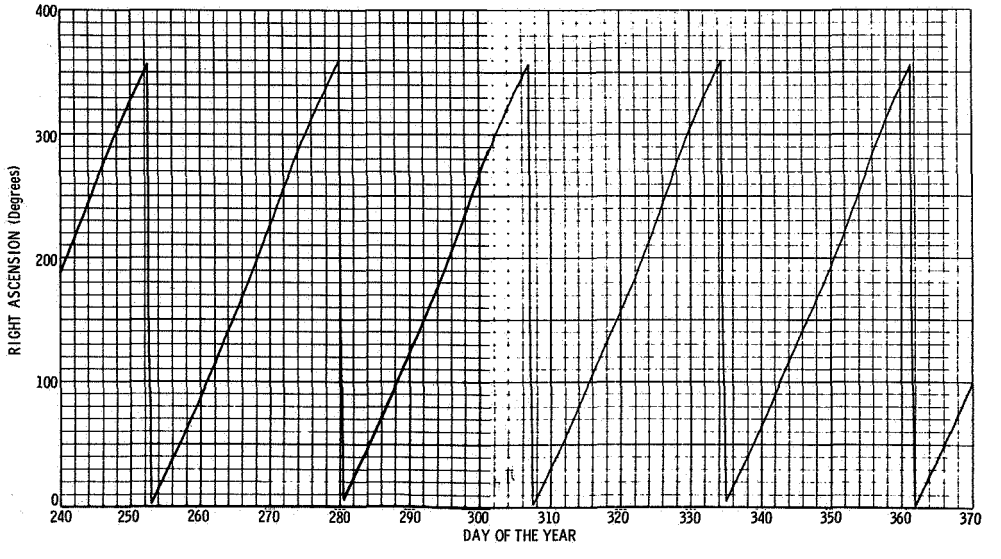
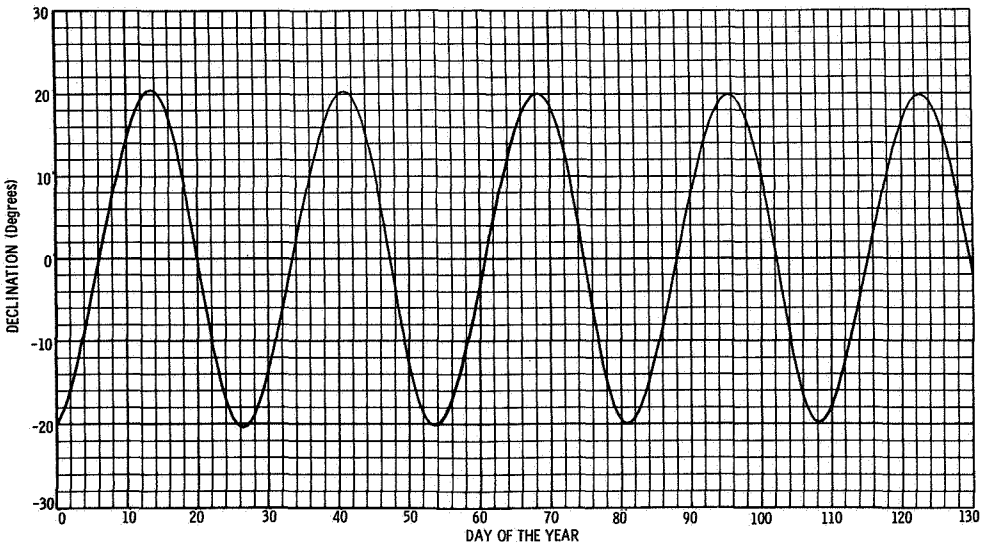
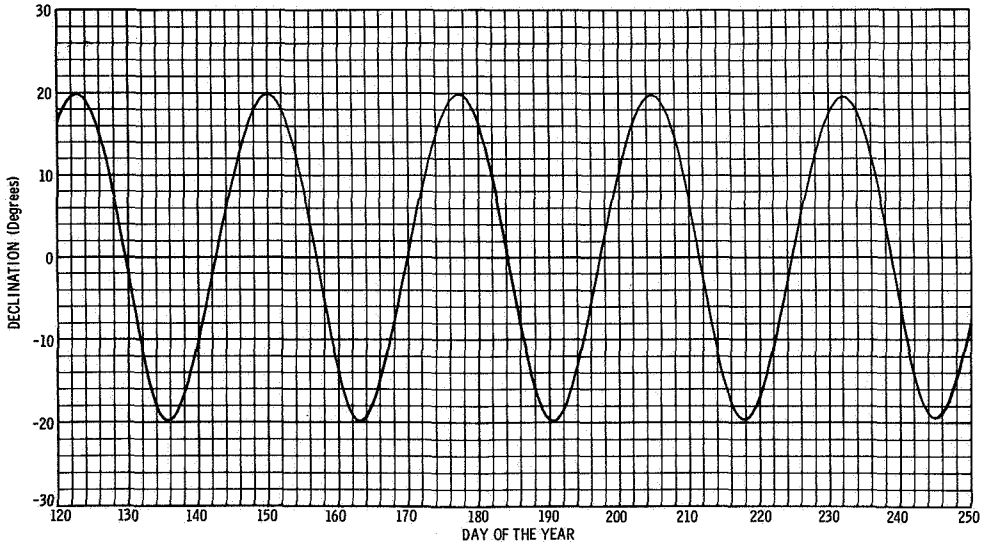


FIGURE B1976-2 RIGHT ASCENSION OF THE MOON





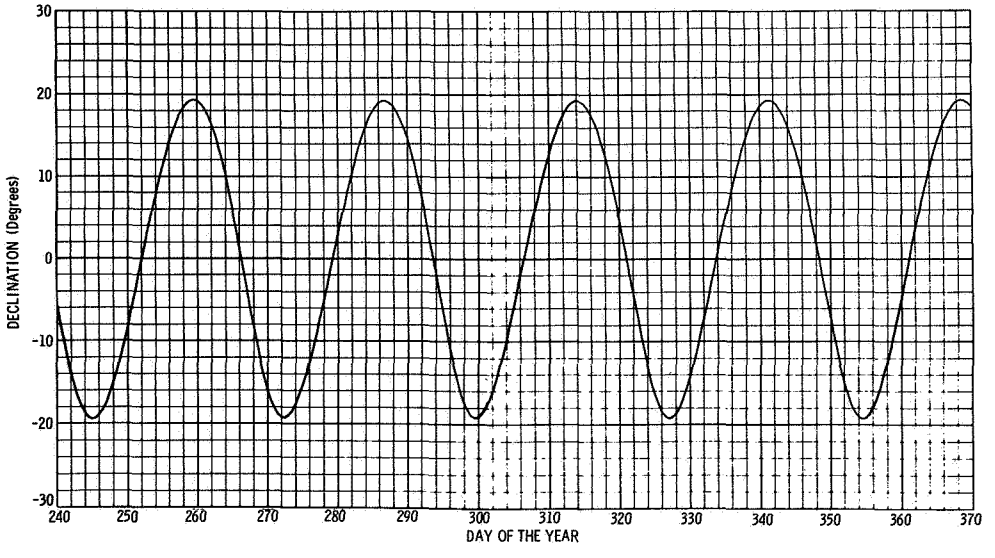
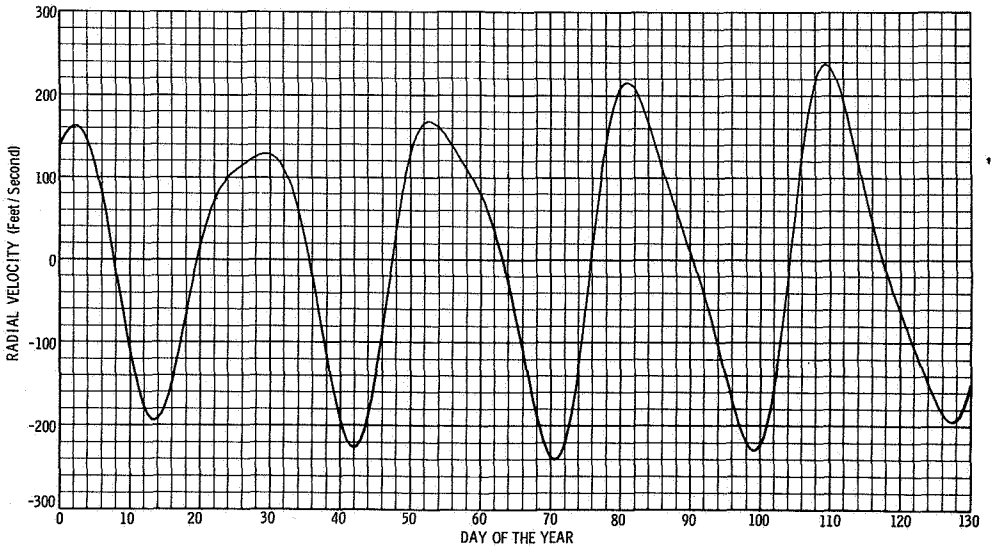
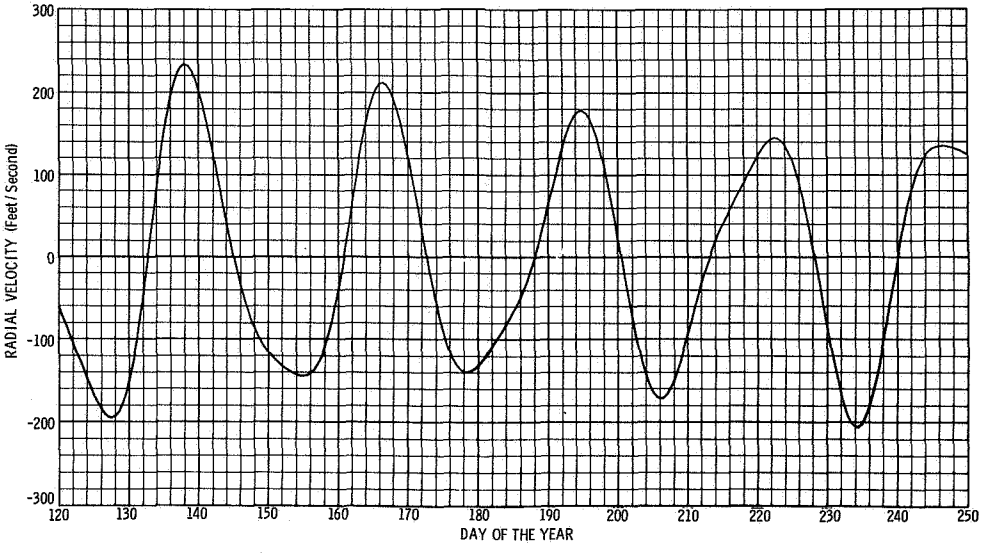


FIGURE B1976-3 DECLINATION OF THE MOON





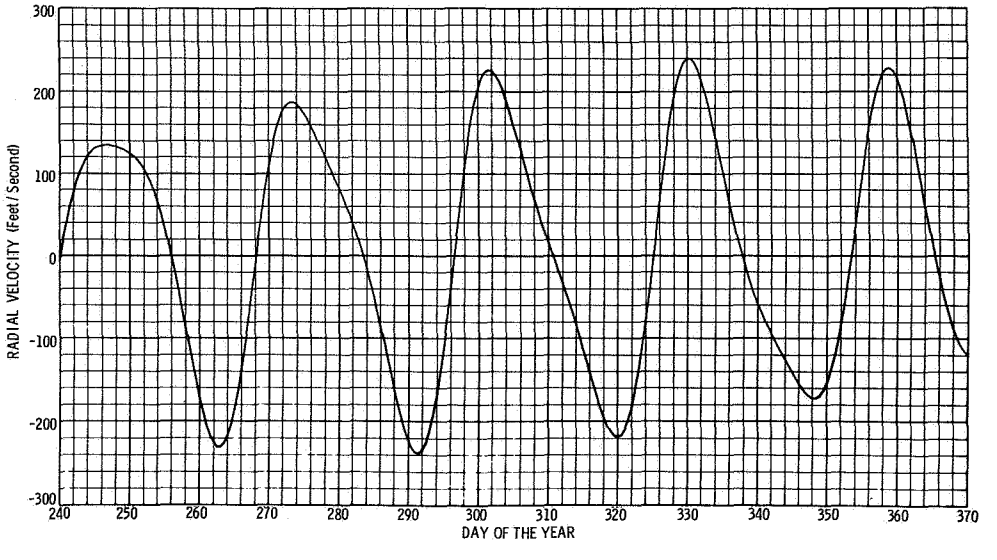
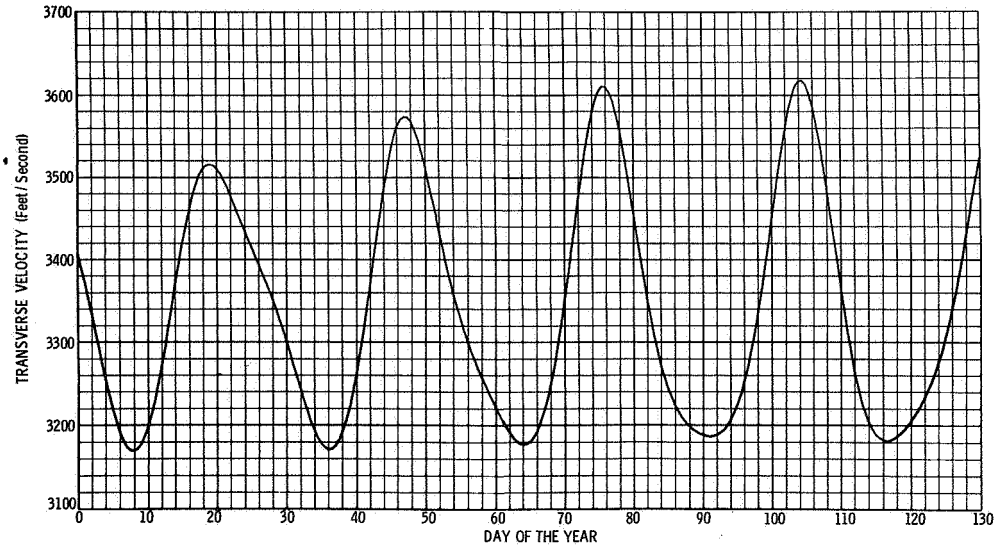
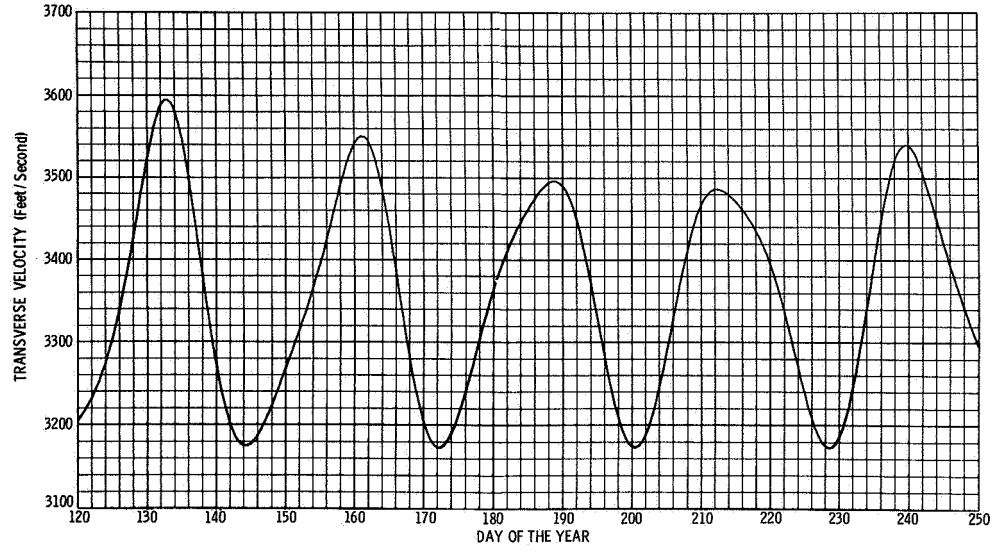


FIGURE B1976-4 RADIAL VELOCITY OF THE MOON





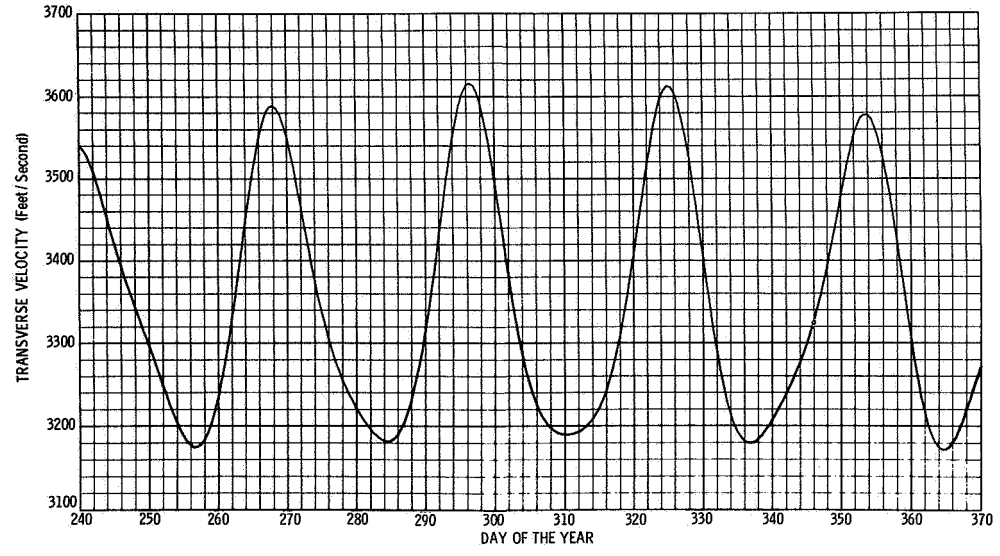


FIGURE B1976-5 TRANSVERSE VELOCITY OF THE MOON

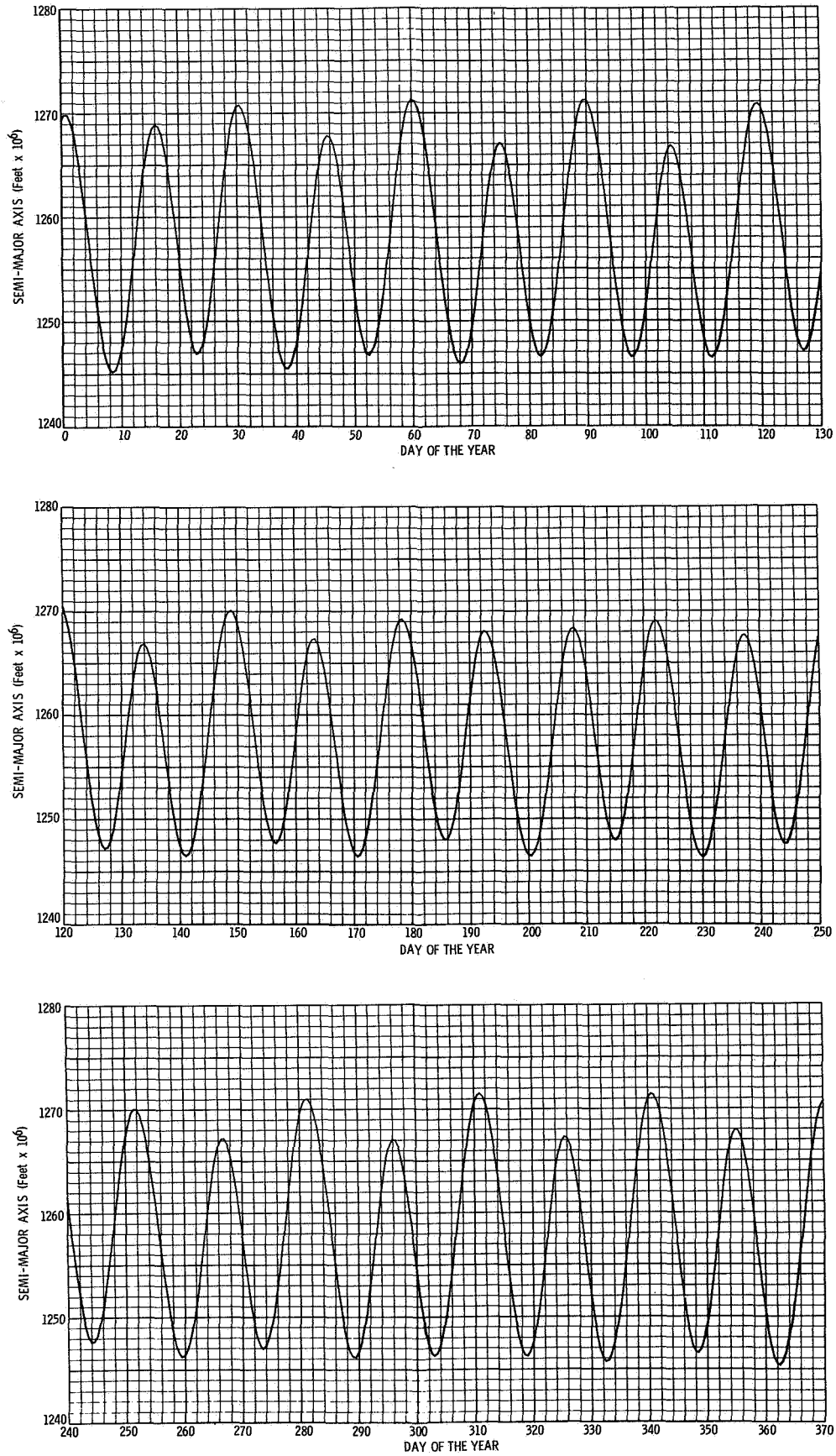


FIGURE B1976-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT

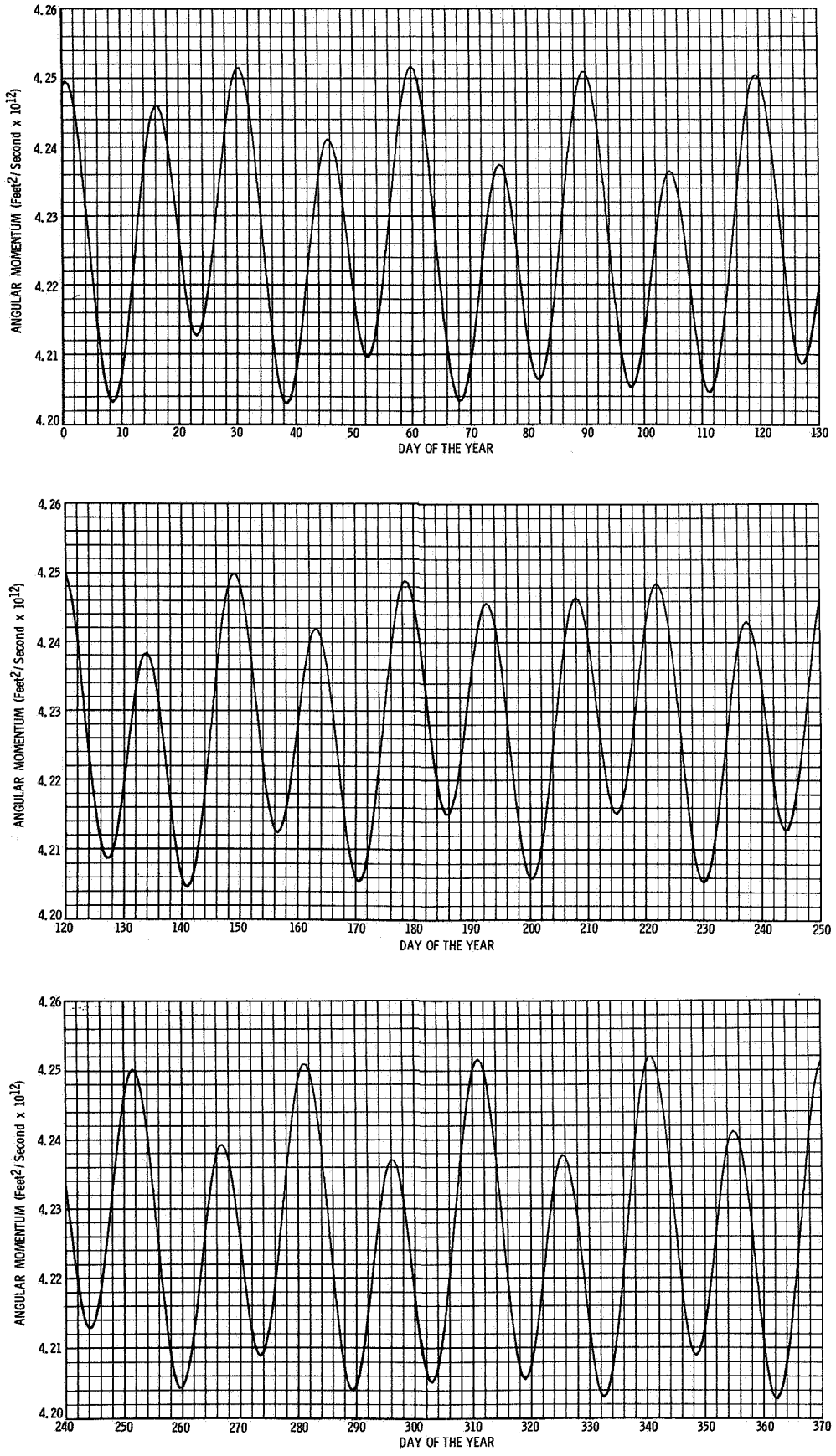


FIGURE B1976-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON

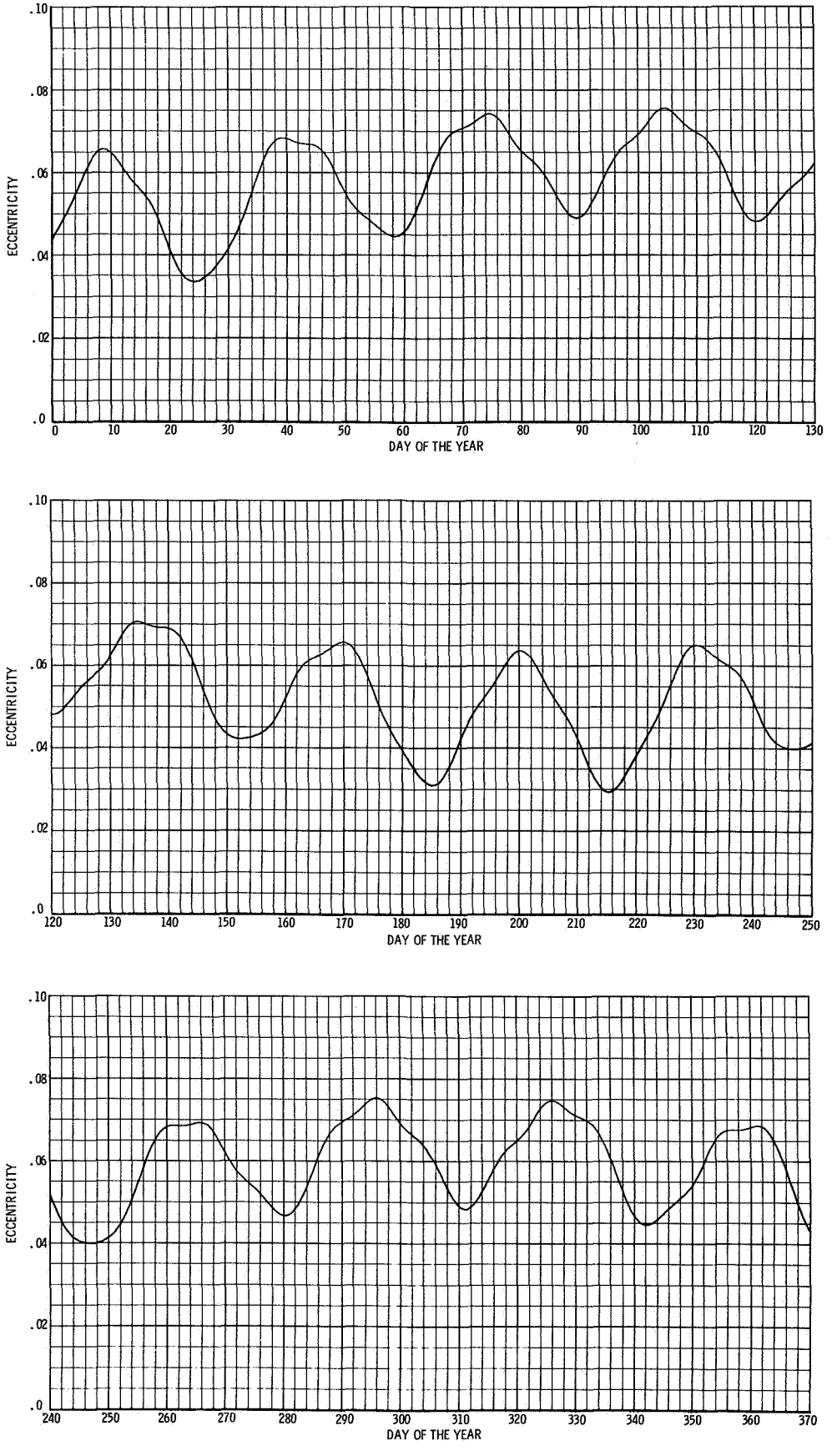


FIGURE B1976-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT

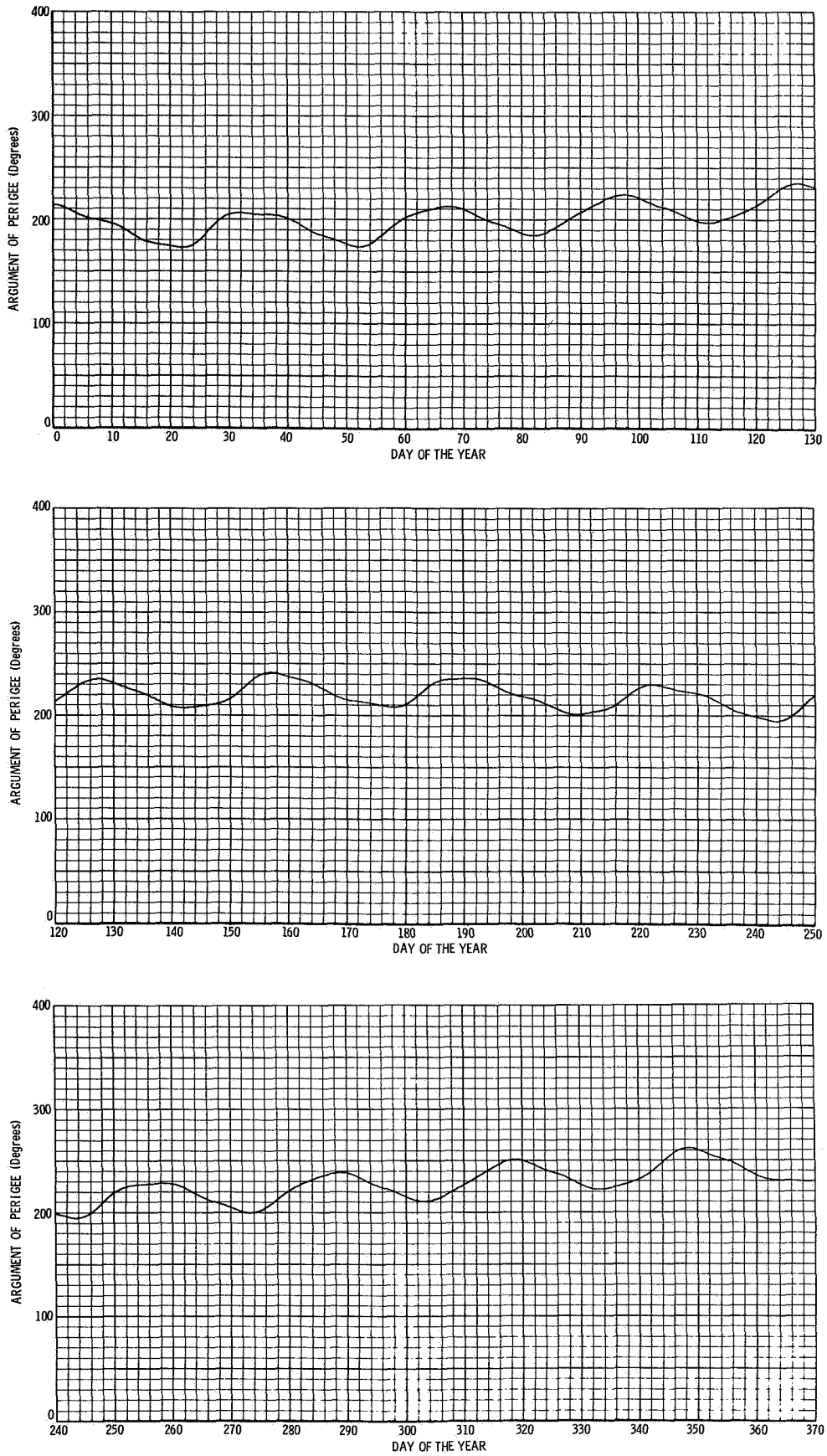
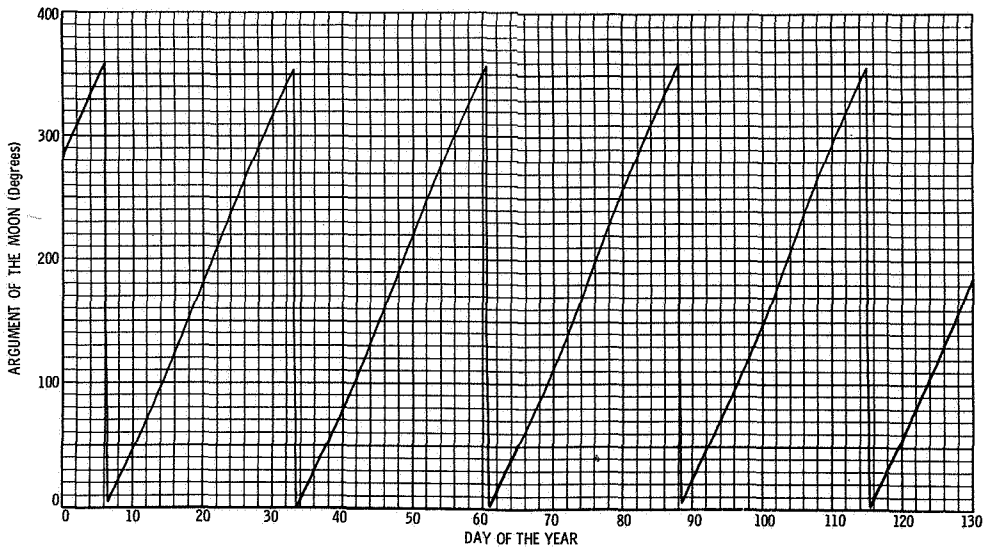
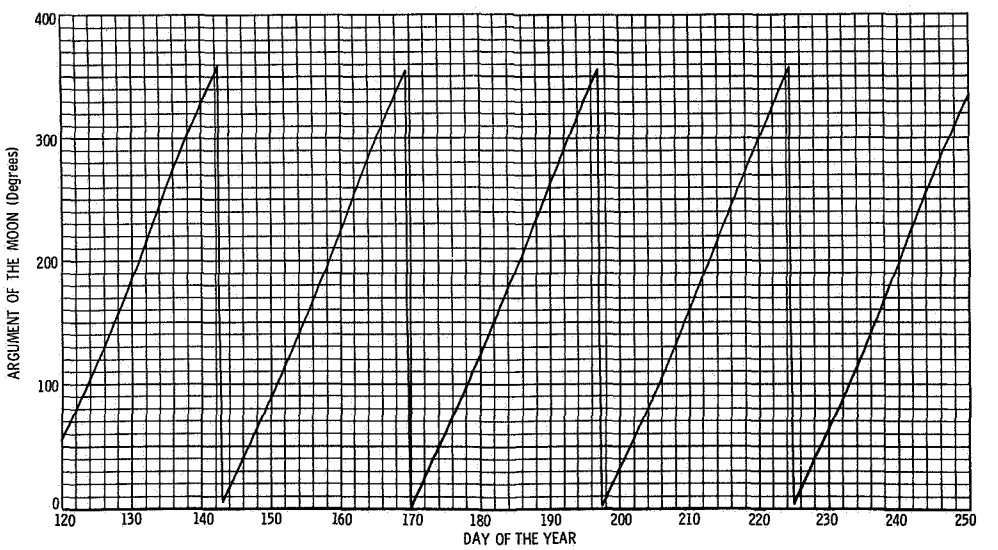


FIGURE B1976-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE





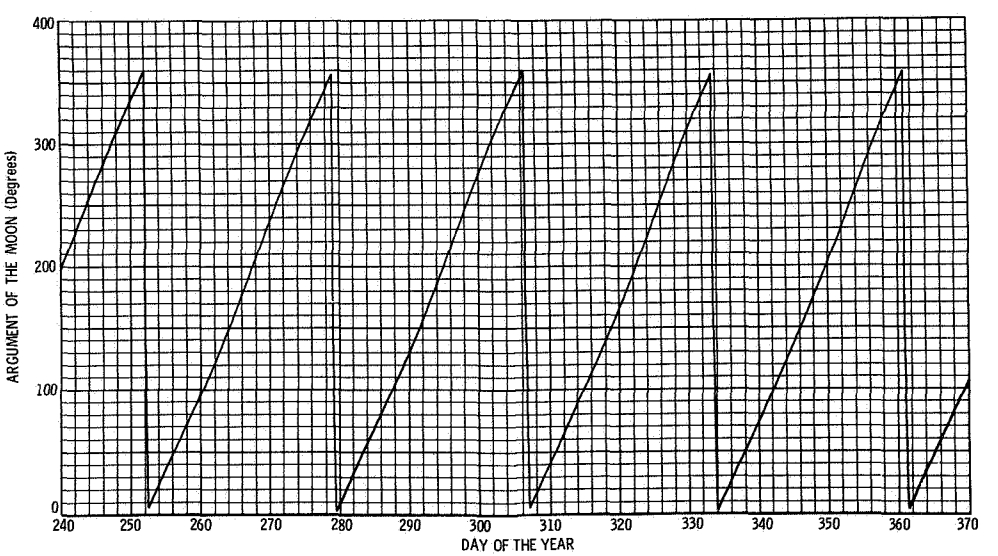
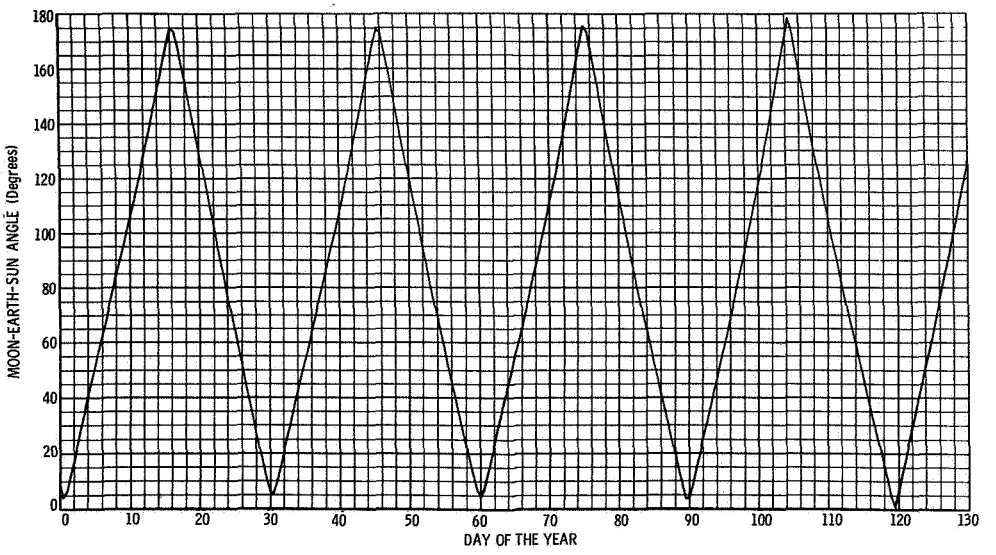
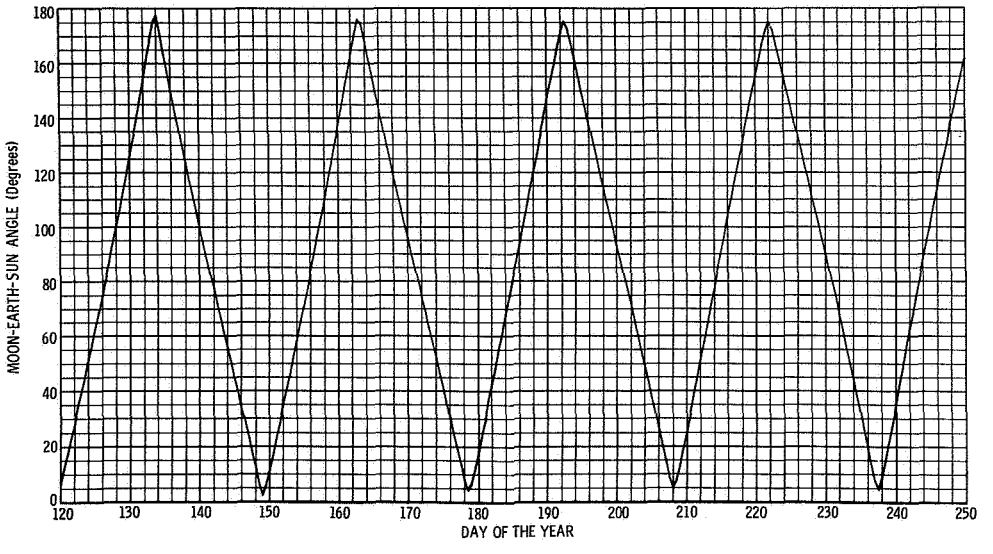


FIGURE B1976-10 ARGUMENT OF THE MOON'S POSITION





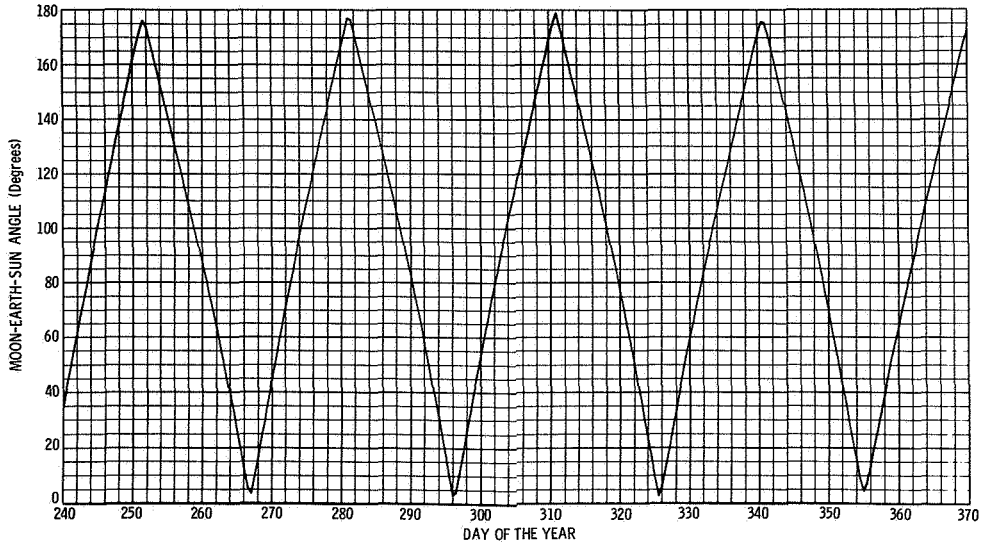
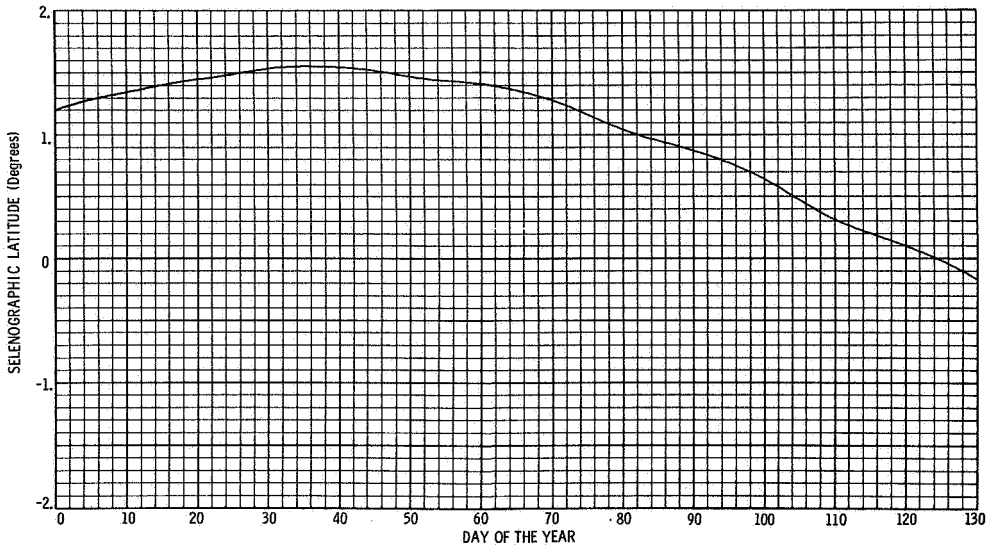
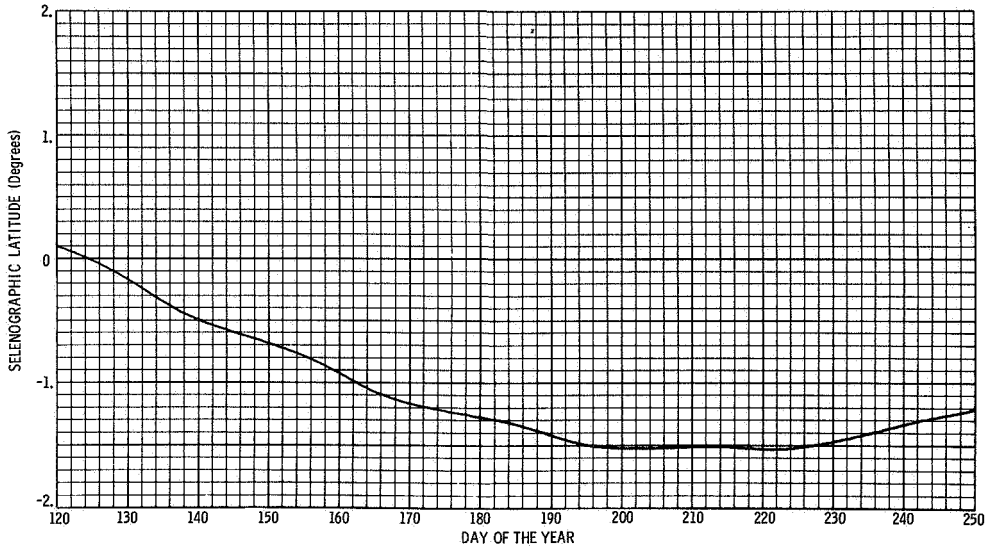


FIGURE B1976-11 MOON-EARTH-SUN ANGLE





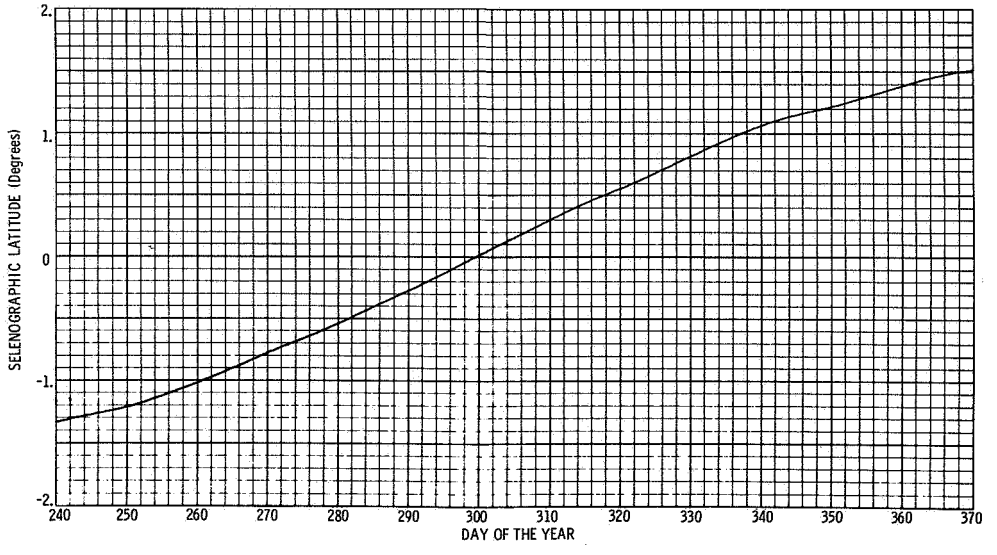


FIGURE B1976-12 SELENOGRAPHIC LATITUDE OF THE SUN

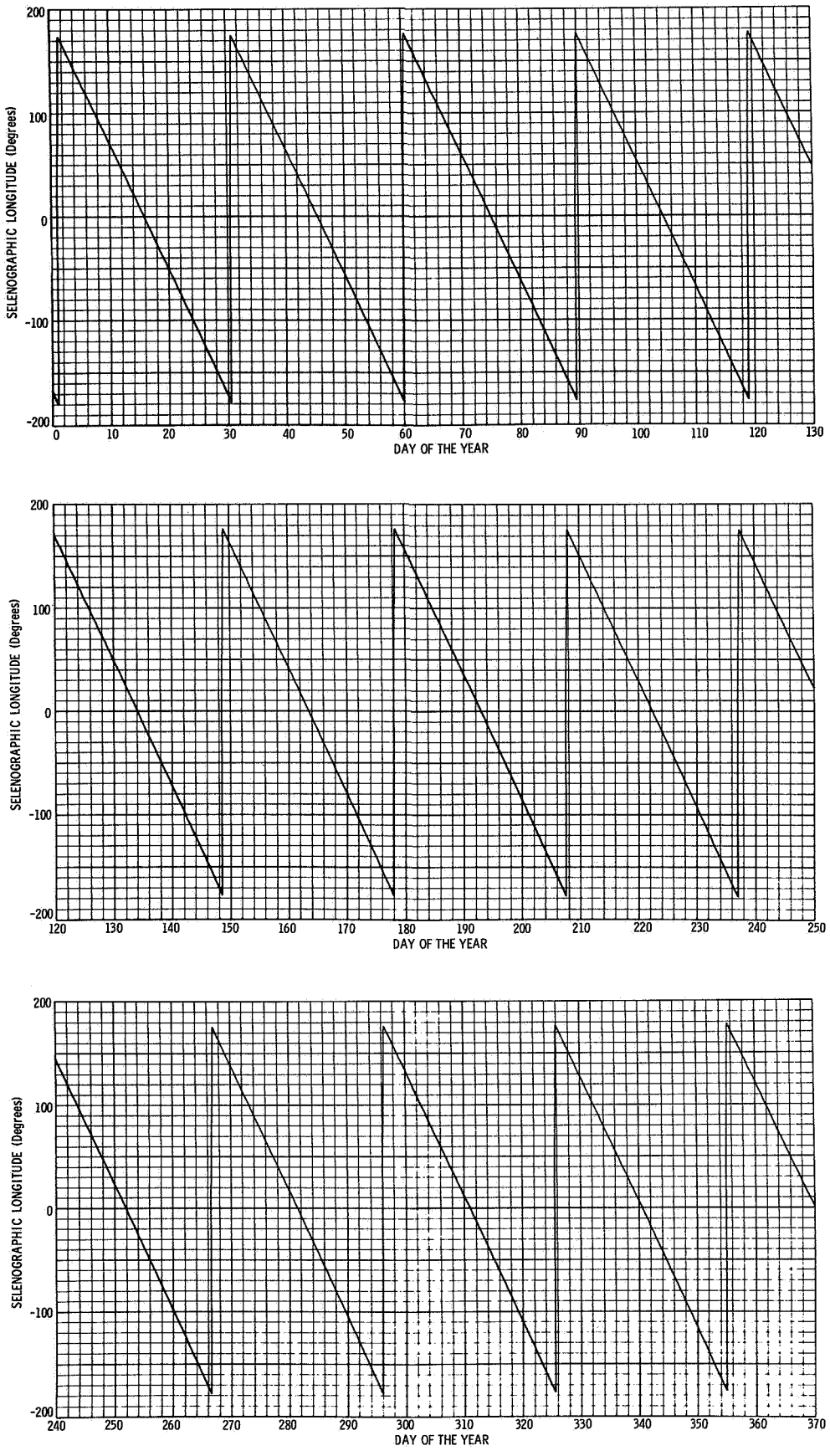
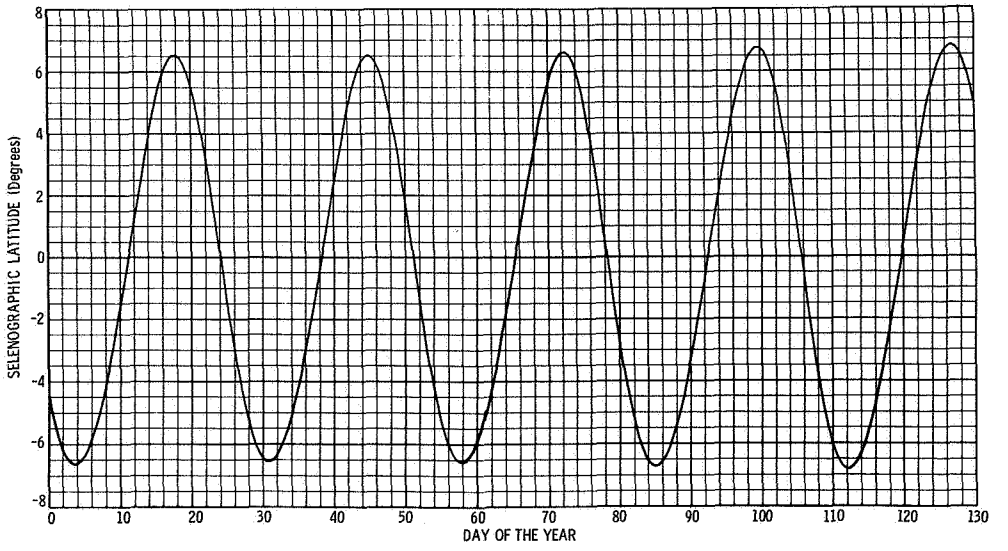
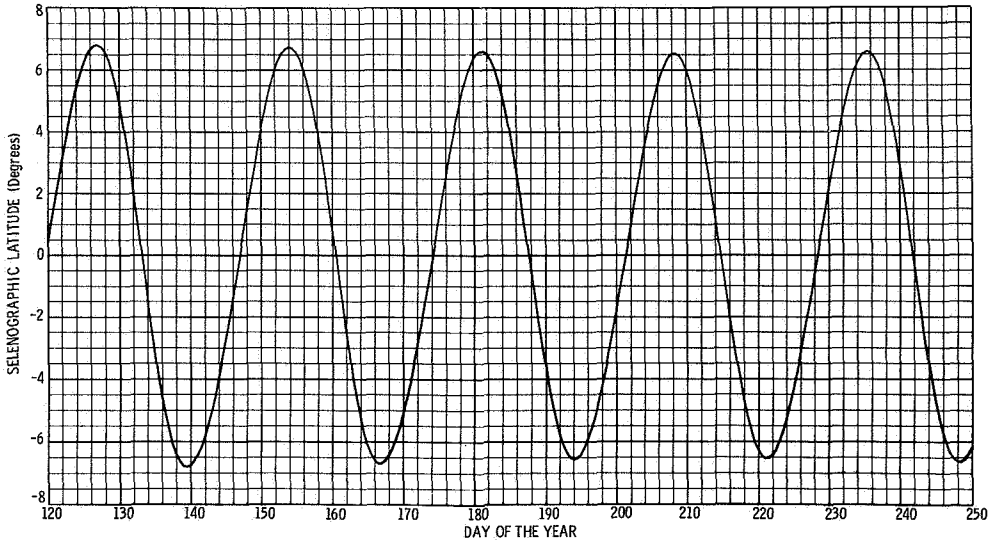


FIGURE B1976-13 SELENOGRAPHIC LONGITUDE OF THE SUN





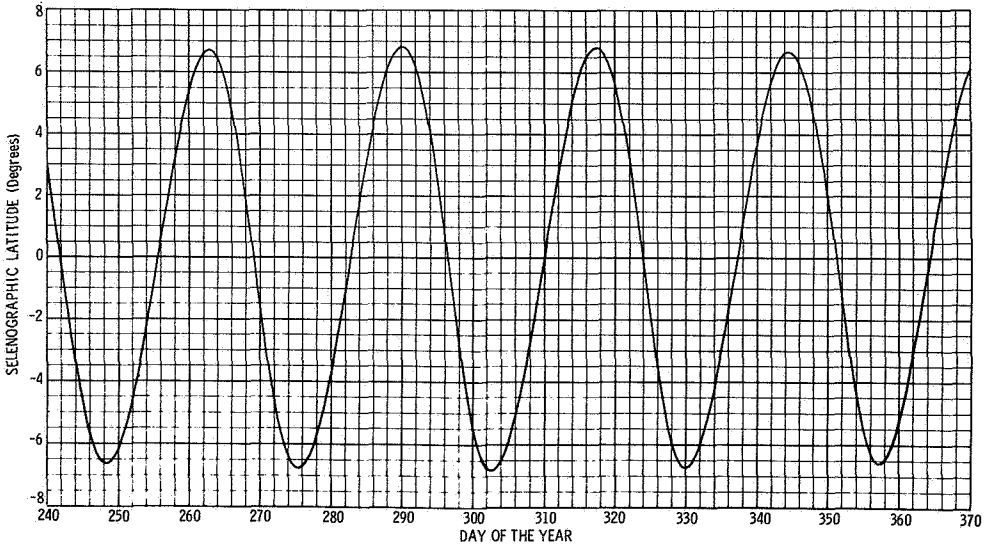
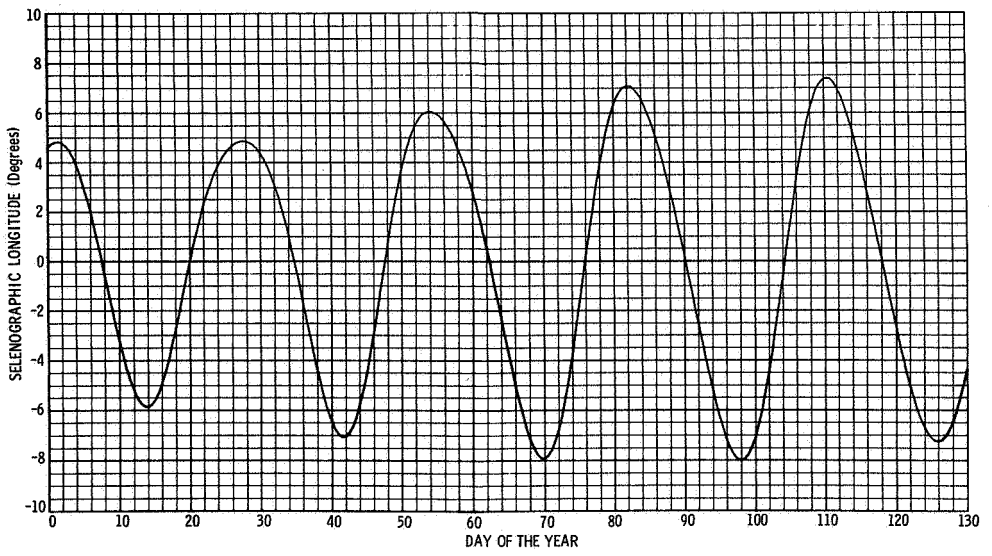
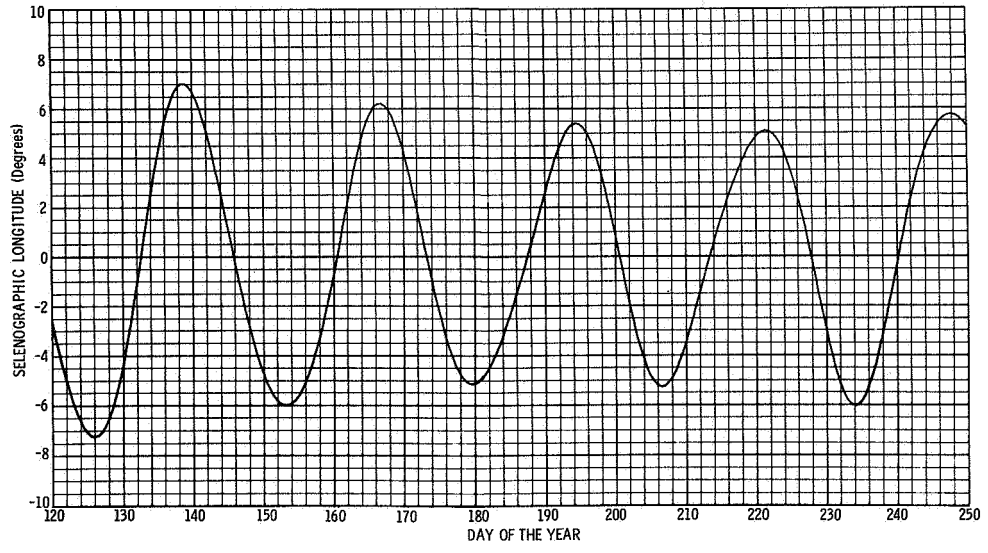


FIGURE B1976-14 SELENOGRAPHIC LATITUDE OF THE EARTH





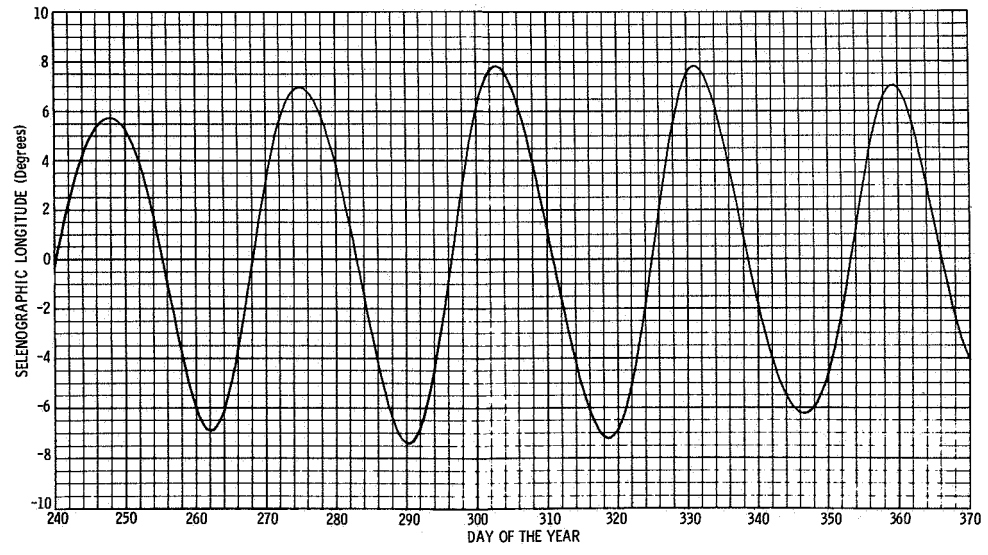
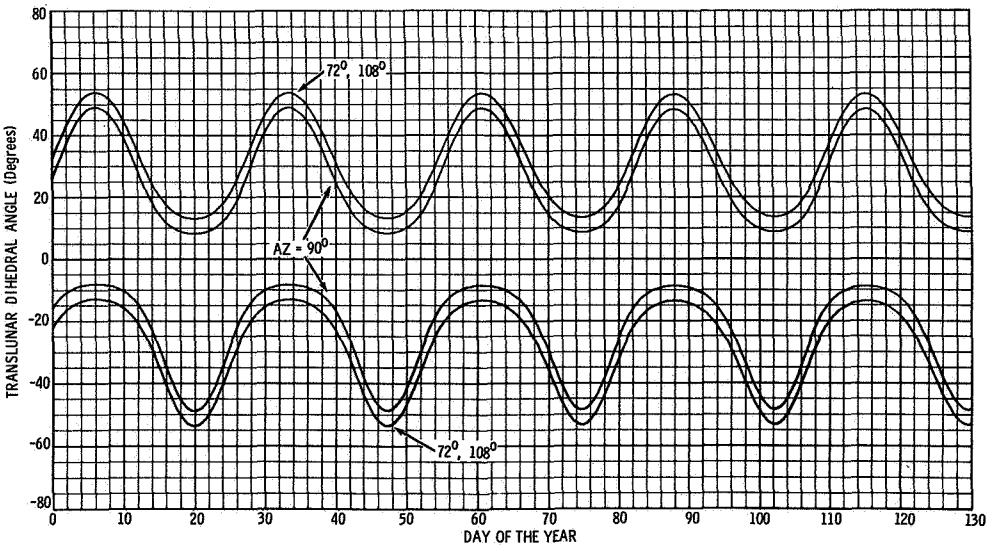
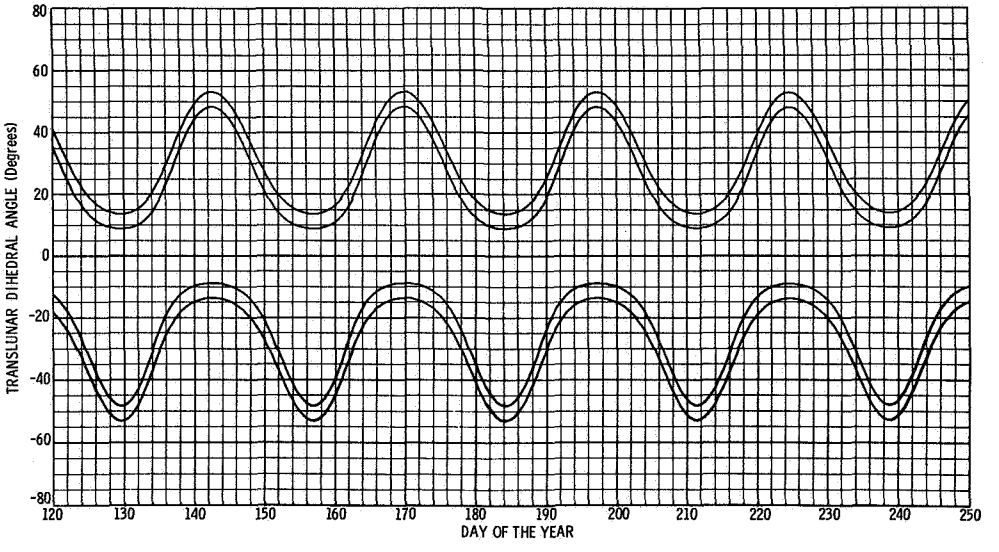


FIGURE B1976-15 SELENOGRAPHIC LONGITUDE OF THE EARTH





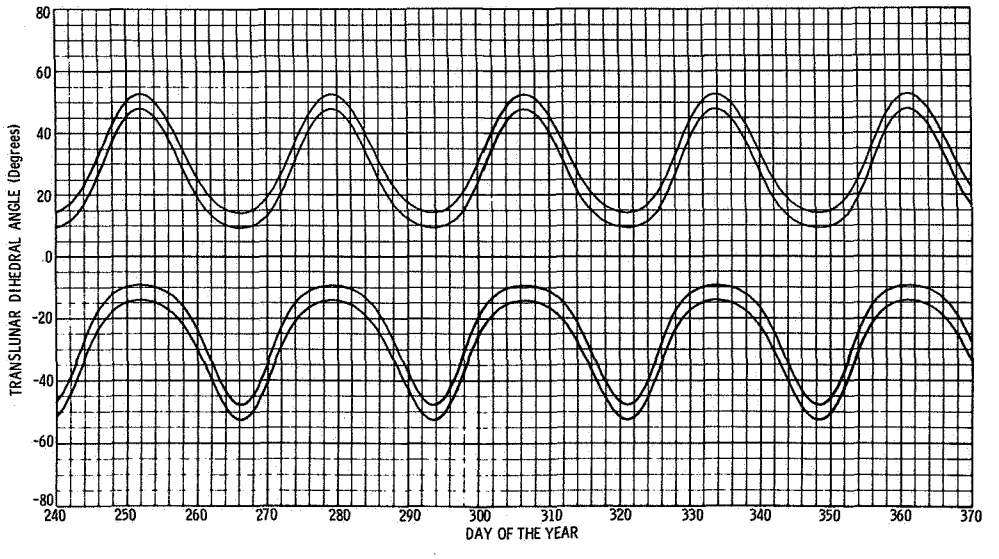
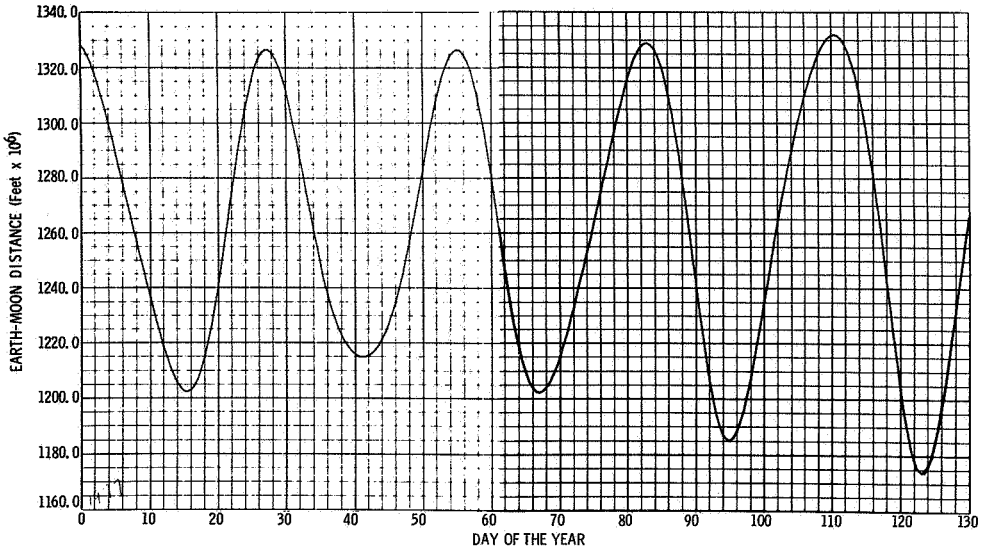
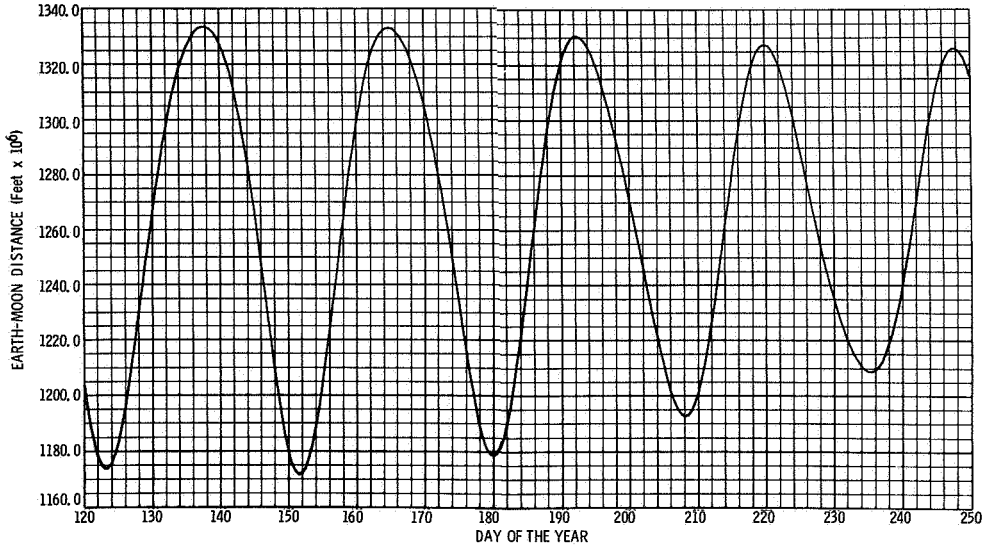


FIGURE B1976-16 TRANSLUNAR DIHEDRAL ANGLES

1977





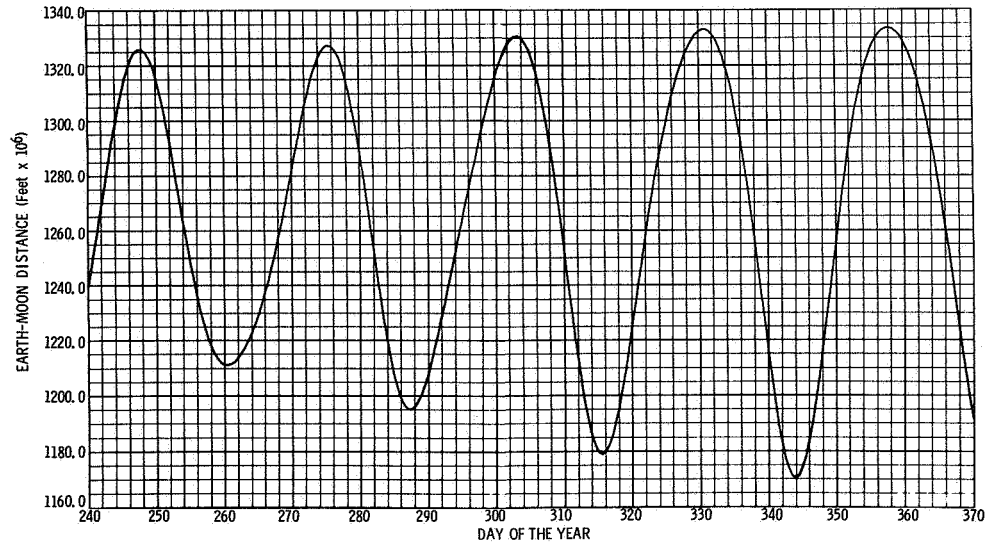
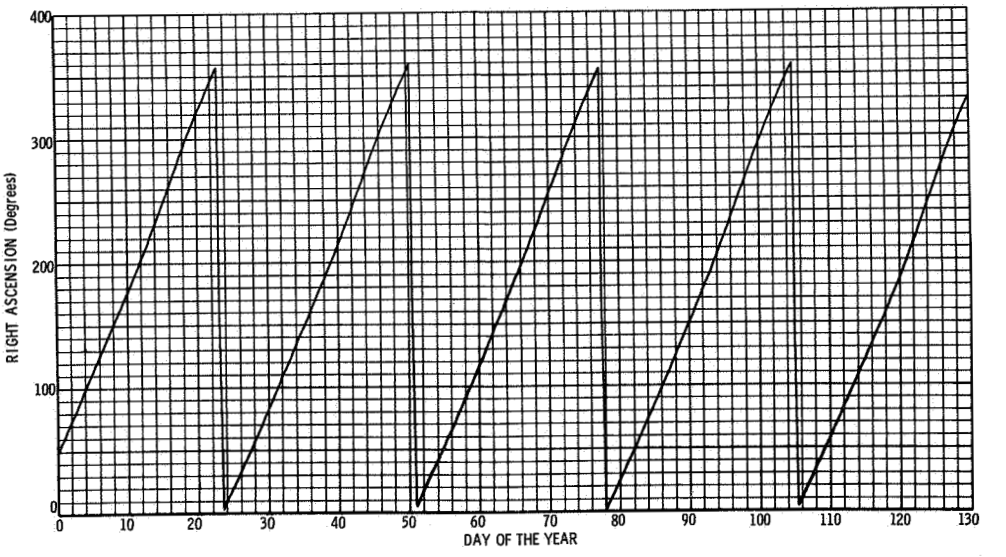
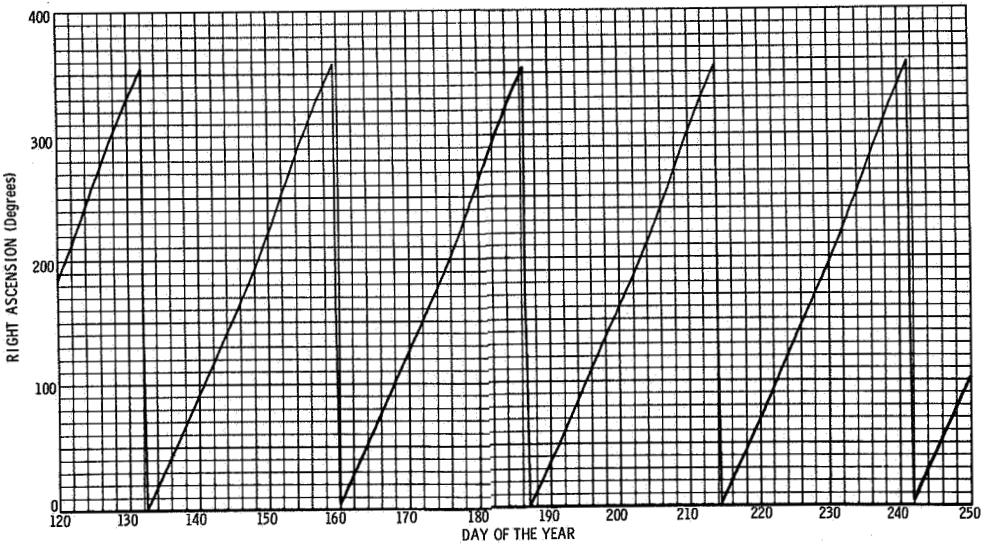


FIGURE B1977-1 EARTH-MOON DISTANCE





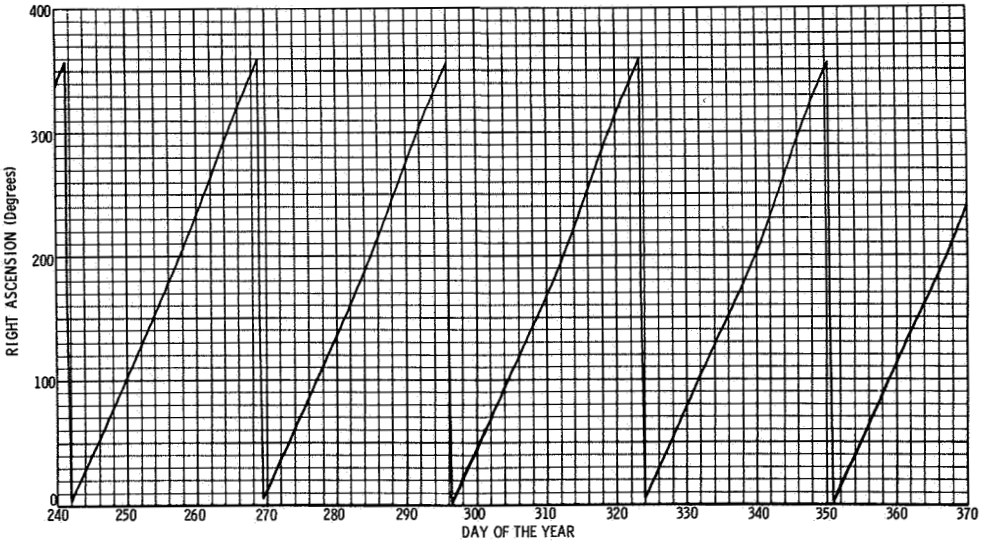
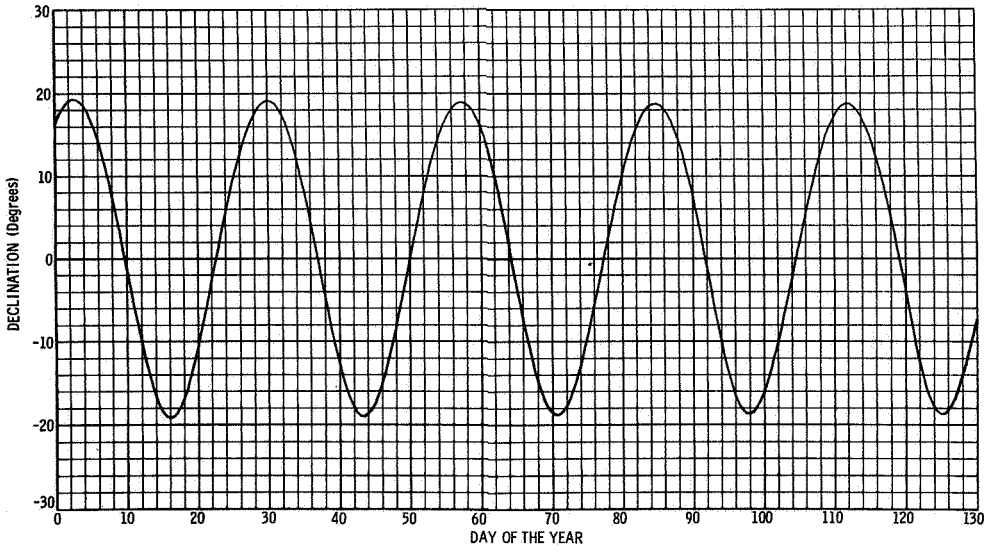
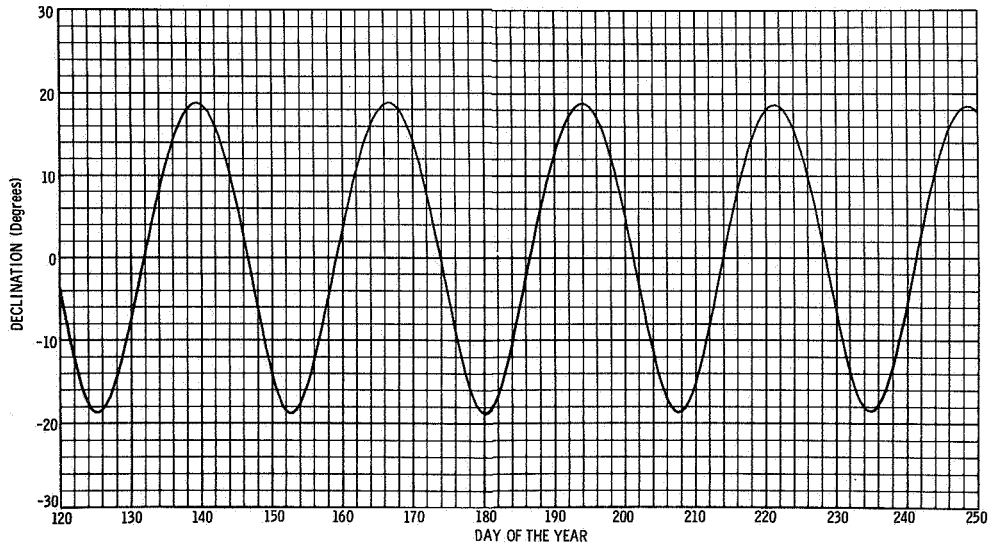


FIGURE B1977-2 RIGHT ASCENSION OF THE MOON





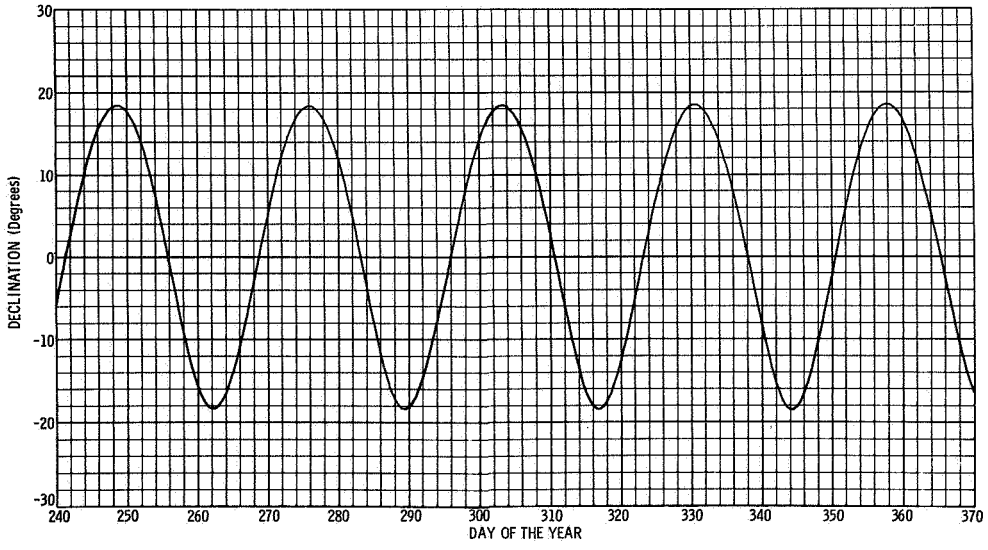


FIGURE B1977-3 DECLINATION OF THE MOON

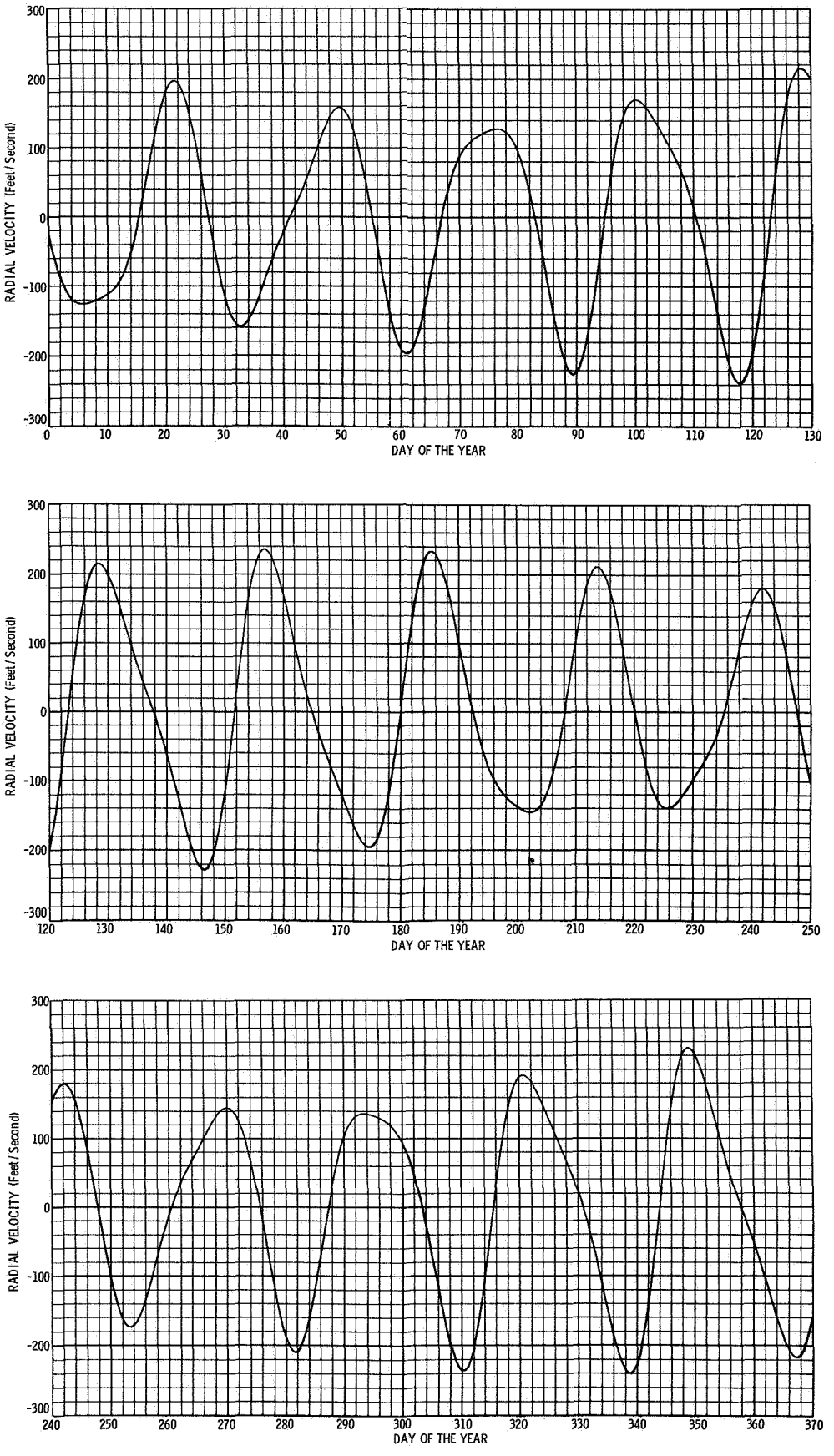
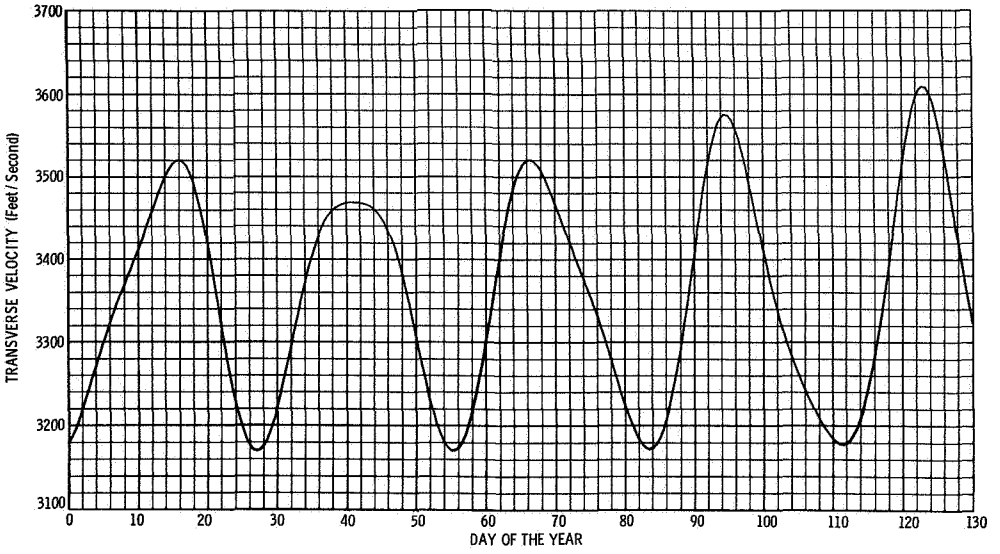
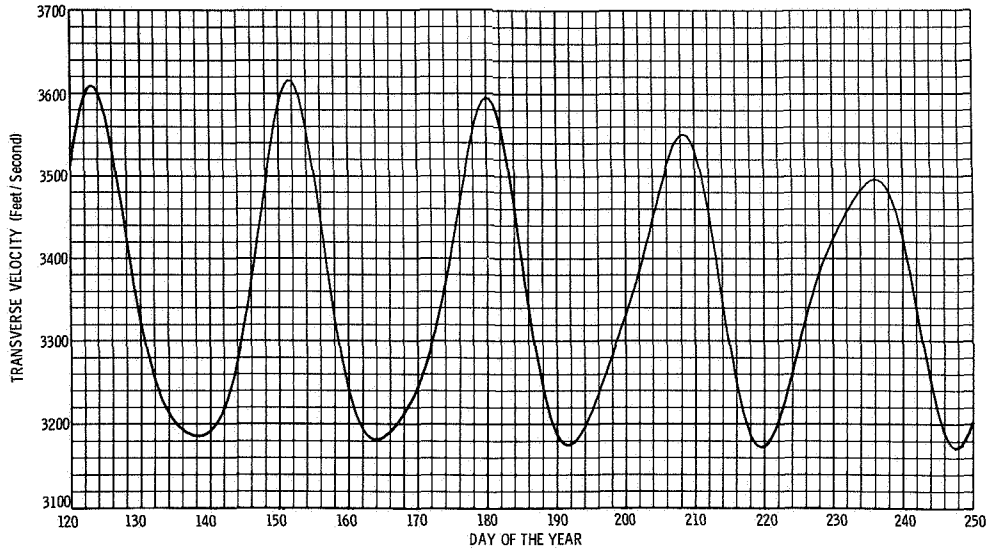


FIGURE B1977-4 RADIAL VELOCITY OF THE MOON





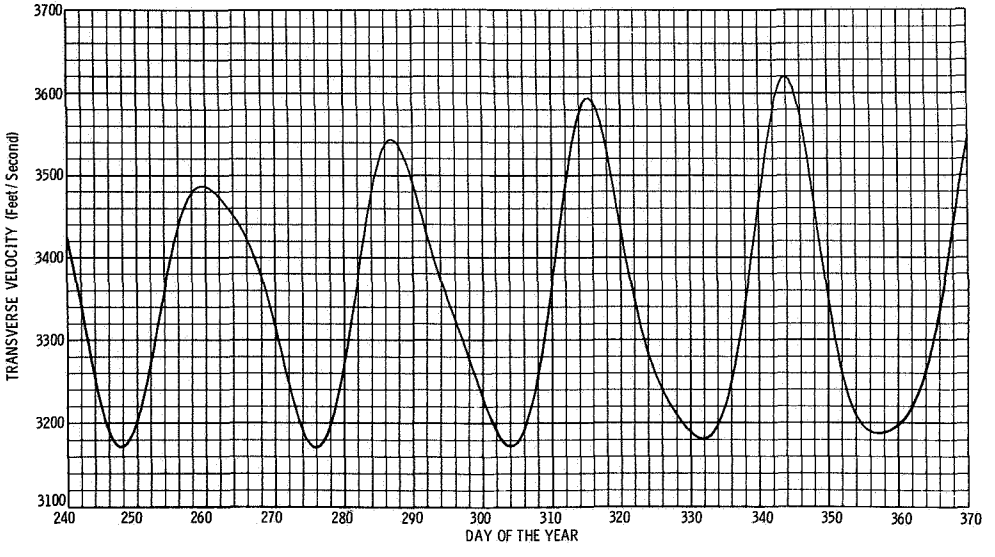


FIGURE B1977-5 TRANSVERSE VELOCITY OF THE MOON

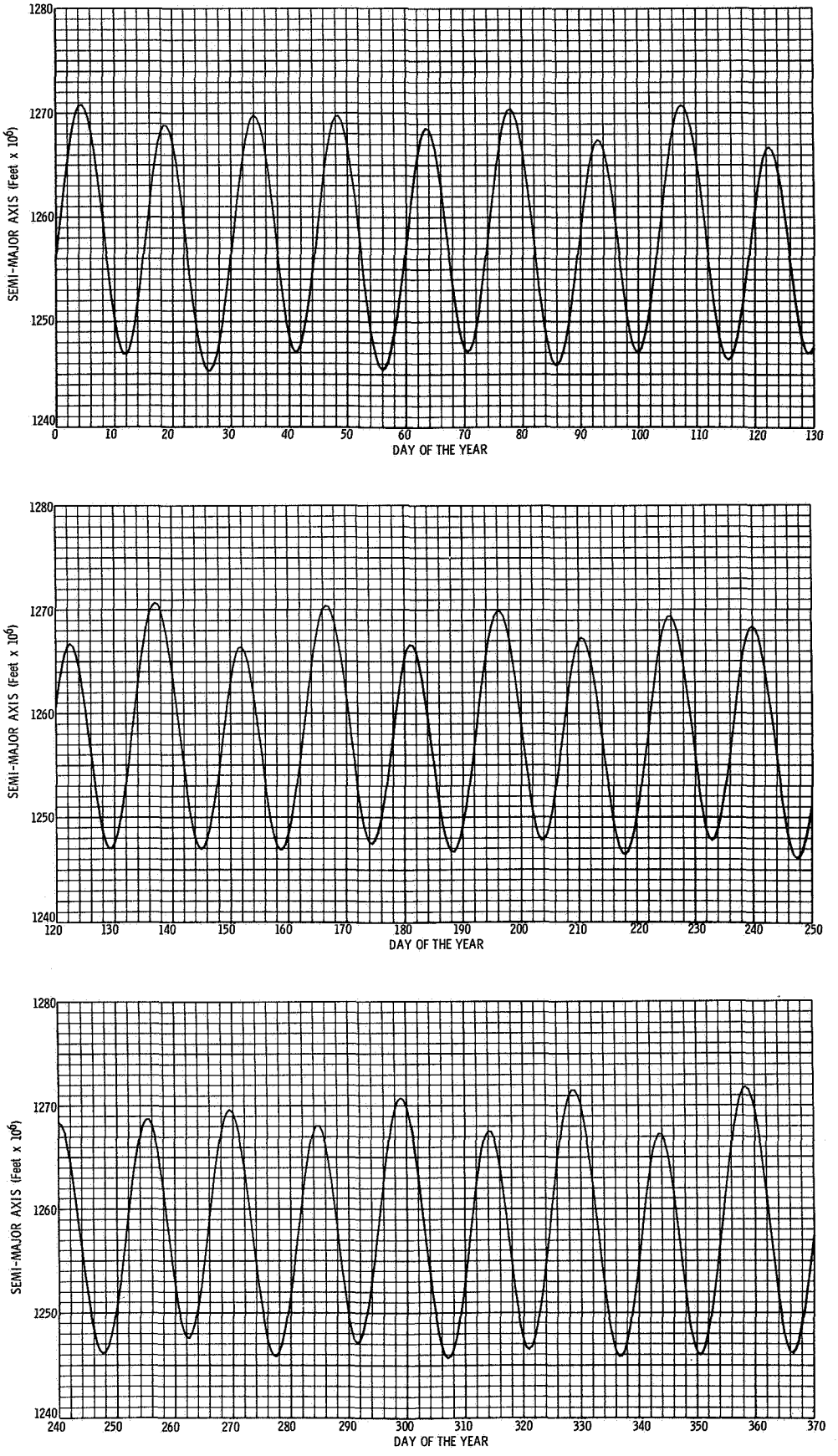


FIGURE B1977-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT

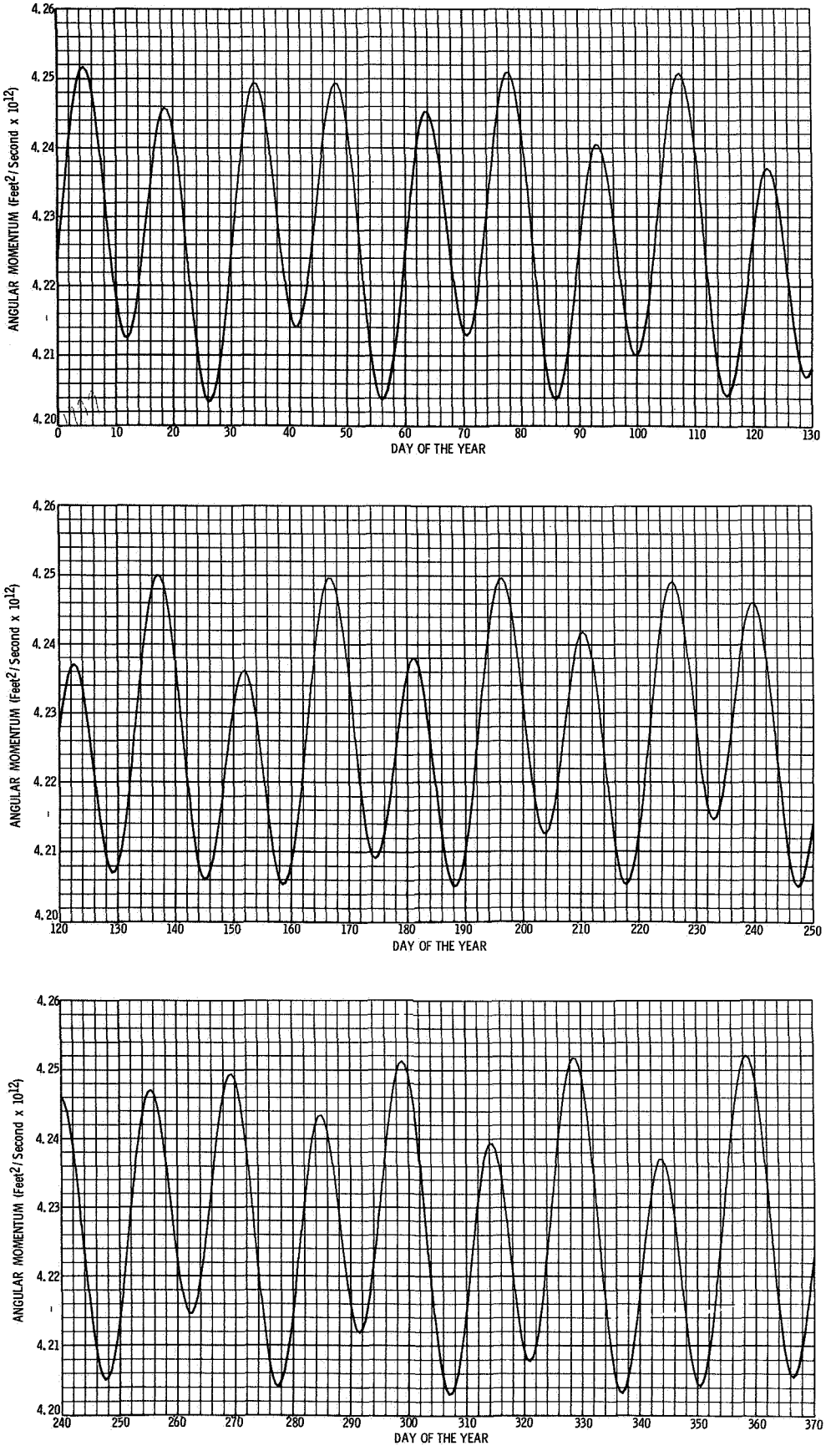
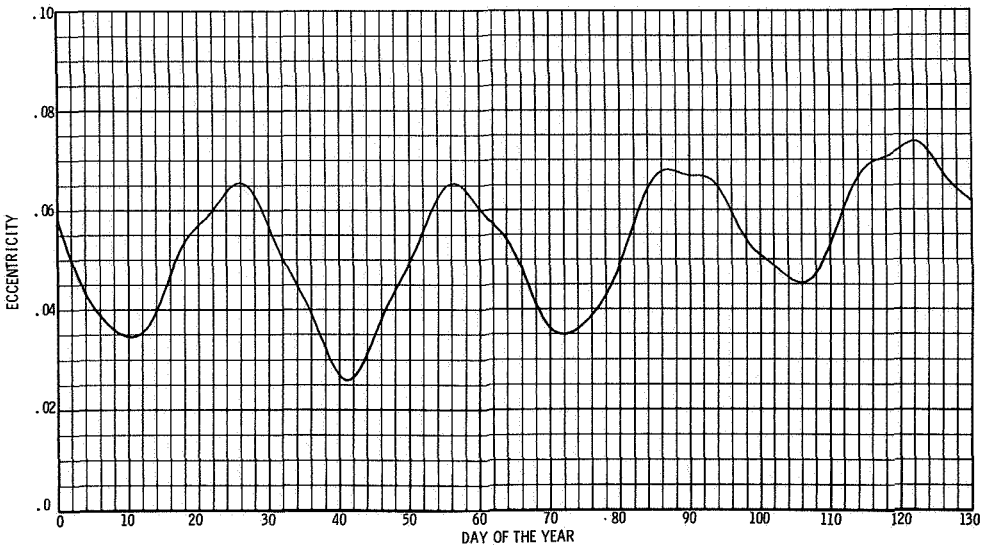
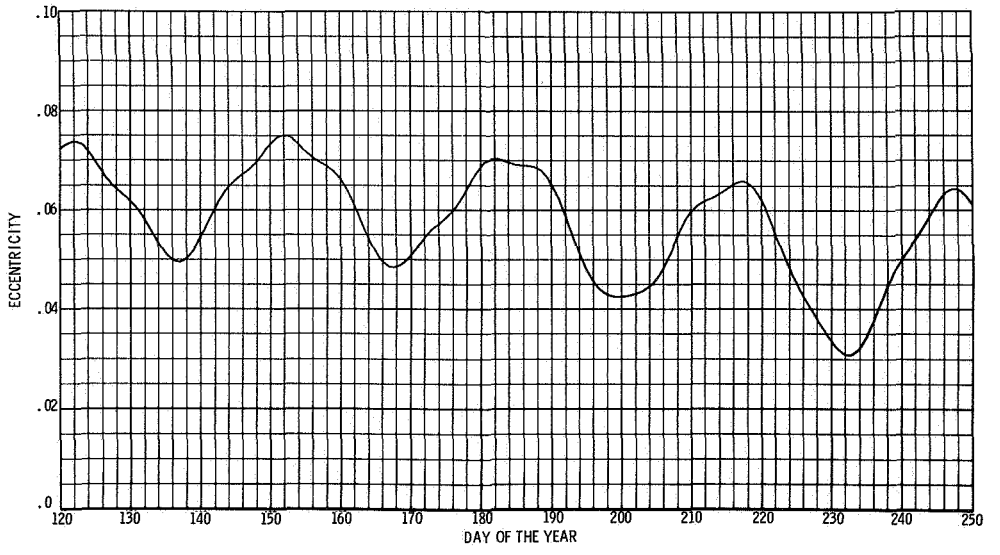


FIGURE B1977-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON





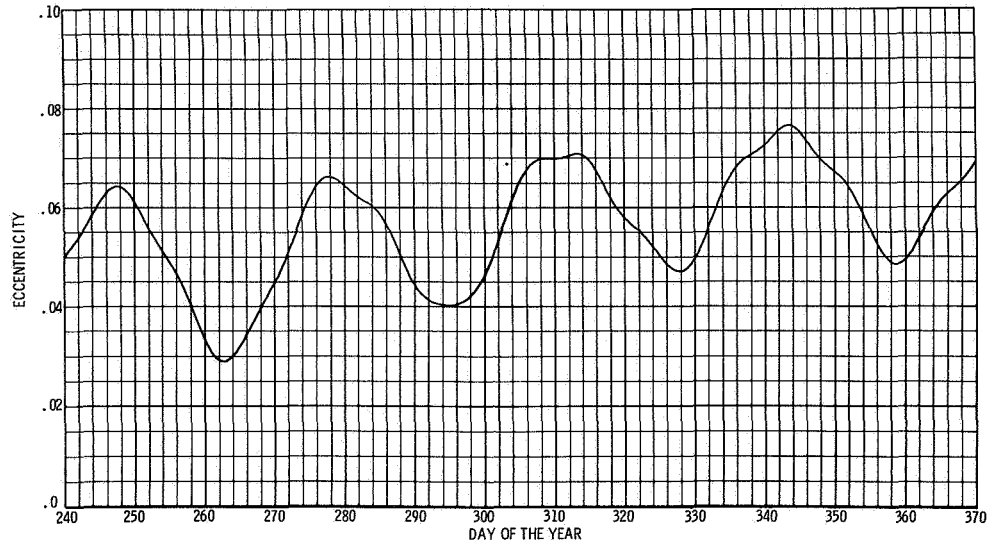


FIGURE B1977-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT

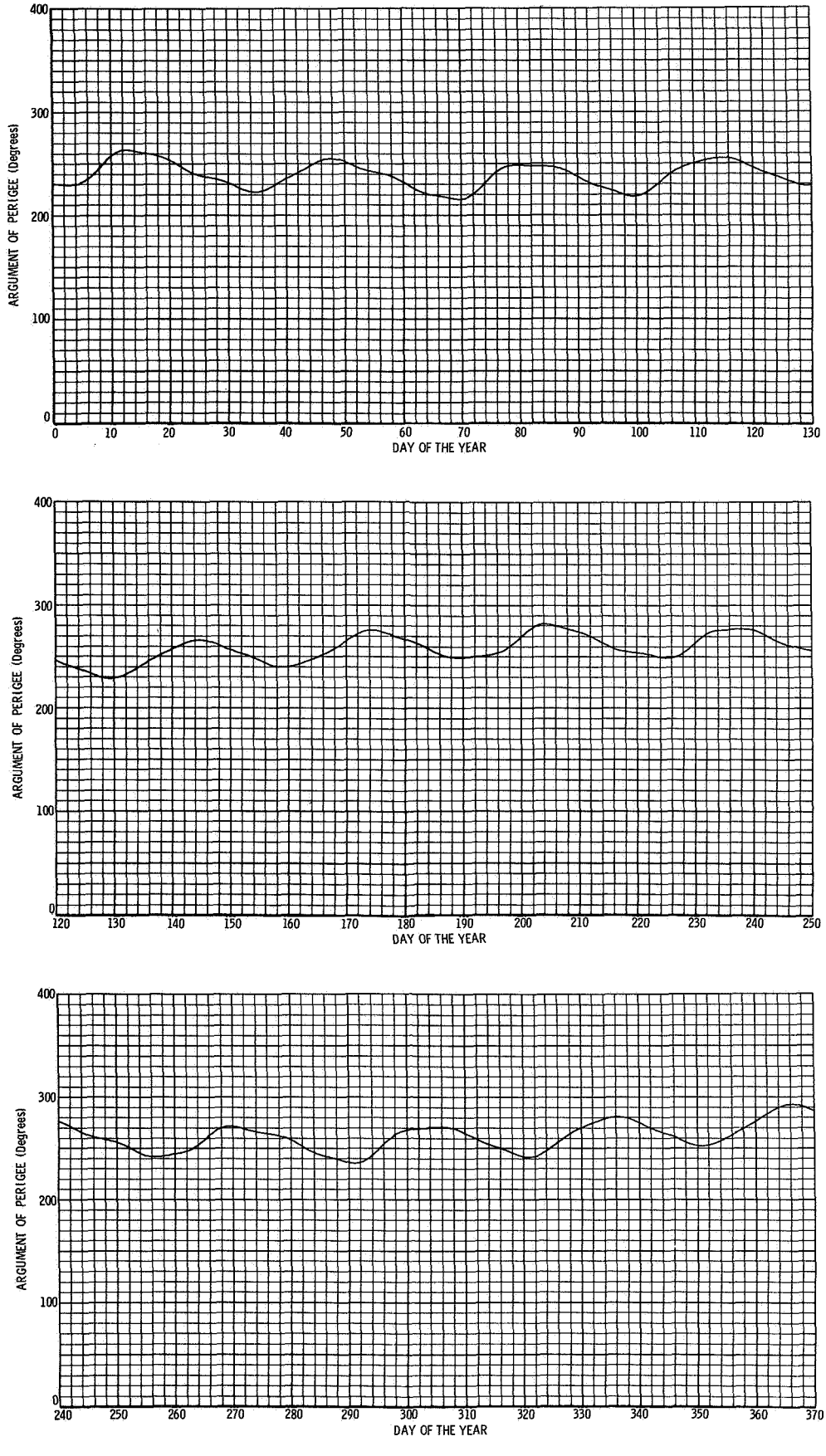
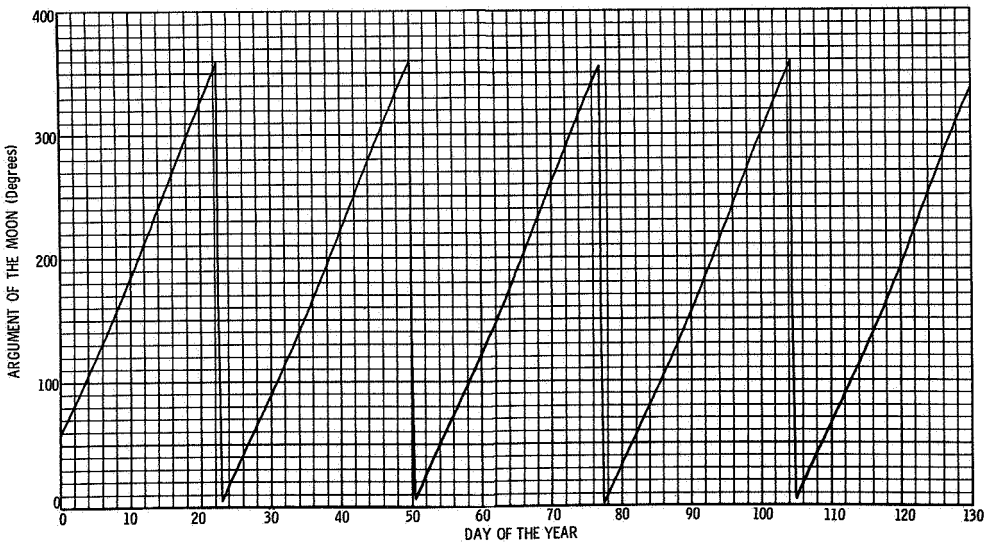
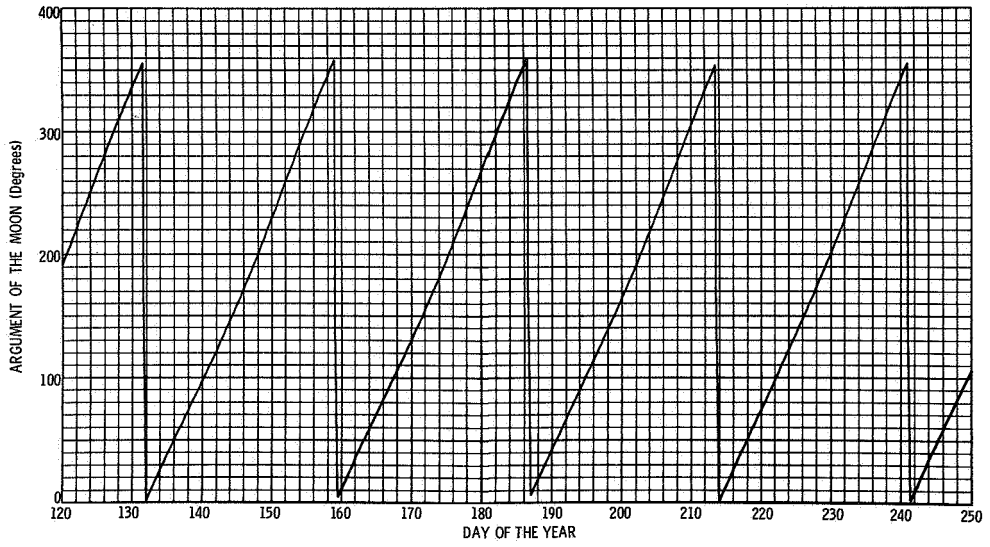


FIGURE B1977-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE





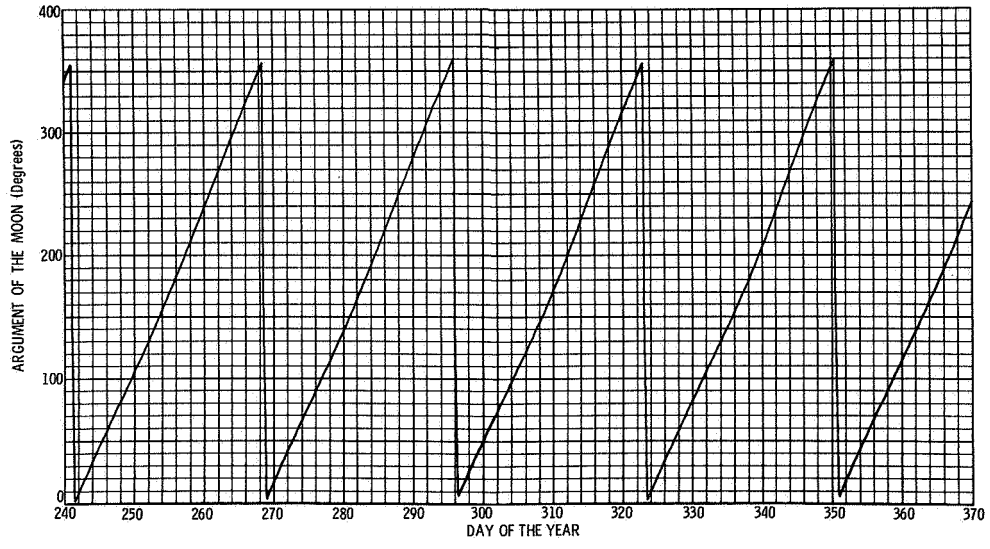


FIGURE B1977-10 ARGUMENT OF THE MOON'S POSITION

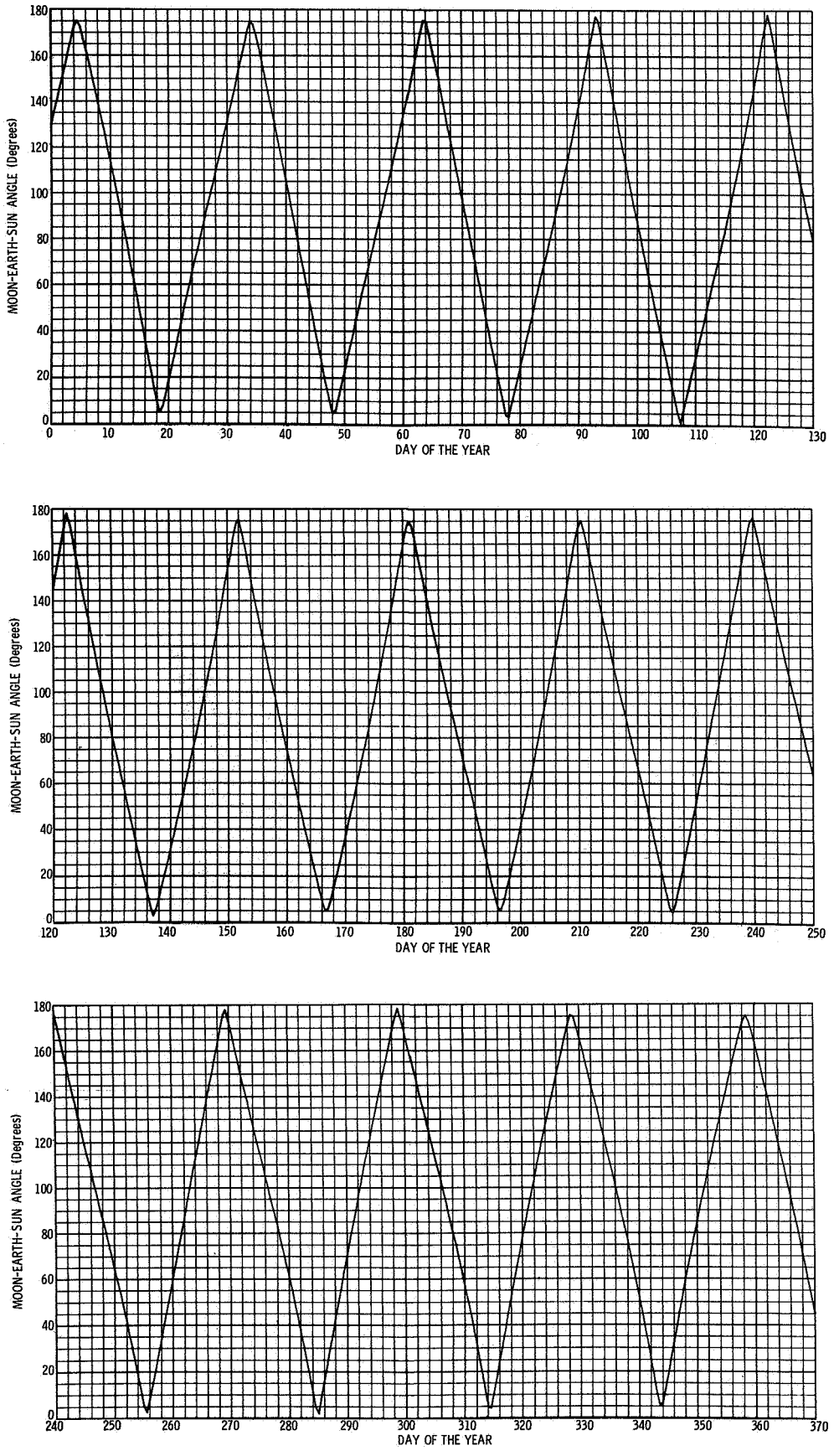
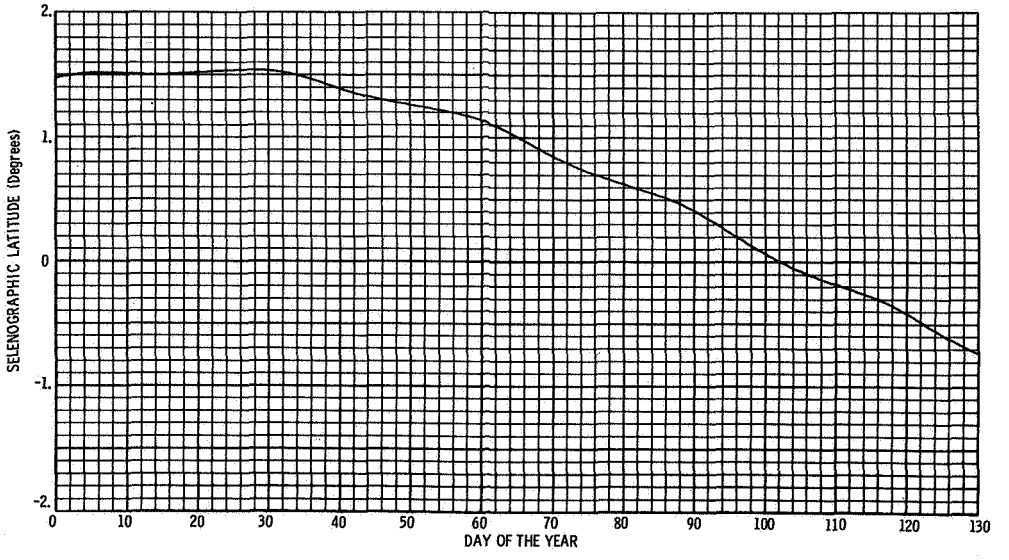
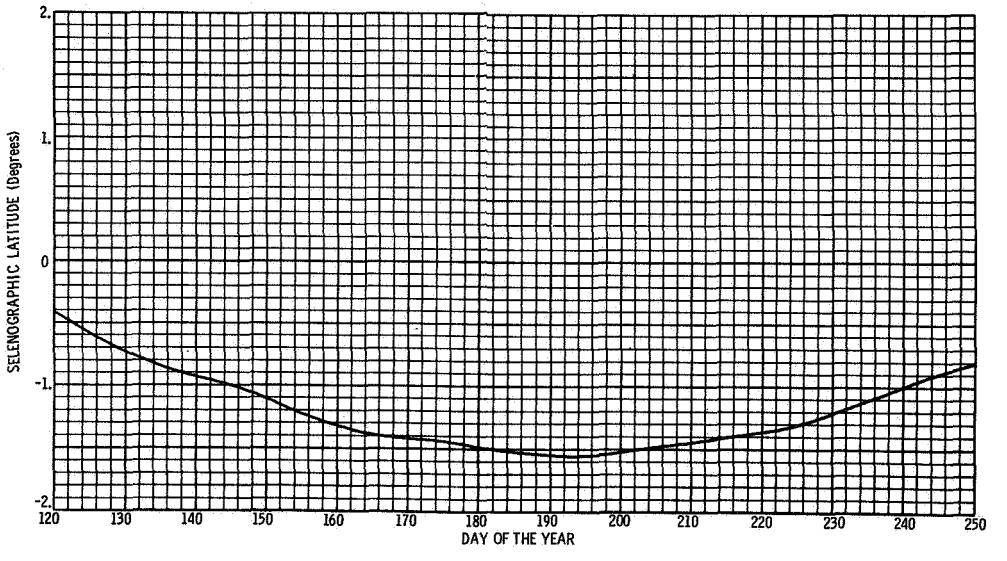


FIGURE B1977-11 MOON-EARTH-SUN ANGLE





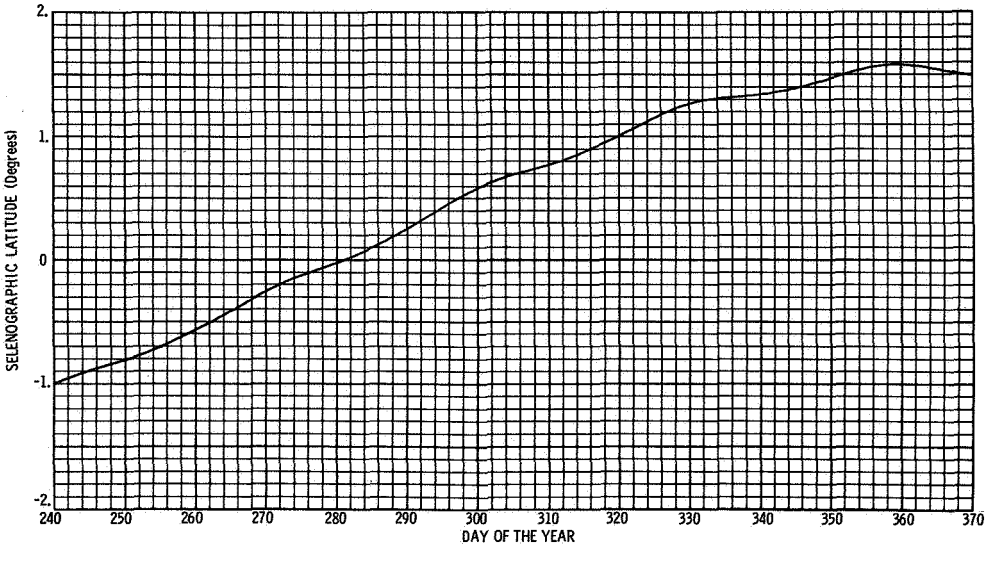


FIGURE B1977-12 SELENOGRAPHIC LATITUDE OF THE SUN

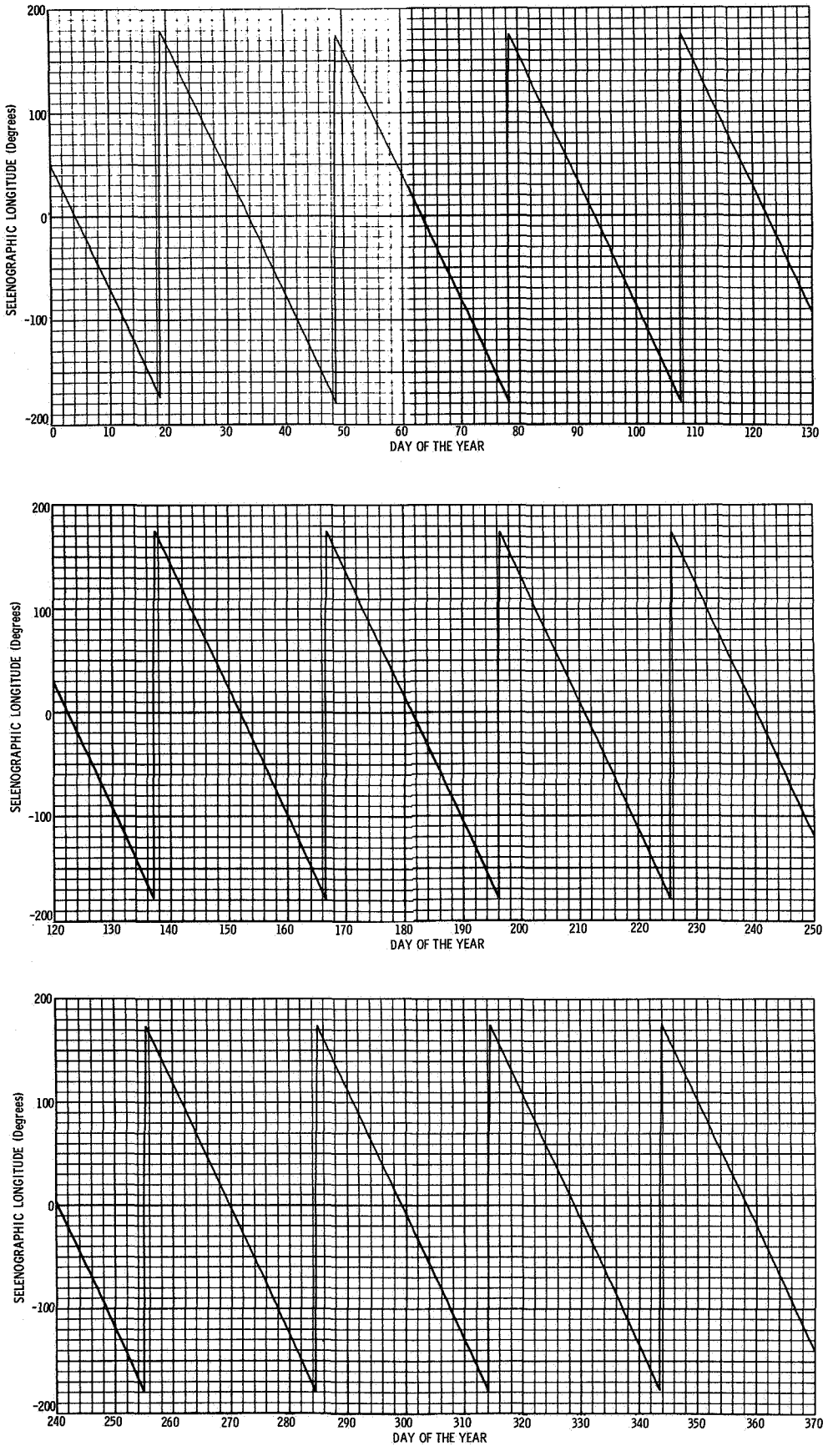
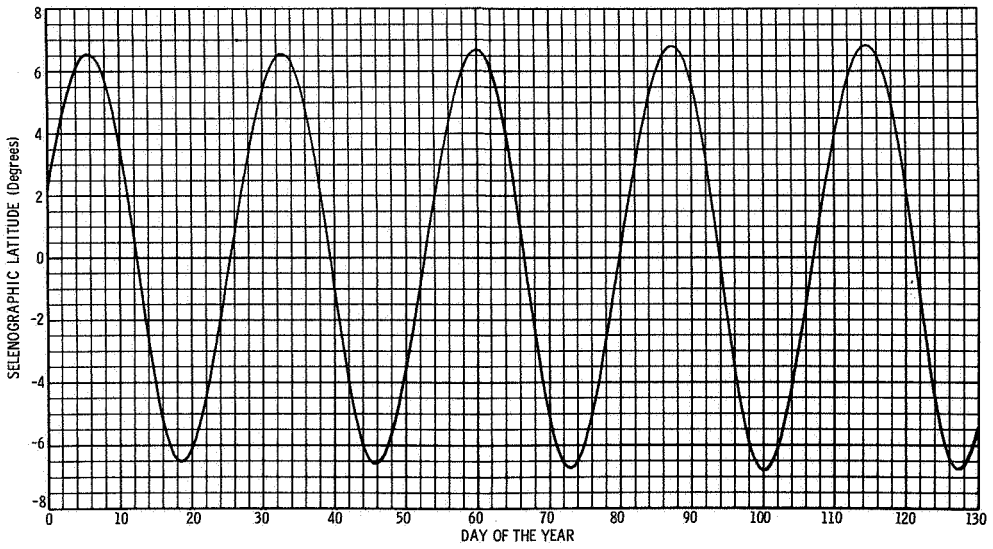
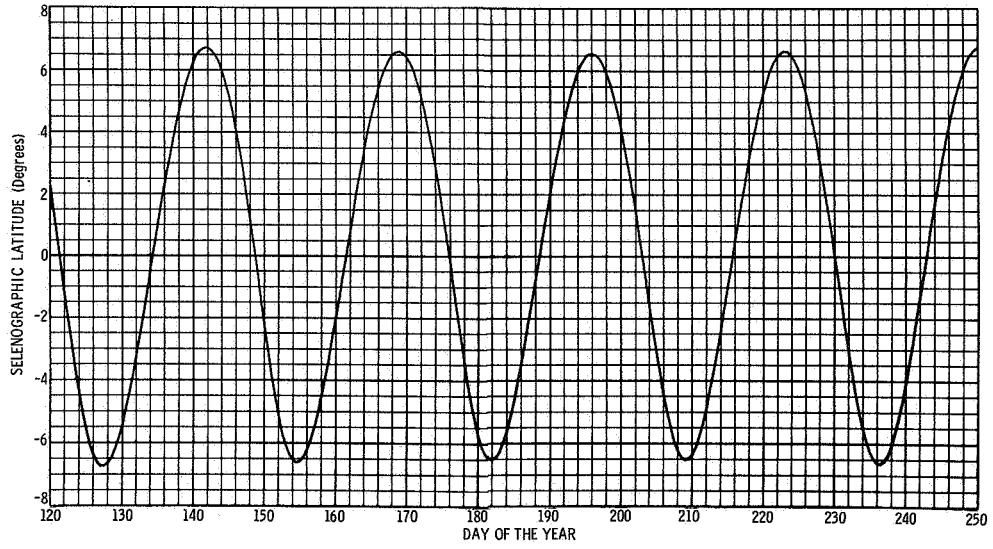


FIGURE B1977-13 SELENOGRAPHIC LONGITUDE OF THE SUN





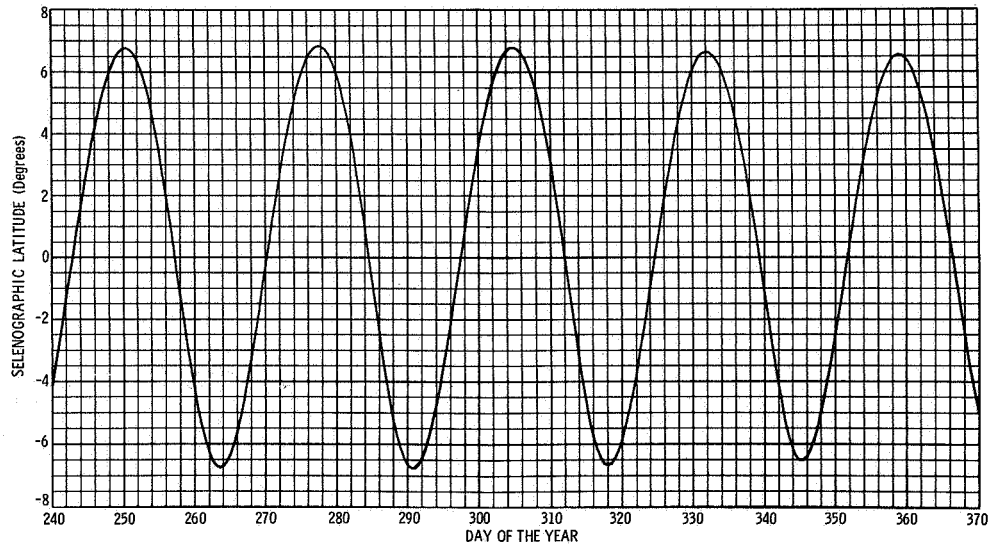
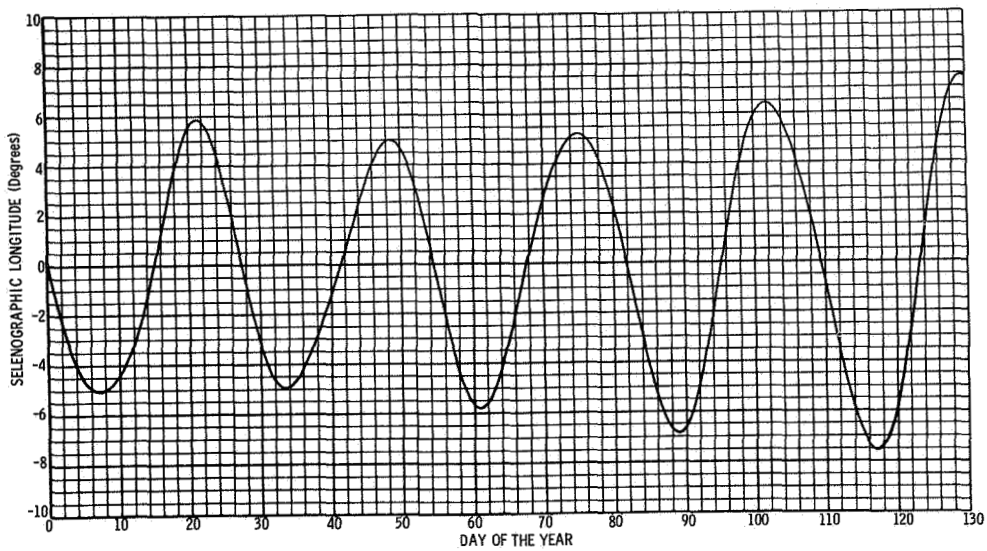
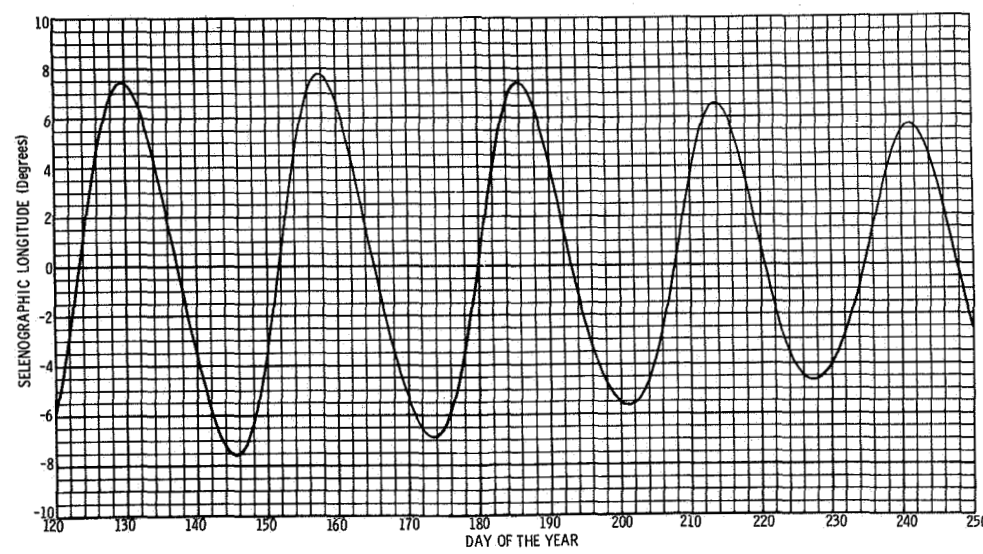
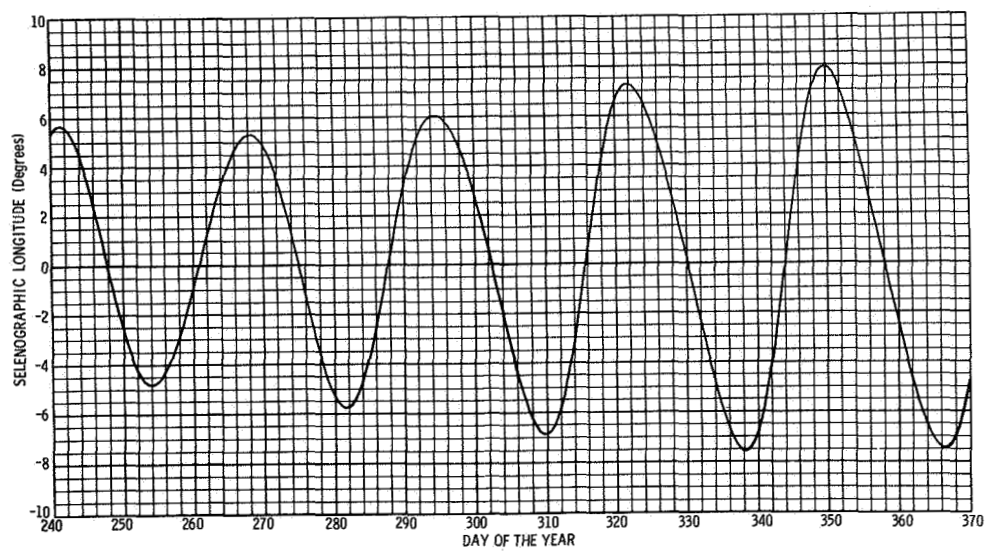


FIGURE B1977-14 SELENOGRAPHIC LATITUDE OF THE EARTH







B1977-15 SELENOGRAPHIC LONGITUDE OF THE EARTH

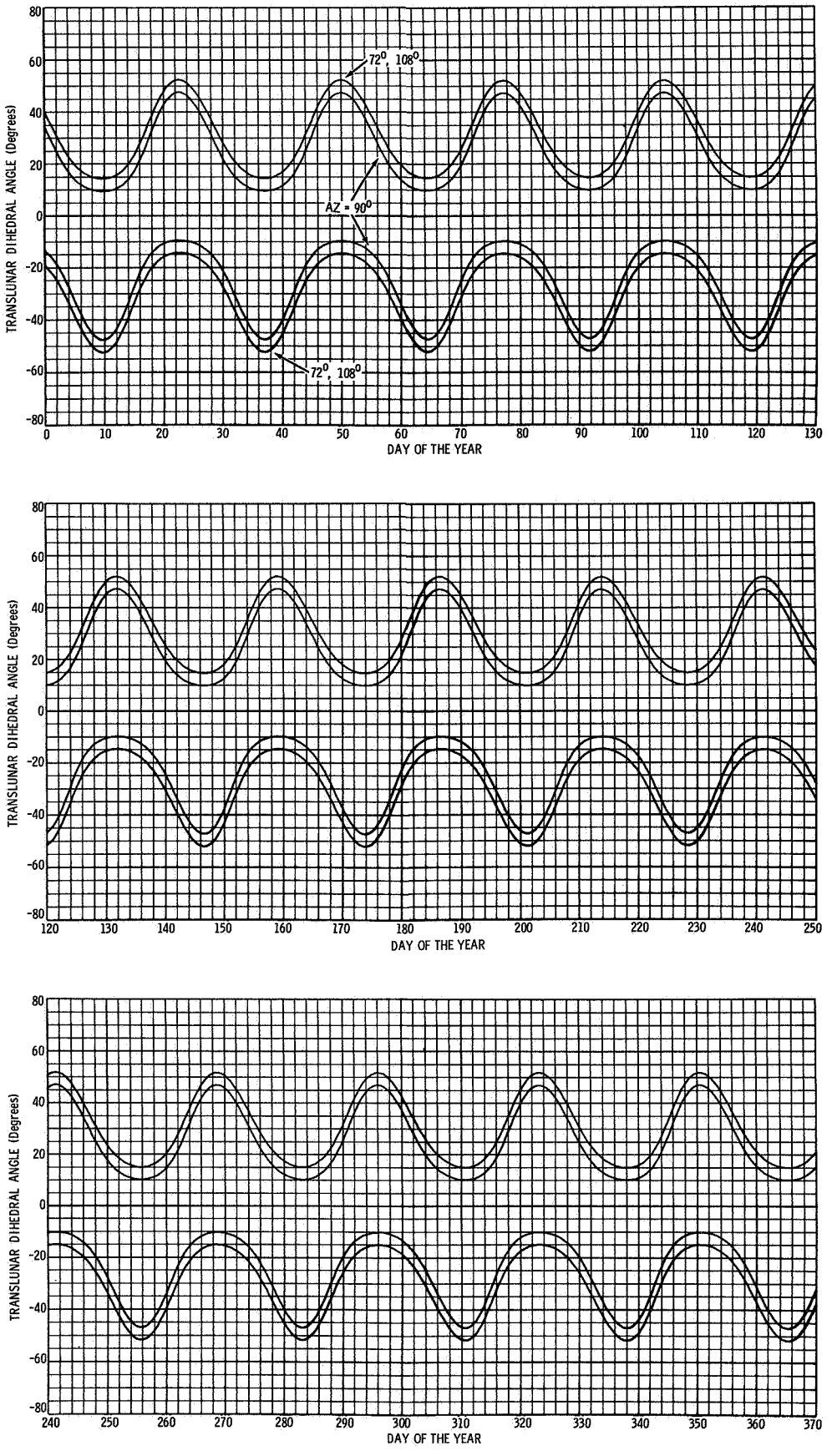
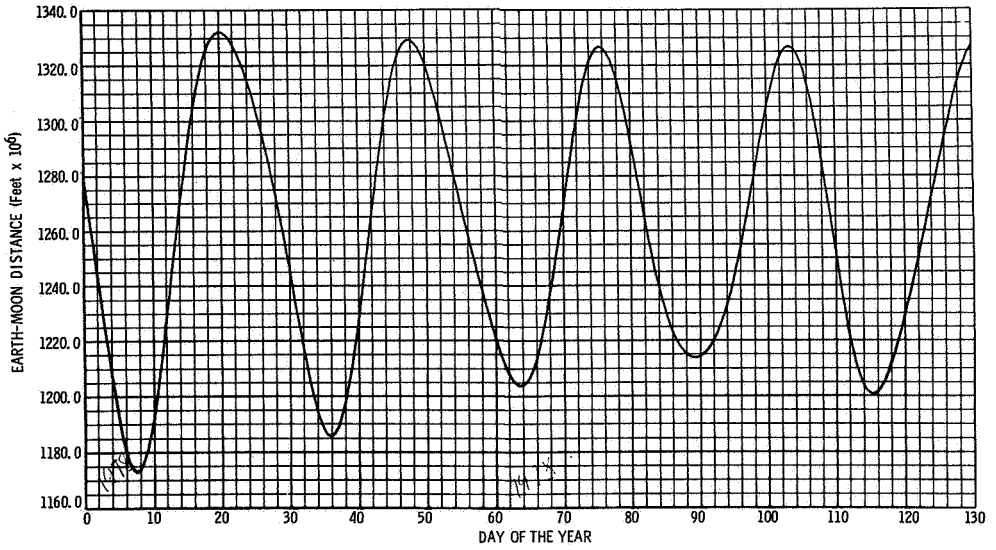
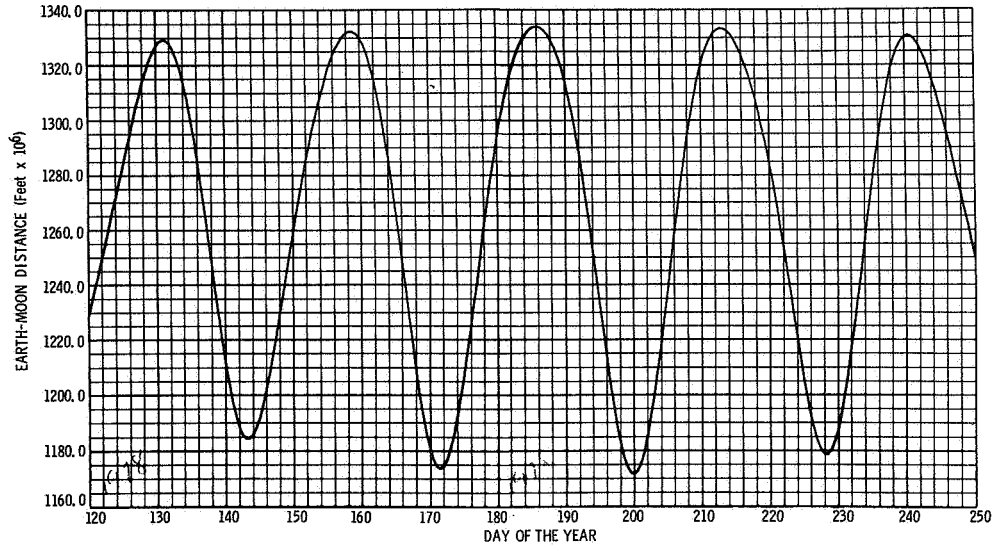


FIGURE B1977-16 TRANSLUNAR DIHEDRAL ANGLES

1978





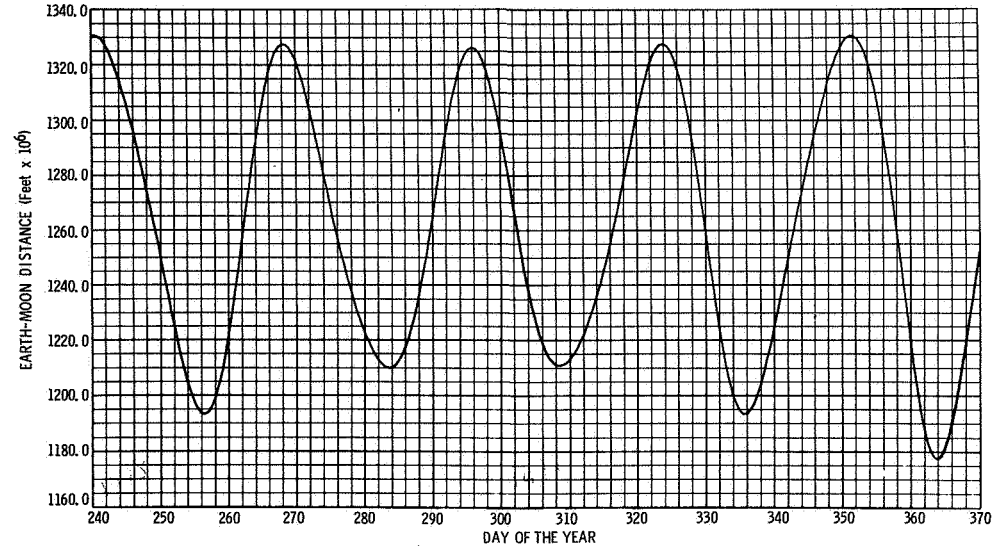
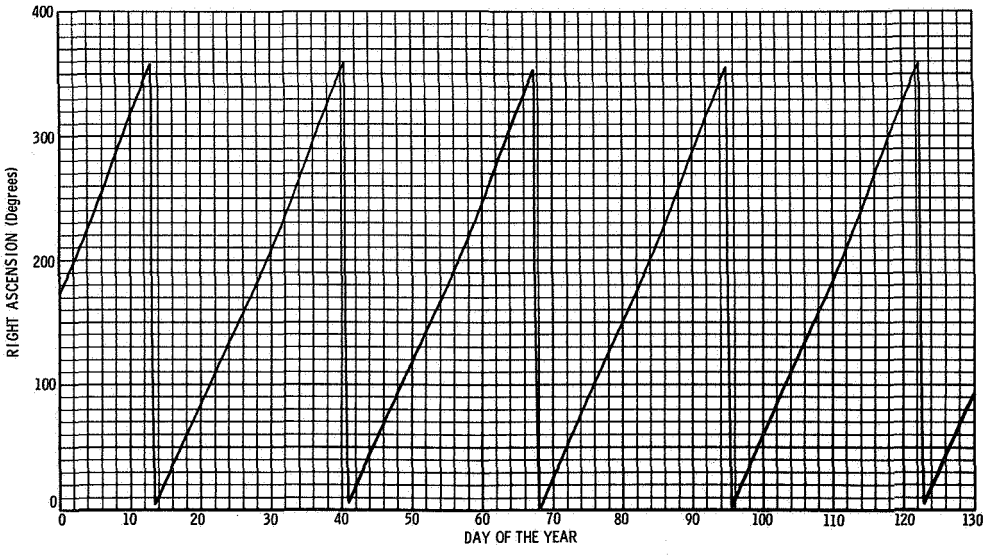
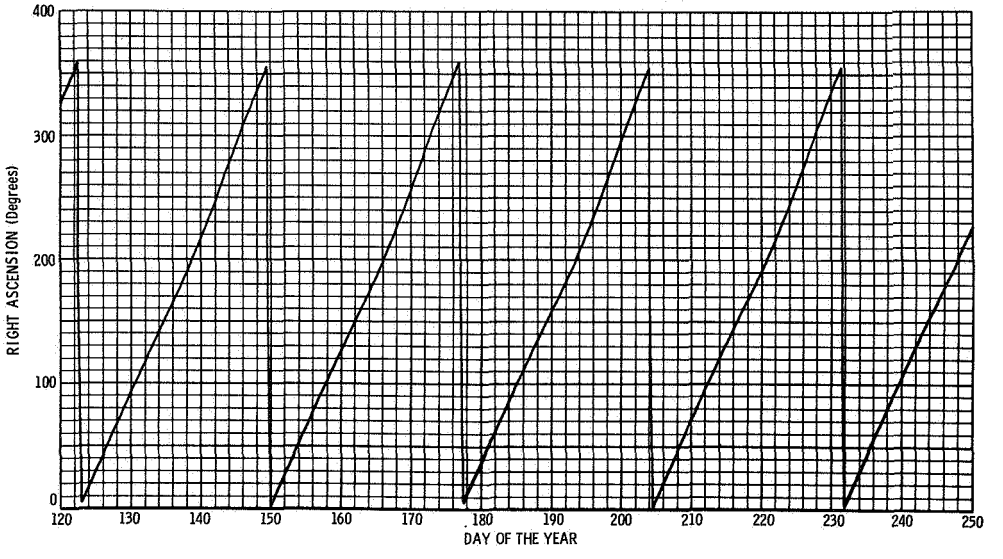


FIGURE B1978-1 EARTH-MOON DISTANCE





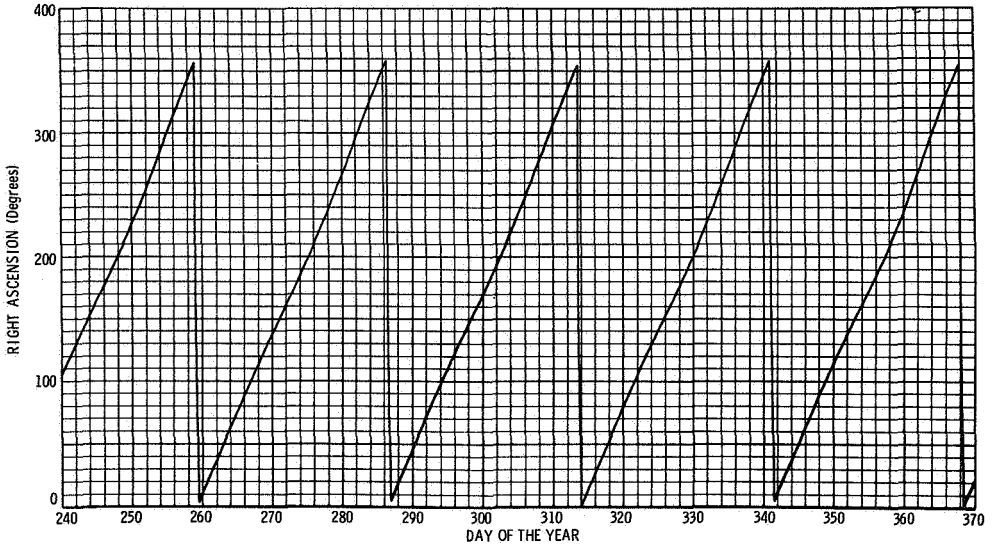
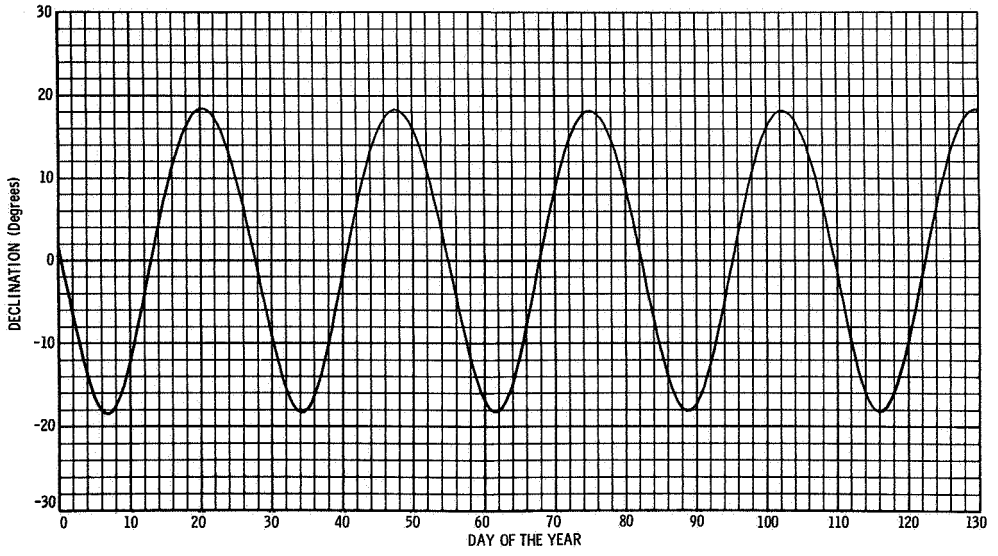
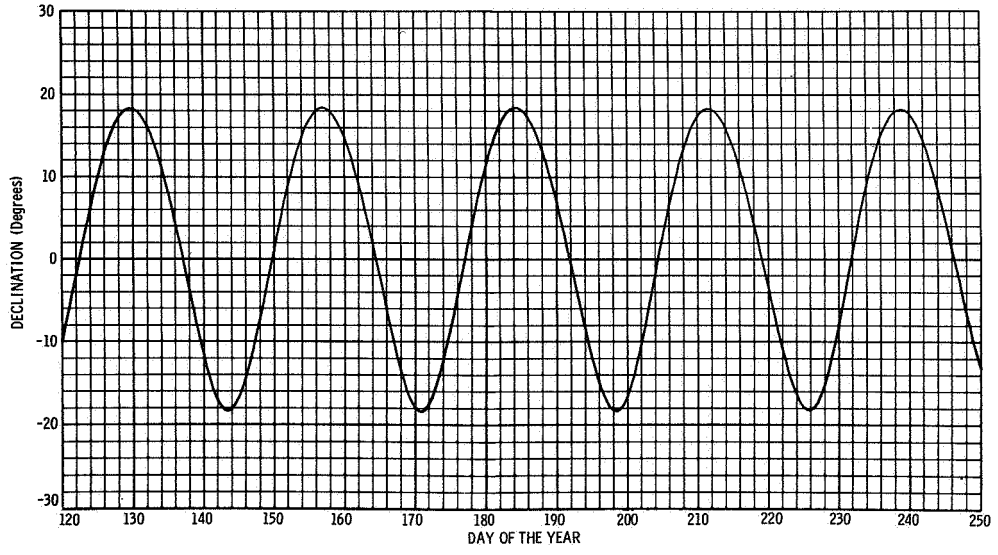


FIGURE B1978-2 RIGHT ASCENSION OF THE MOON





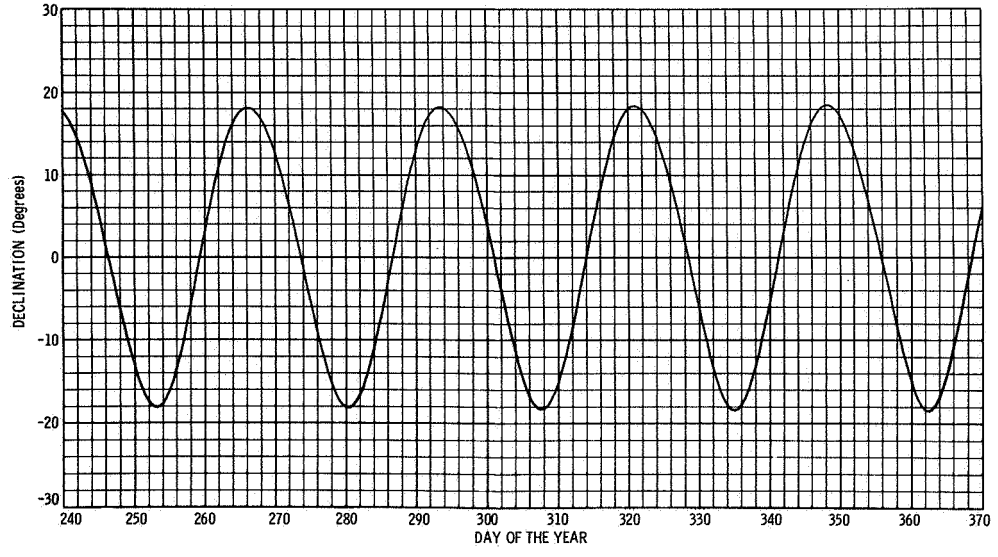
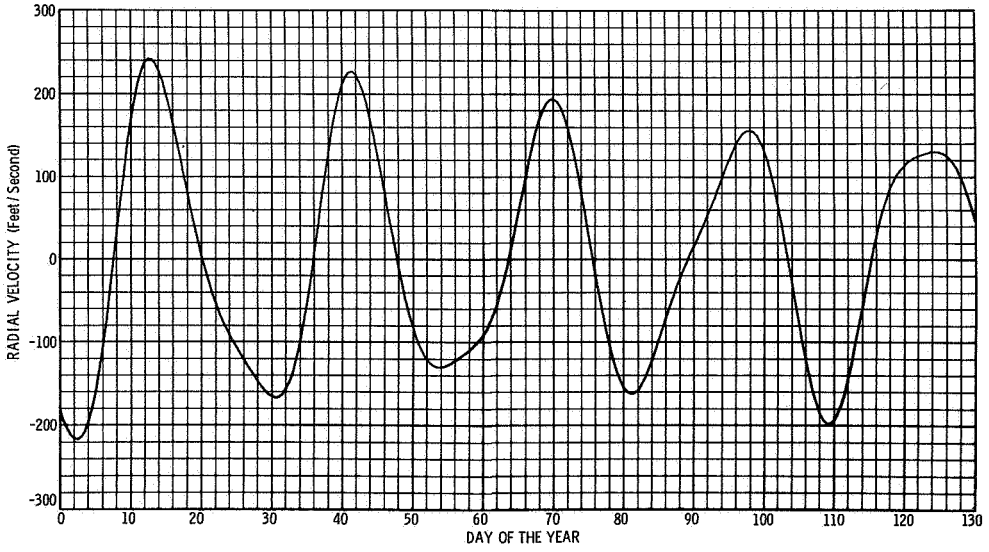
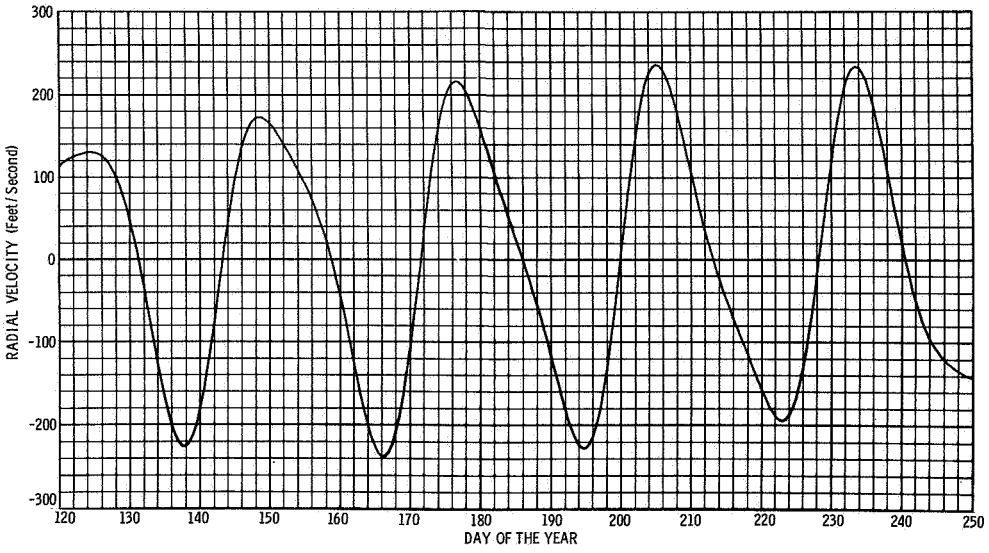


FIGURE B1978-3 DECLINATION OF THE MOON





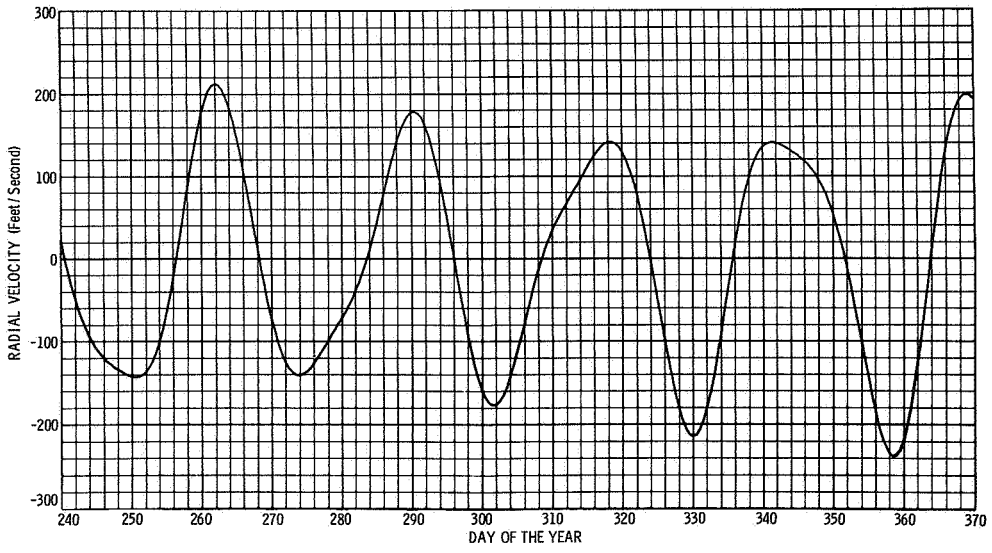
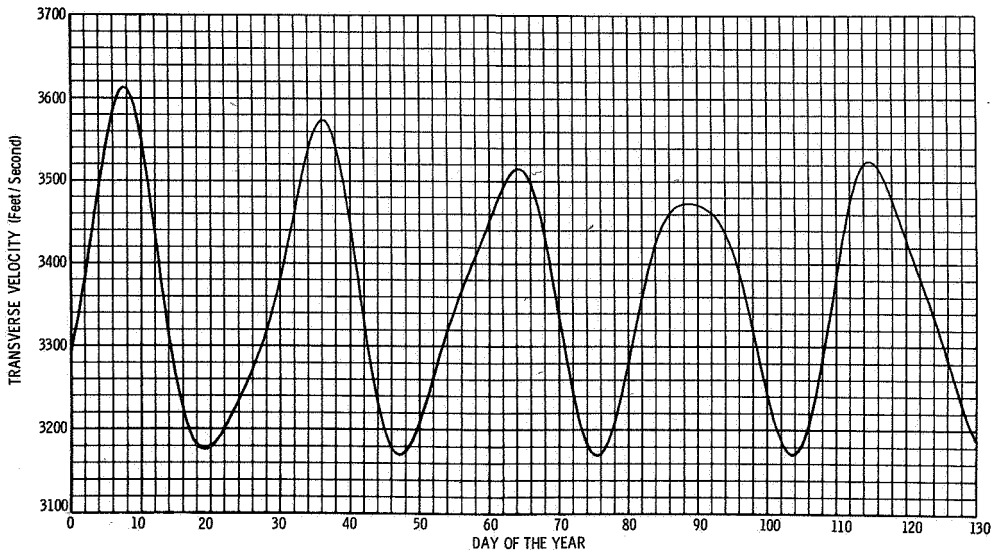
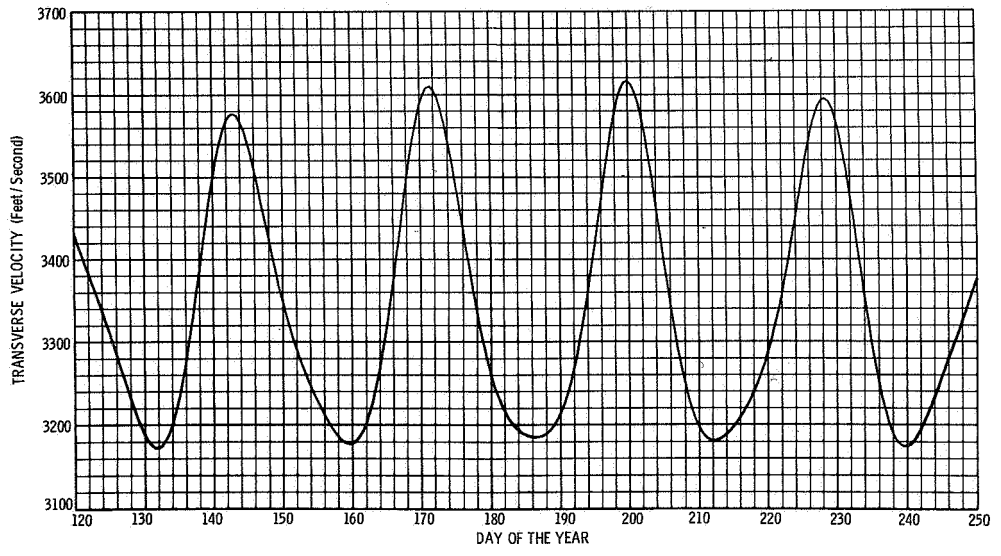


FIGURE B1978-4 RADIAL VELOCITY OF THE MOON





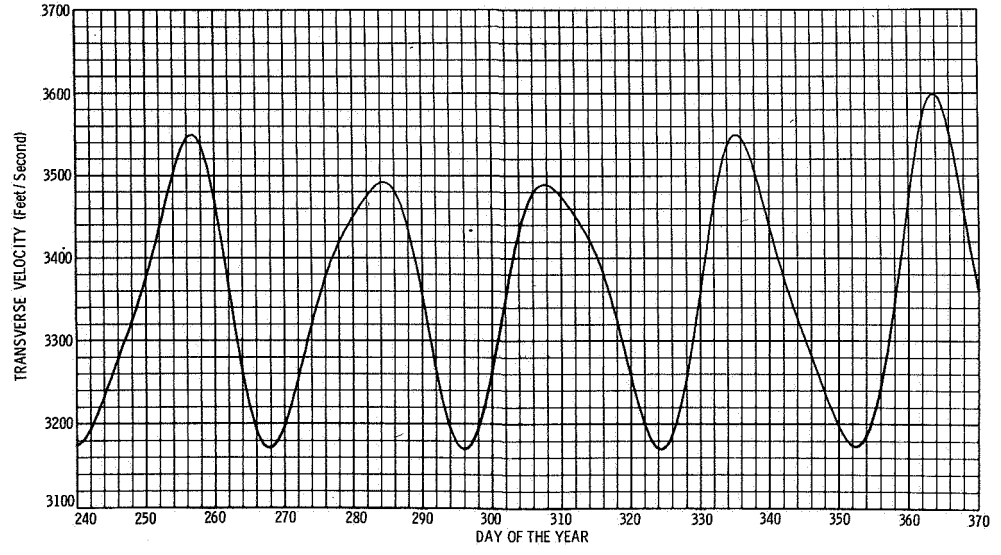


FIGURE B1978-5 TRANSVERSE VELOCITY OF THE MOON

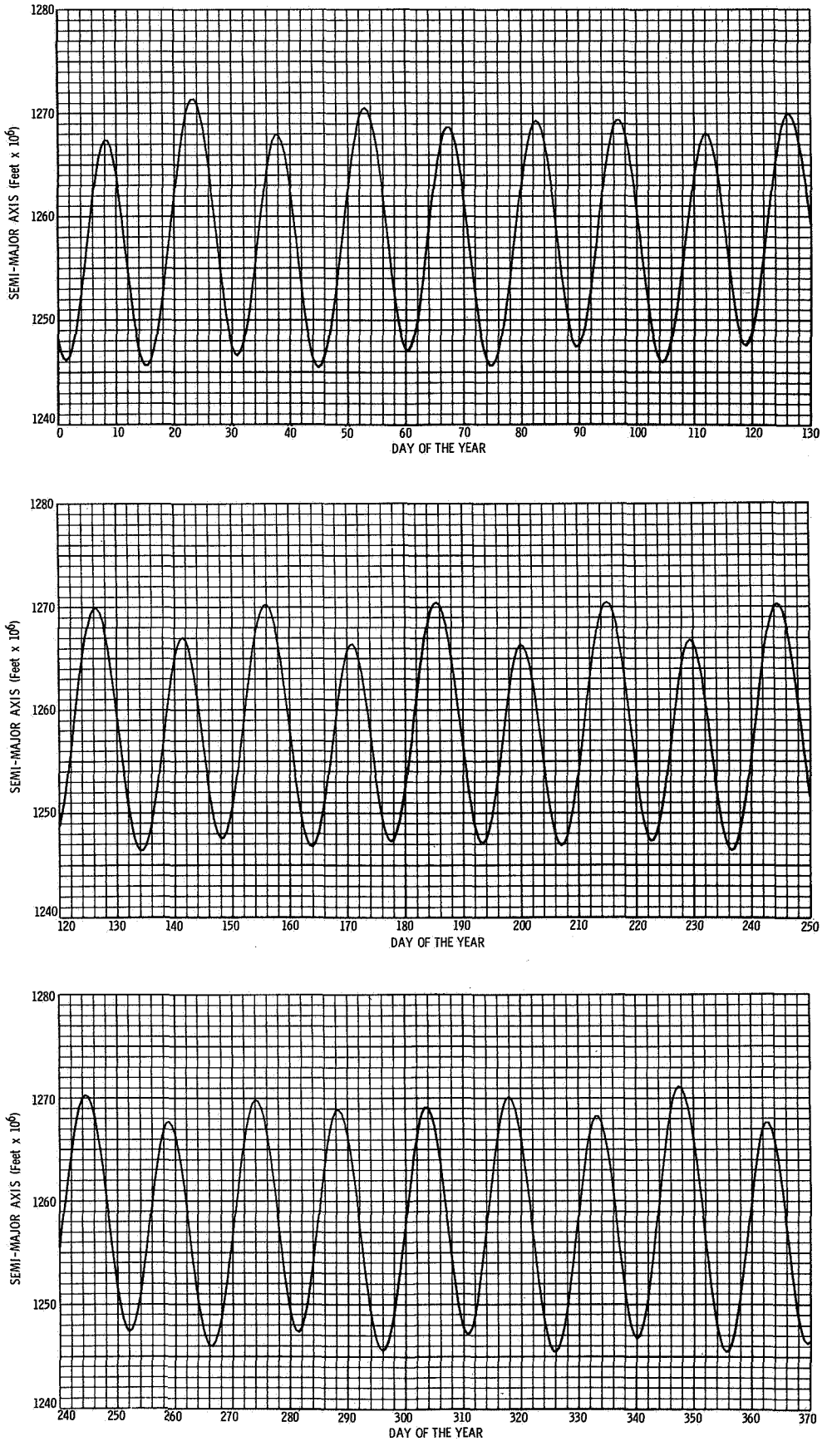


FIGURE B1978-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT

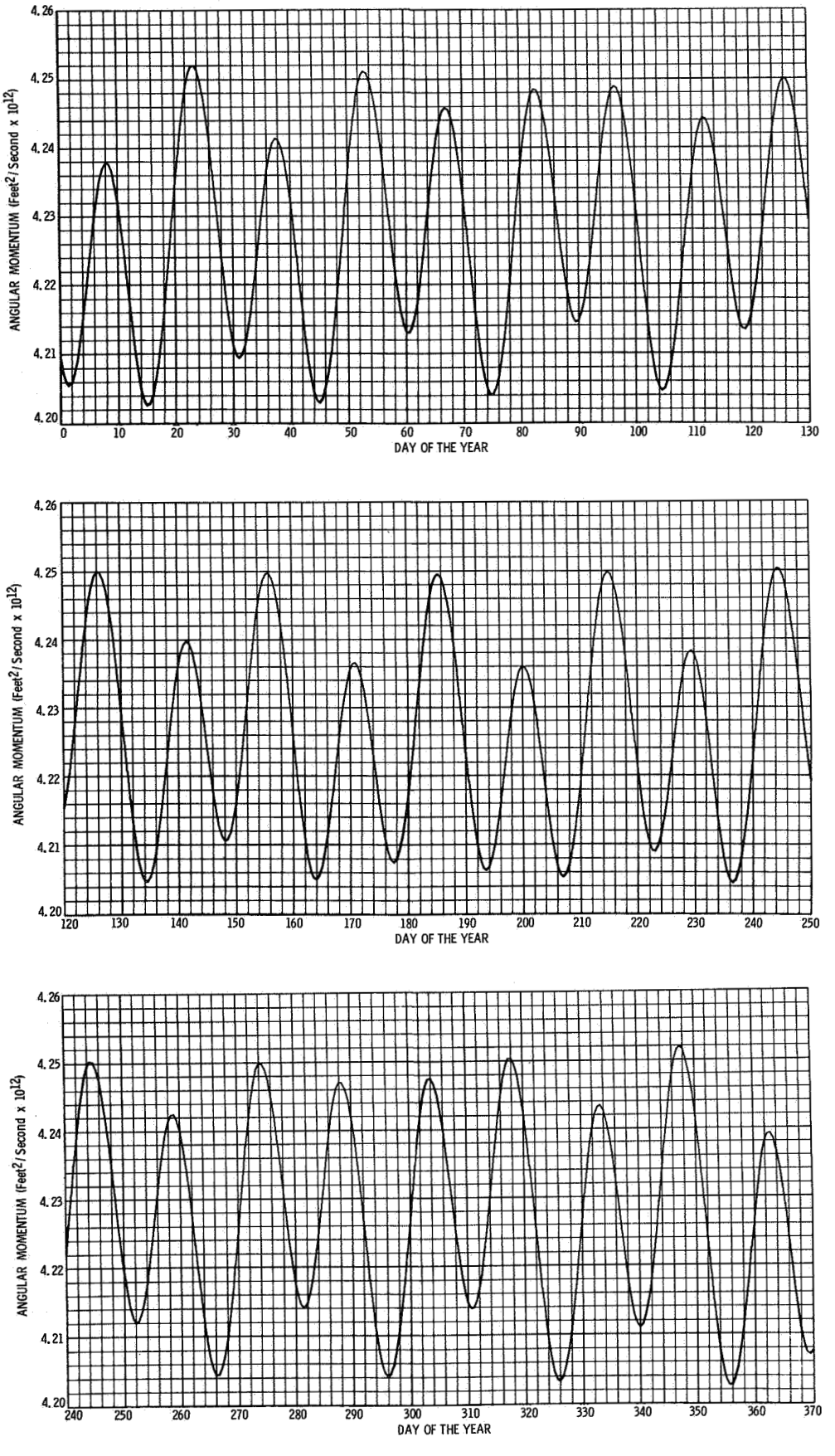
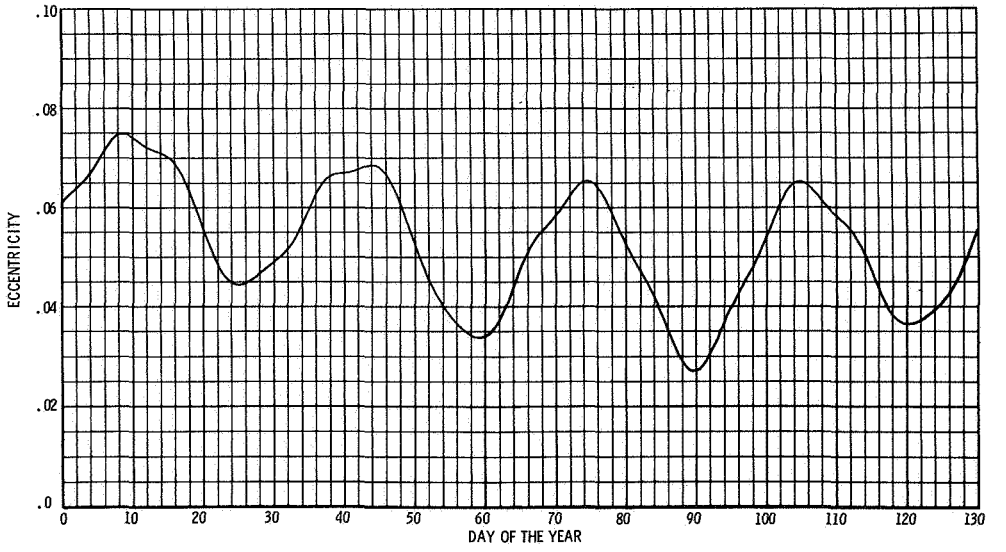
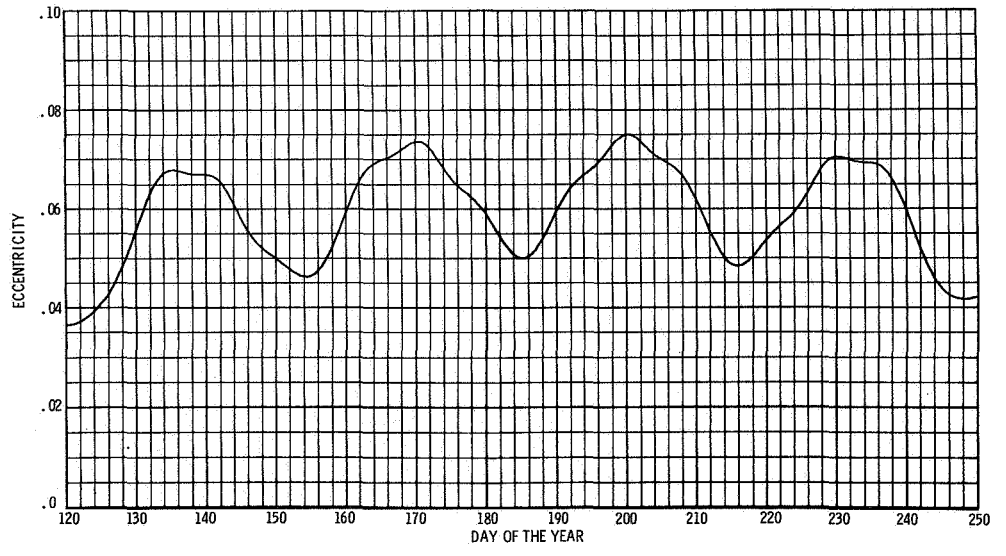


FIGURE B1978-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON





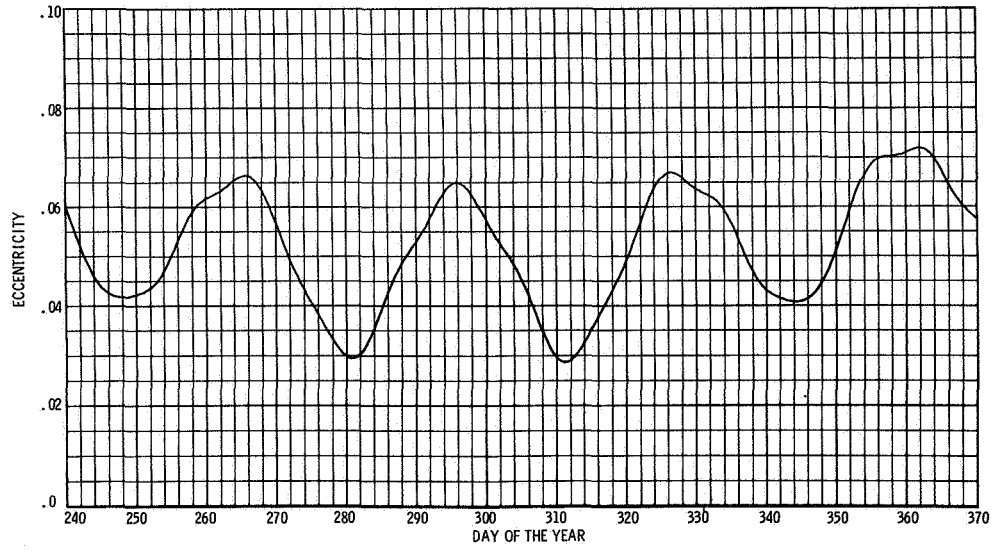
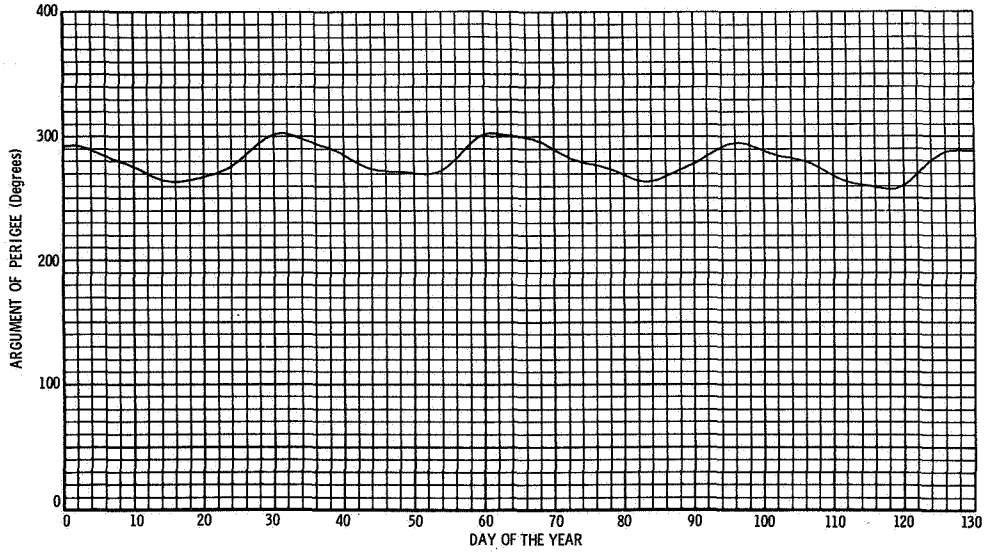
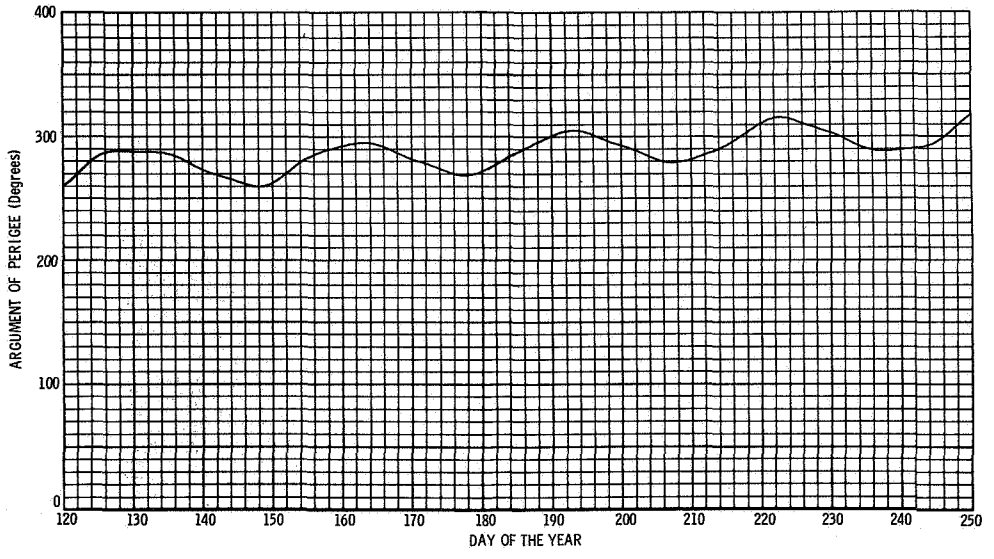


FIGURE B1978-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT





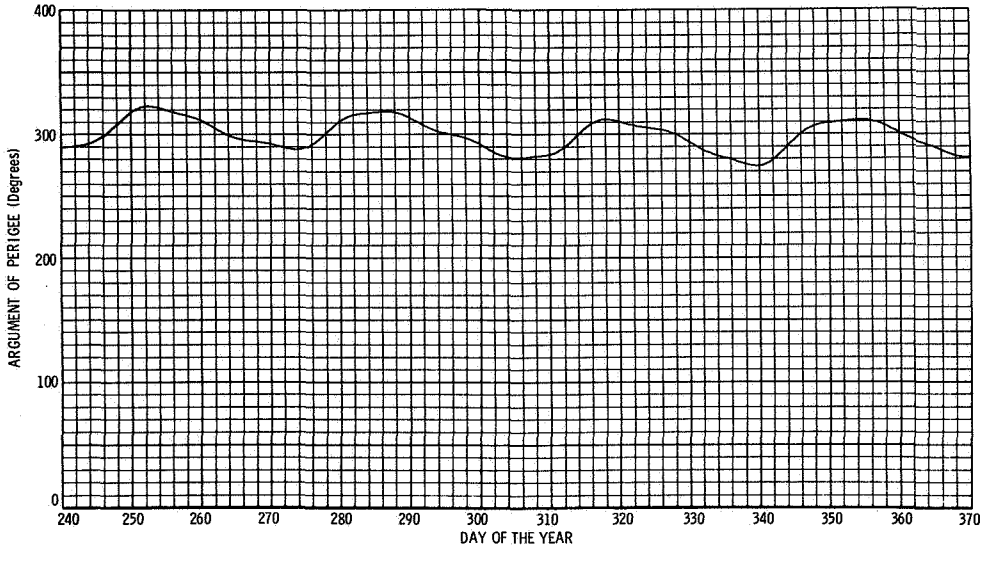


FIGURE B1978-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE

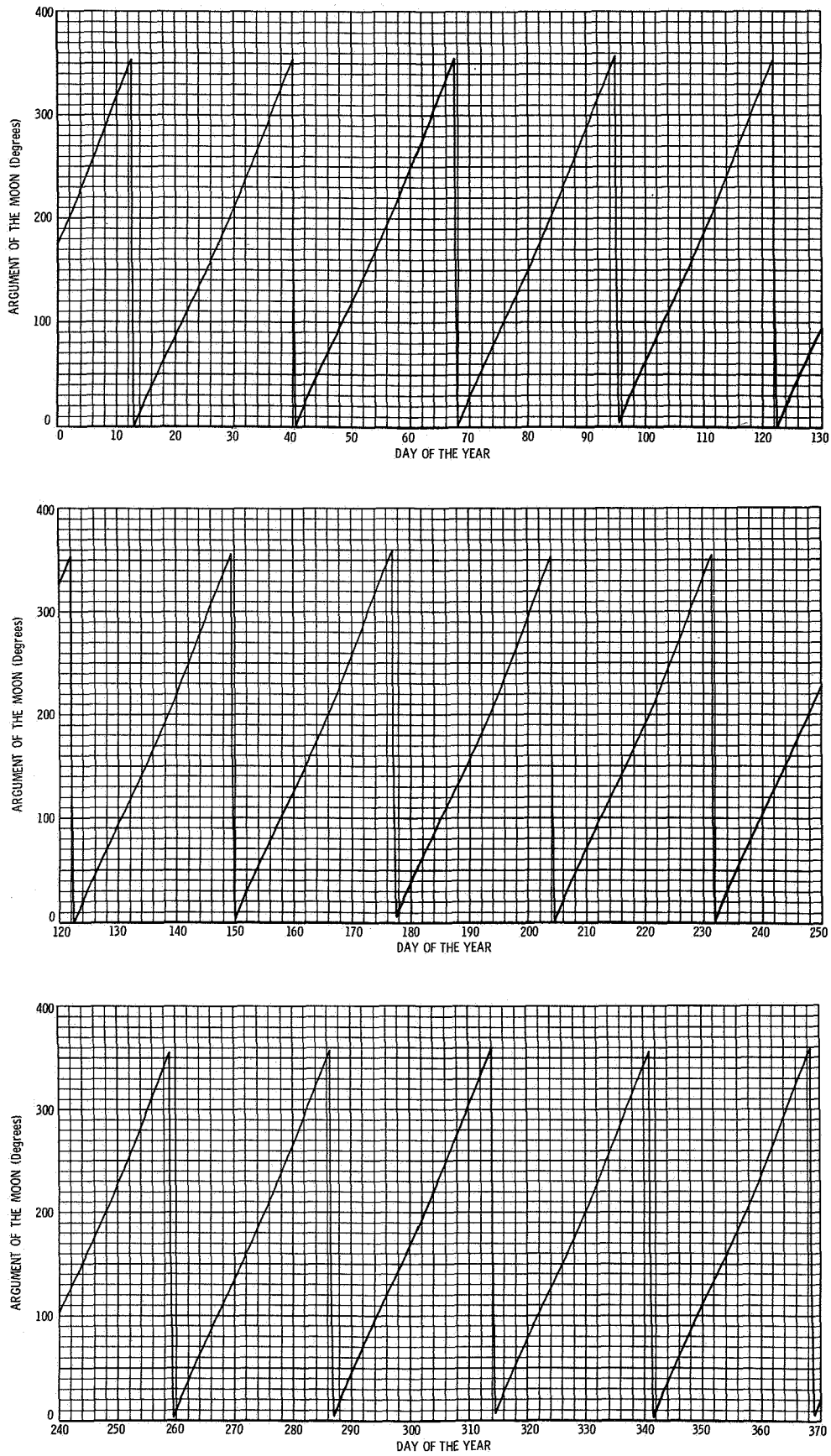


FIGURE B1978-10 ARGUMENT OF THE MOON'S POSITION

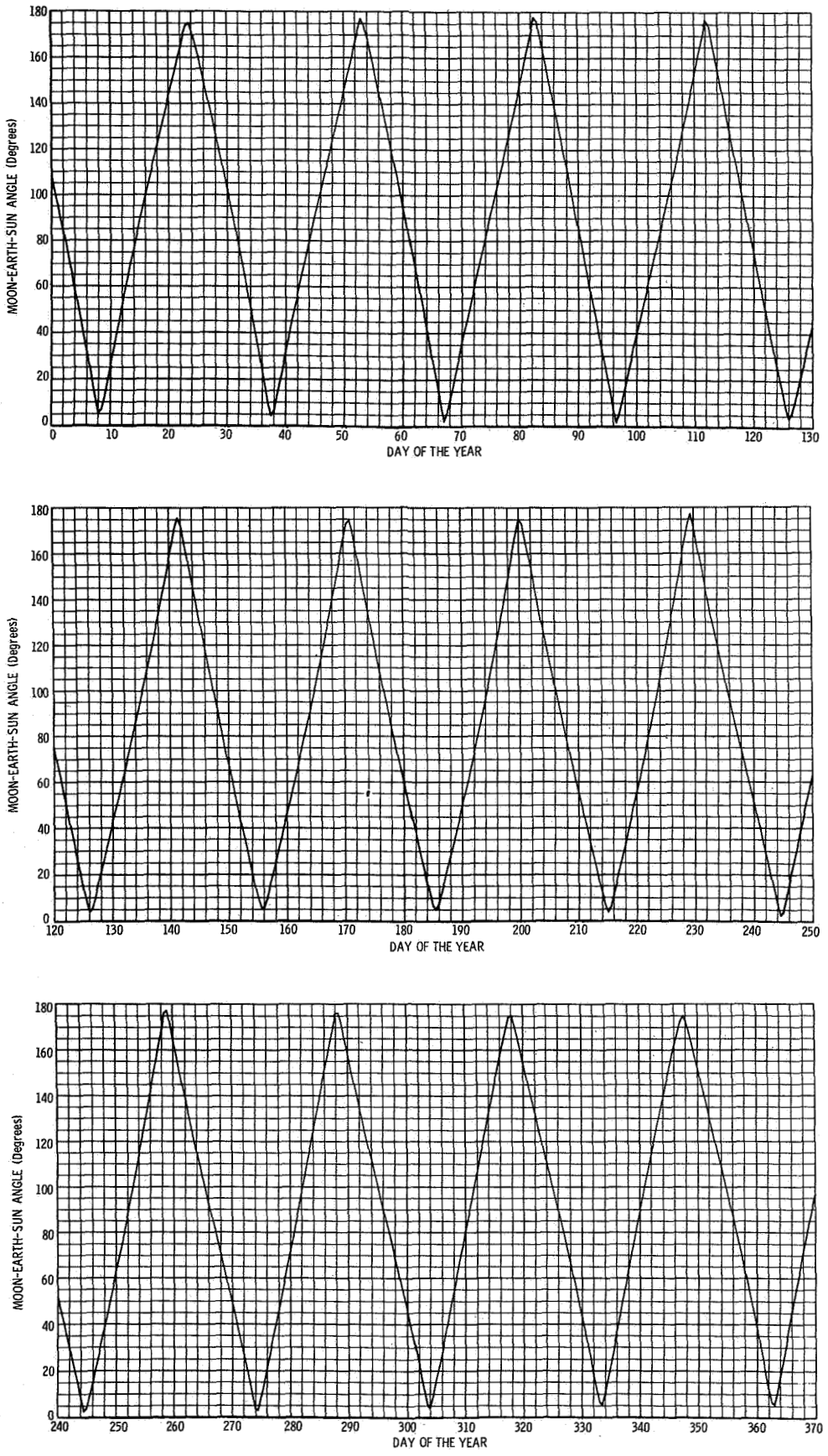
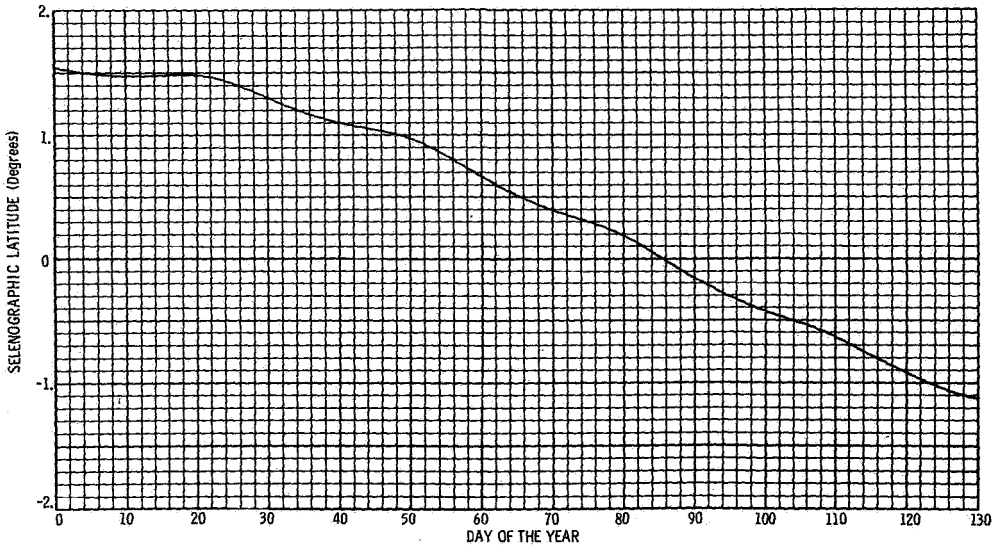
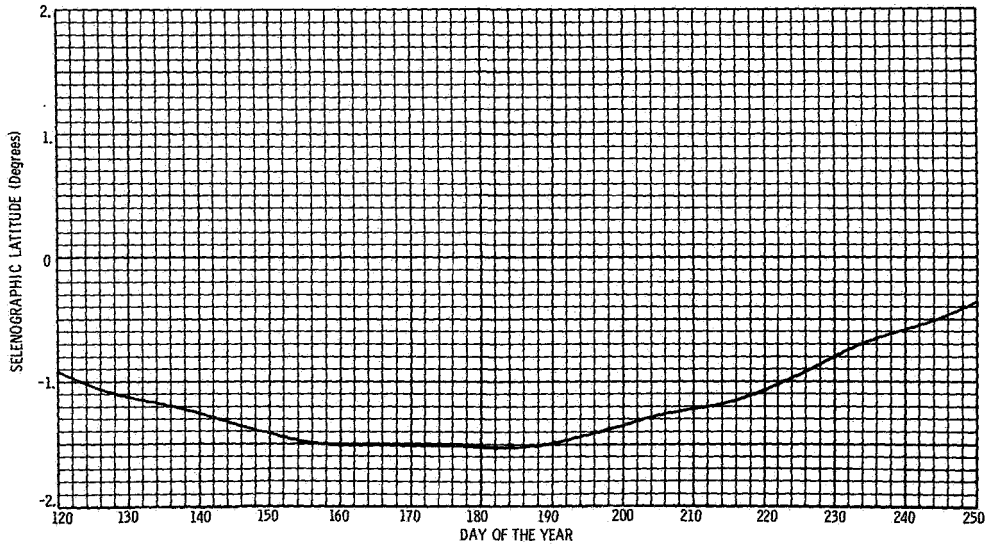


FIGURE B1978-11 MOON-EARTH-SUN ANGLE





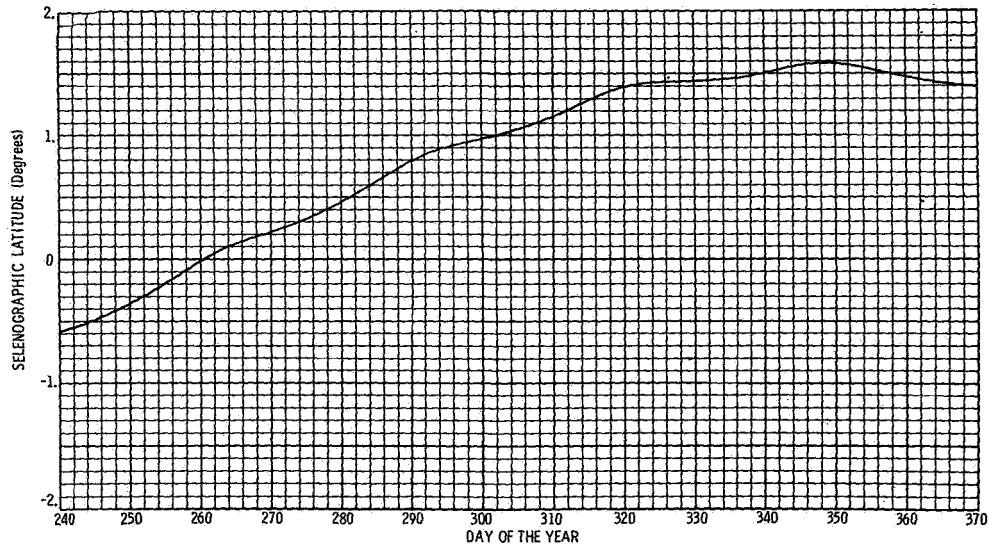
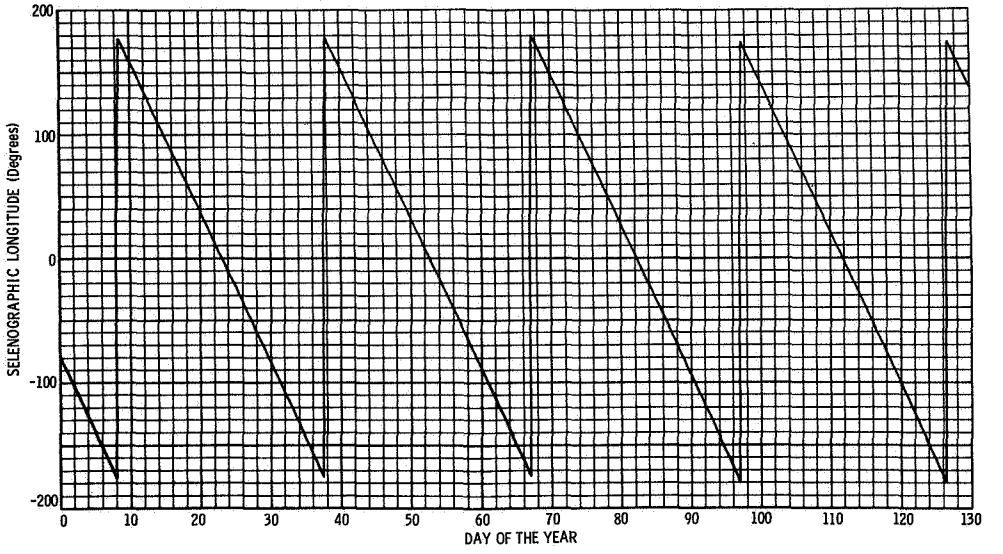
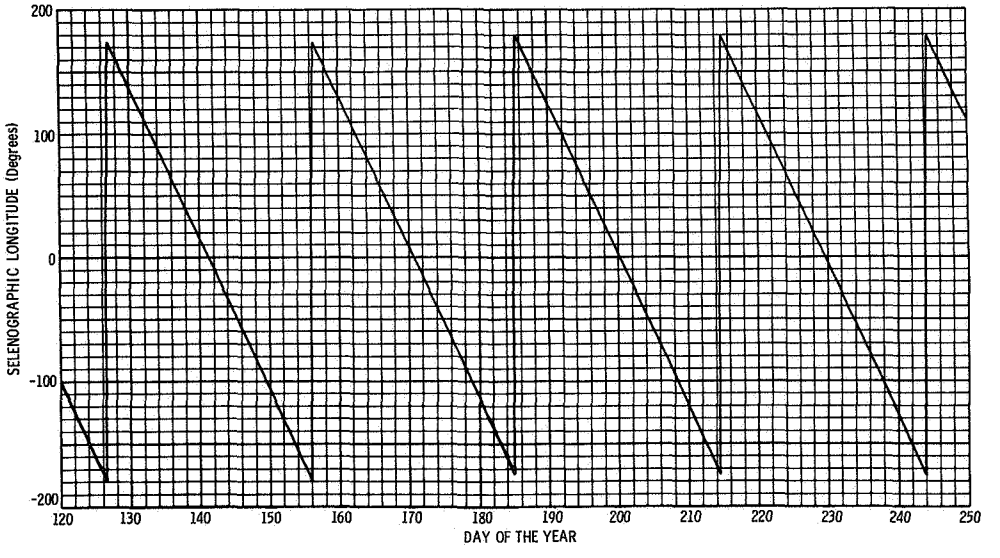


FIGURE B1978-12 SELENOGRAPHIC LATITUDE OF THE SUN





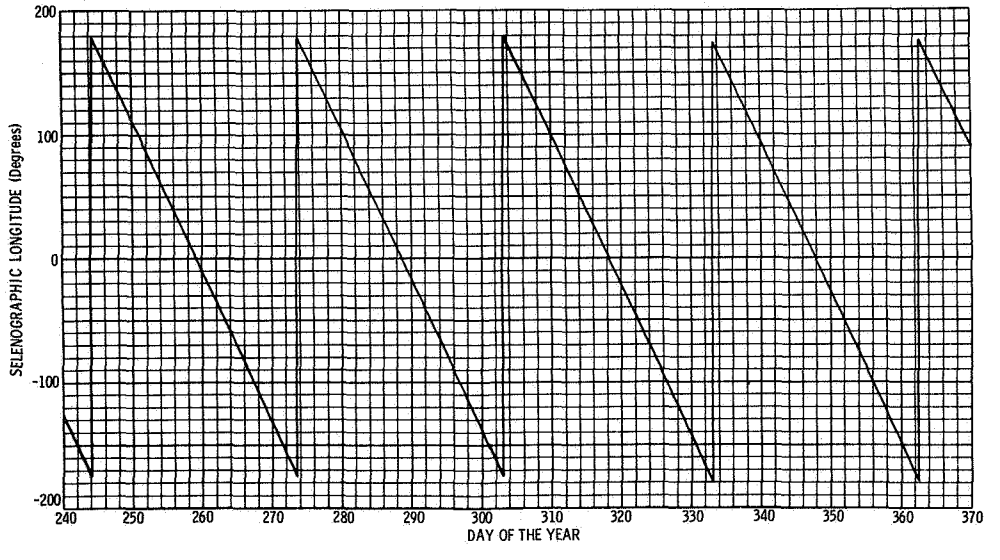
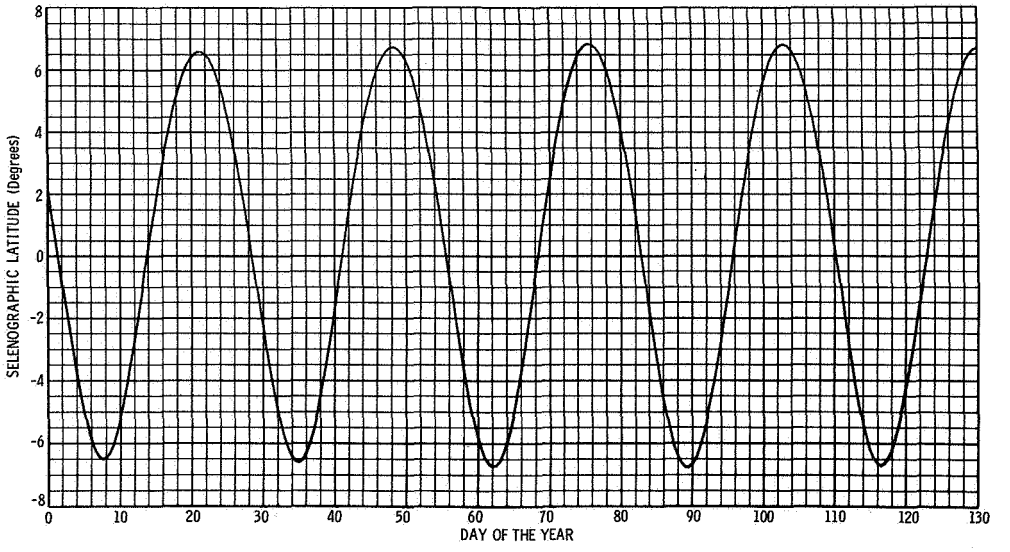
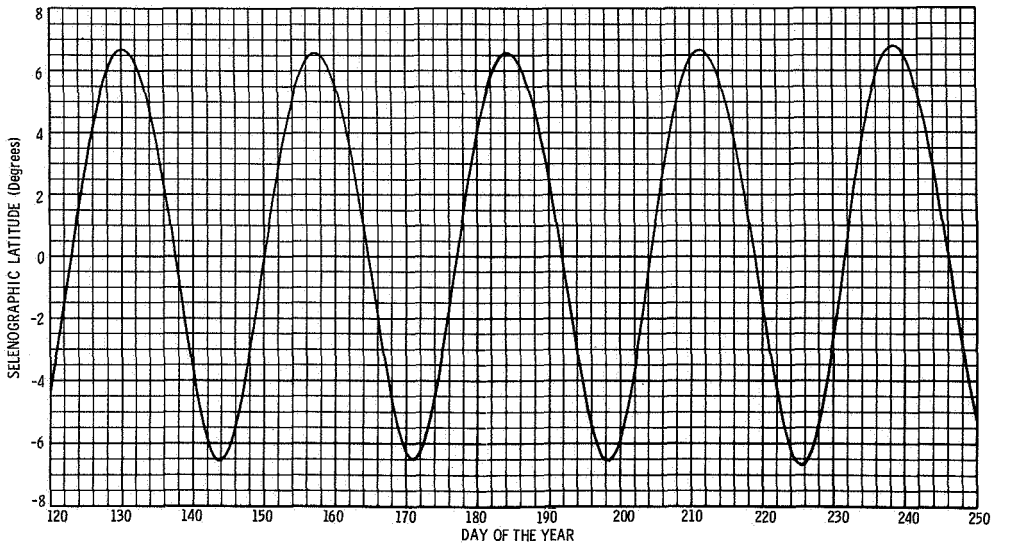


FIGURE B1978-13 SELENOGRAPHIC LONGITUDE OF THE SUN





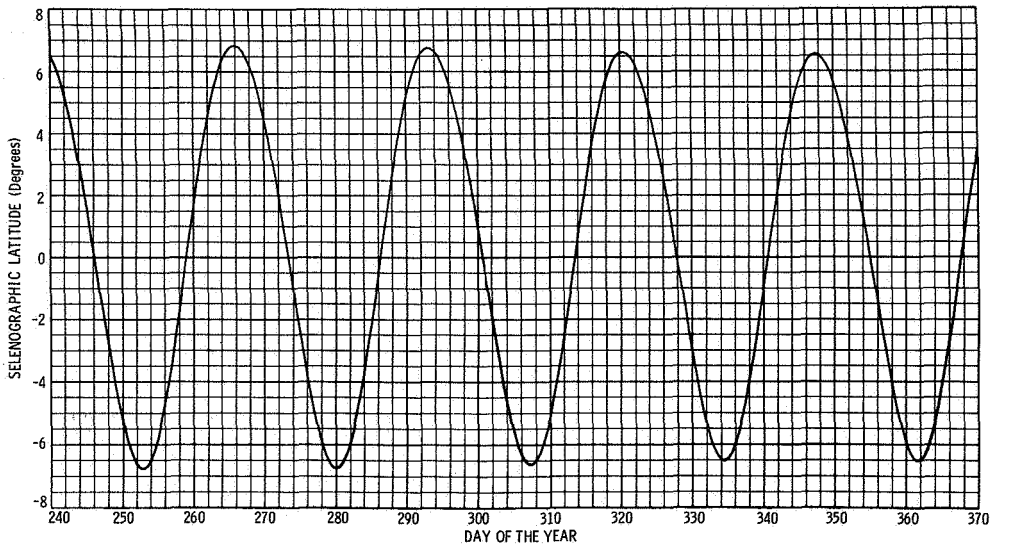


FIGURE B1978-14 SELENOGRAPHIC LATITUDE OF THE EARTH

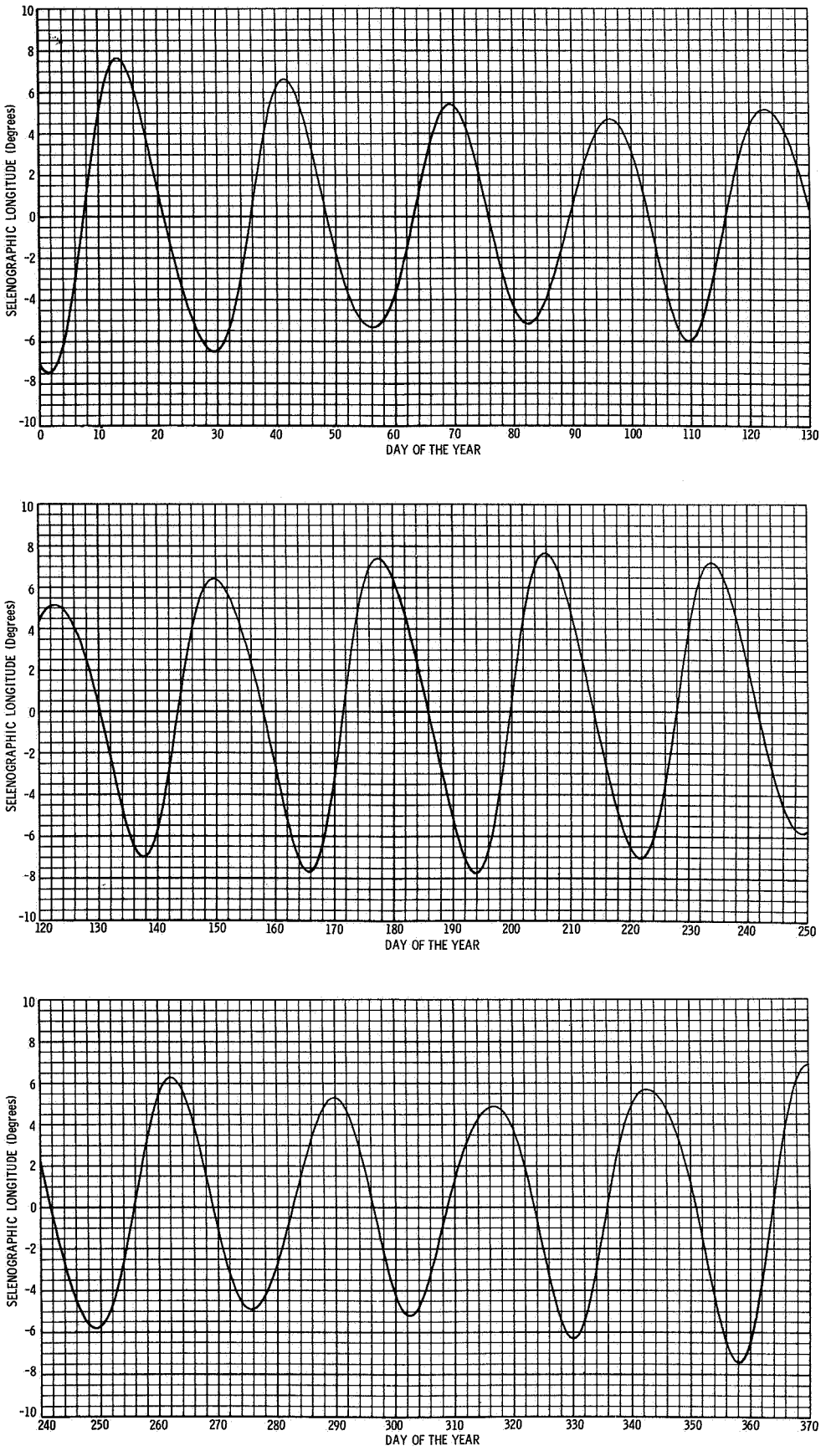


FIGURE B1978-15 SELENOGRAPHIC LONGITUDE OF THE EARTH

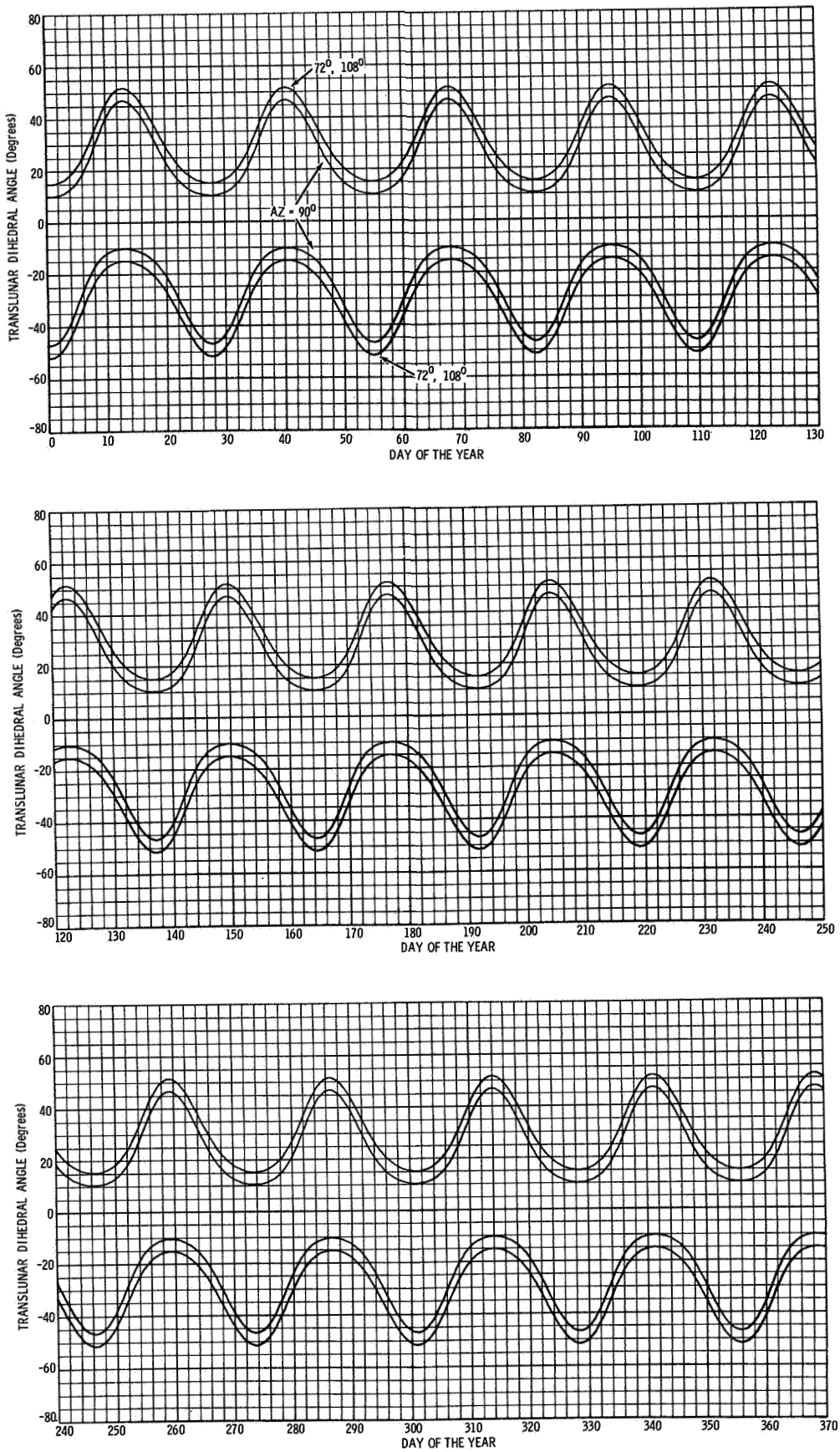
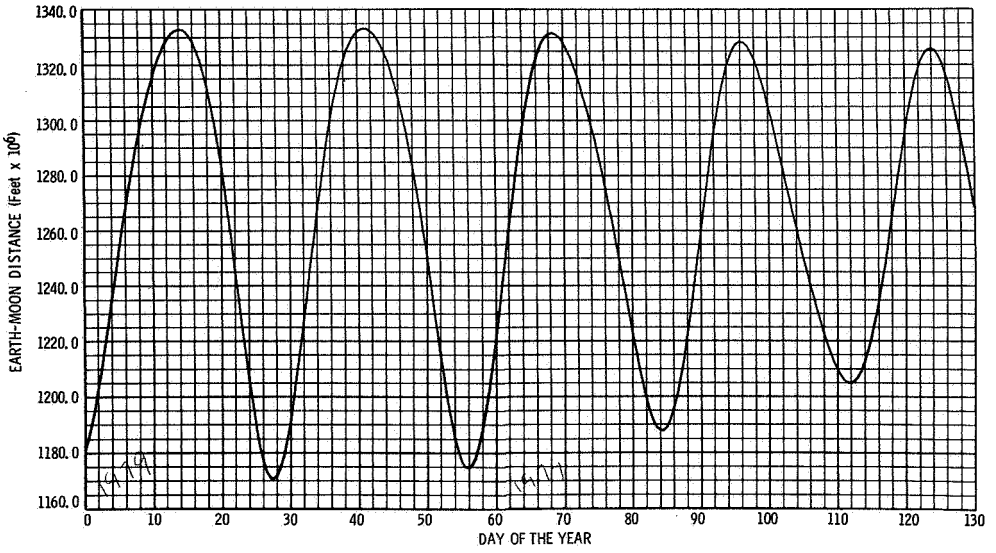
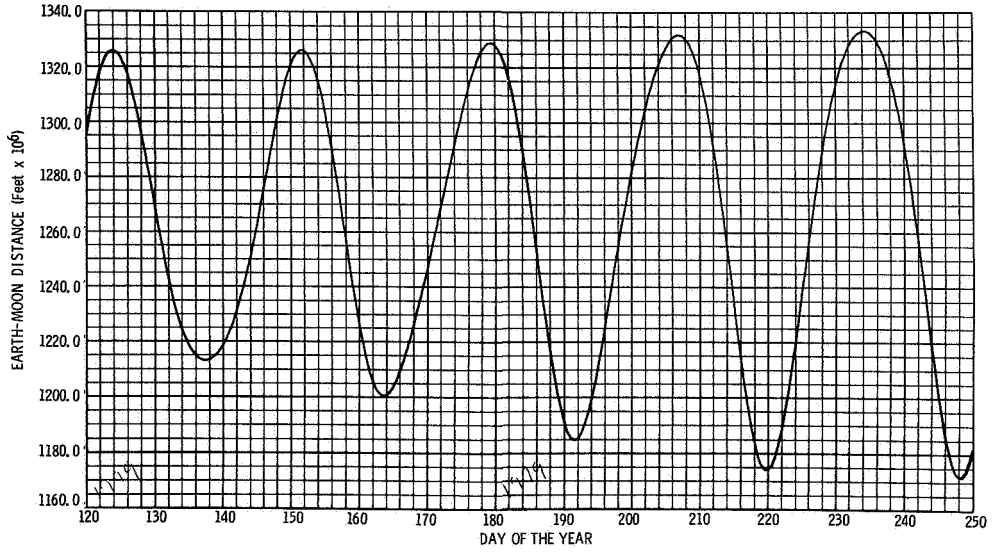


FIGURE B1978-16 TRANSLUNAR DIHEDRAL ANGLES

1979





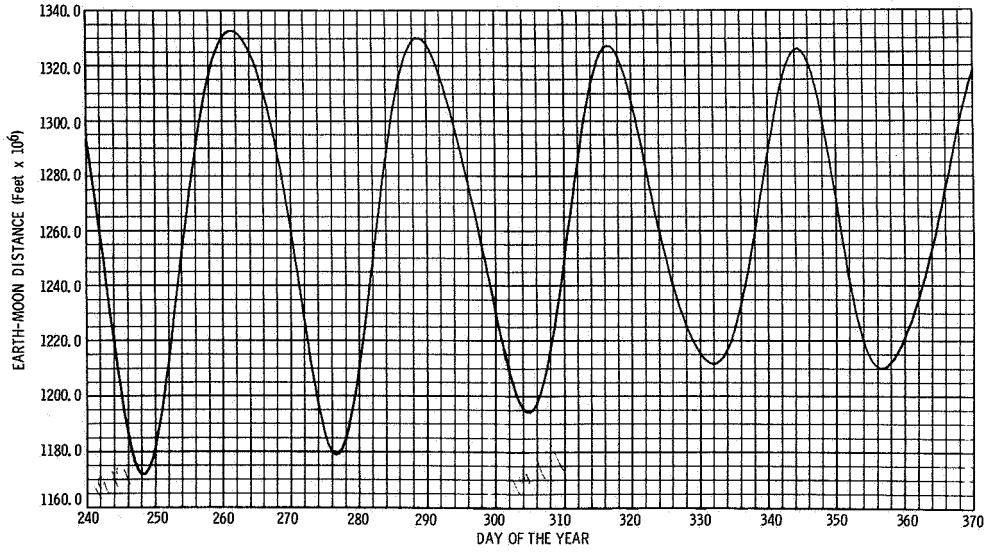
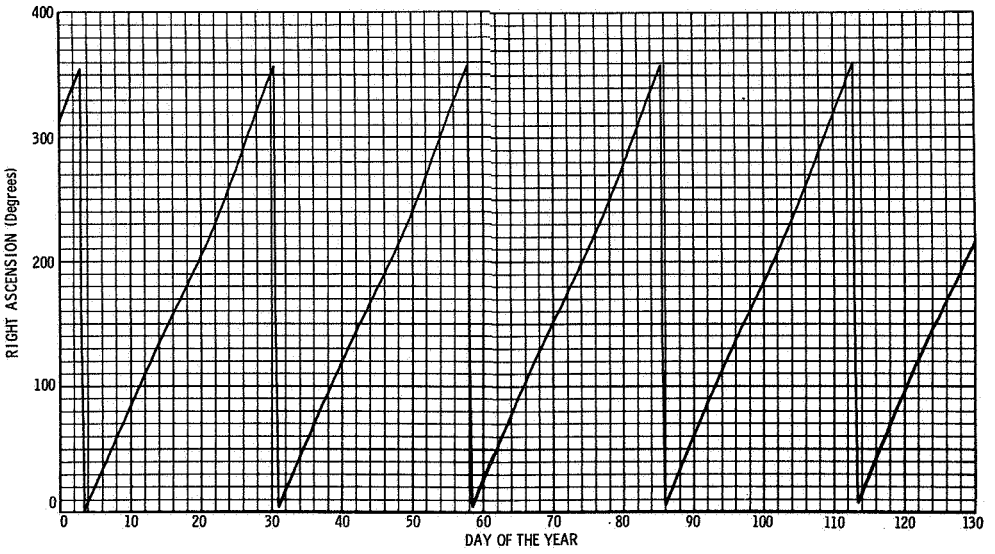
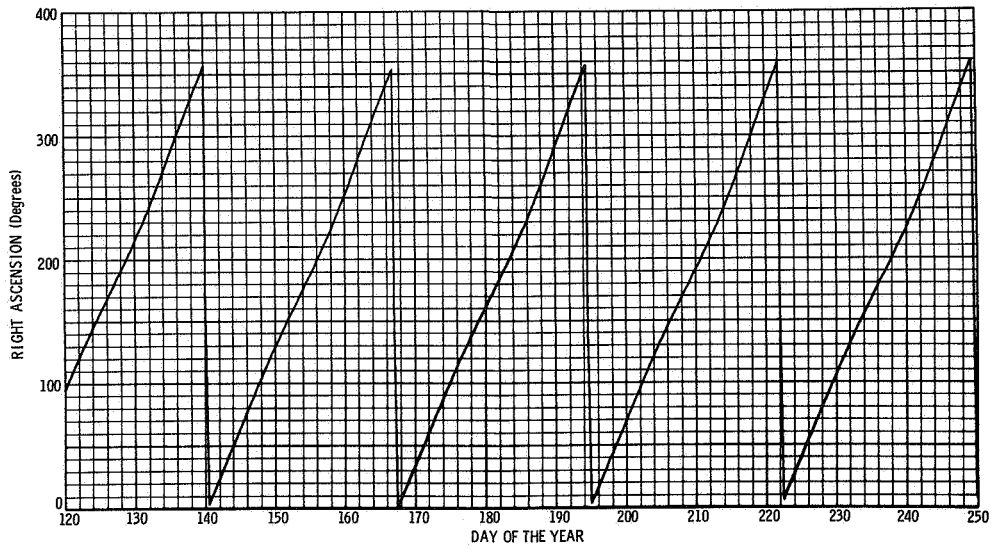


FIGURE B1979-1 EARTH-MOON DISTANCE





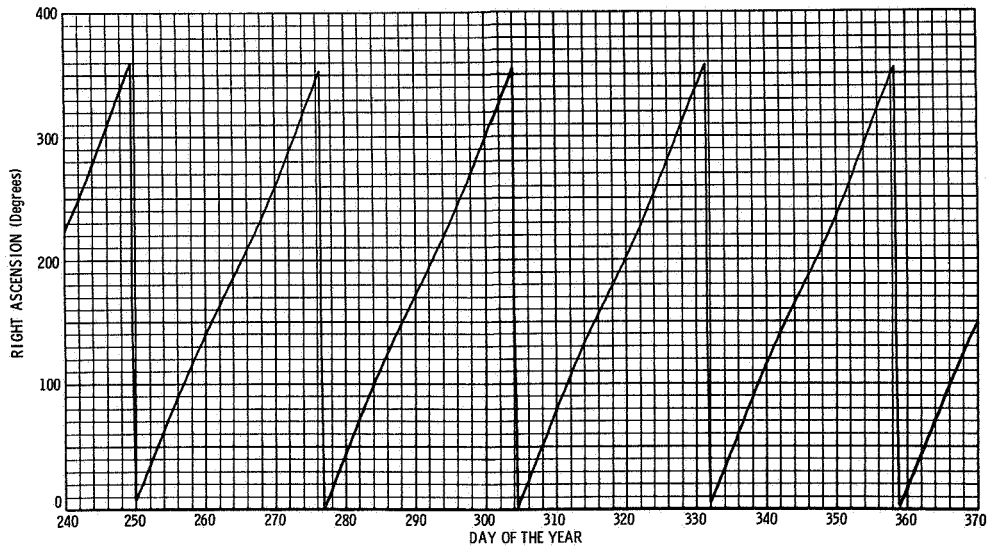
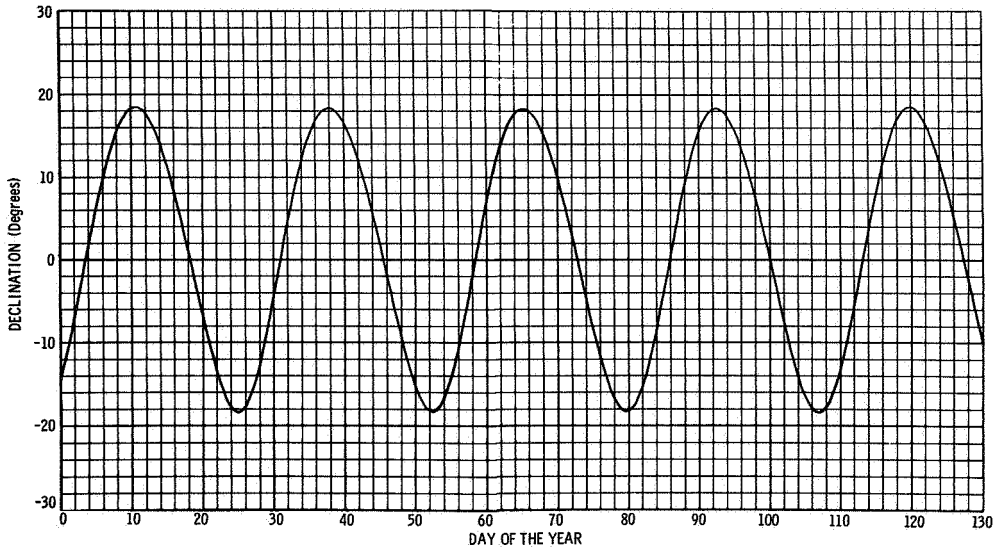
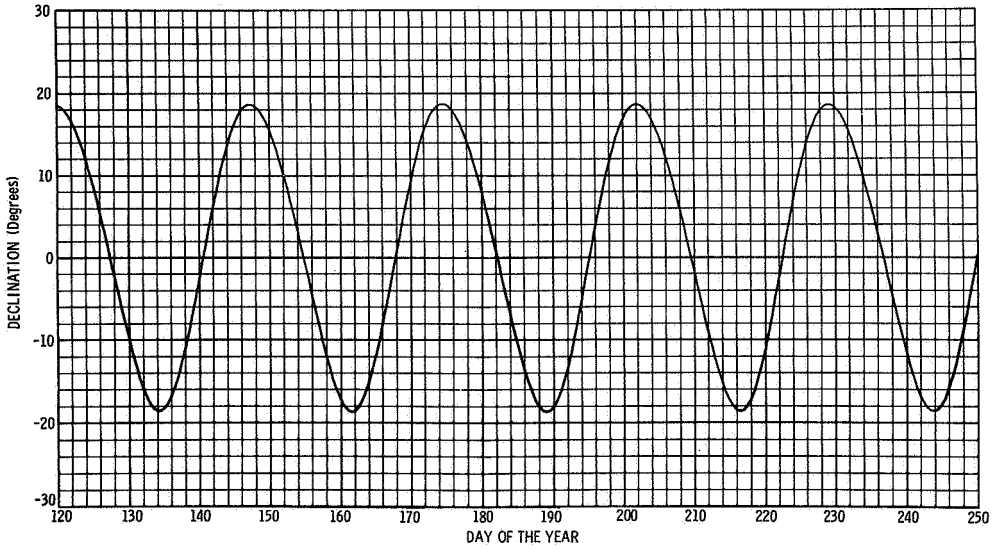


FIGURE B1979-2 RIGHT ASCENSION OF THE MOON





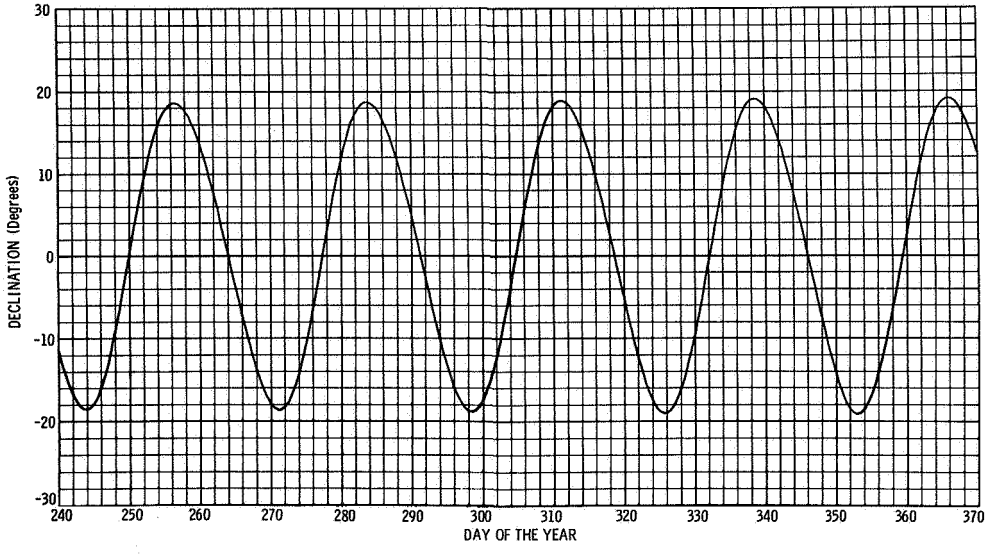


FIGURE B1979-3 DECLINATION OF THE MOON

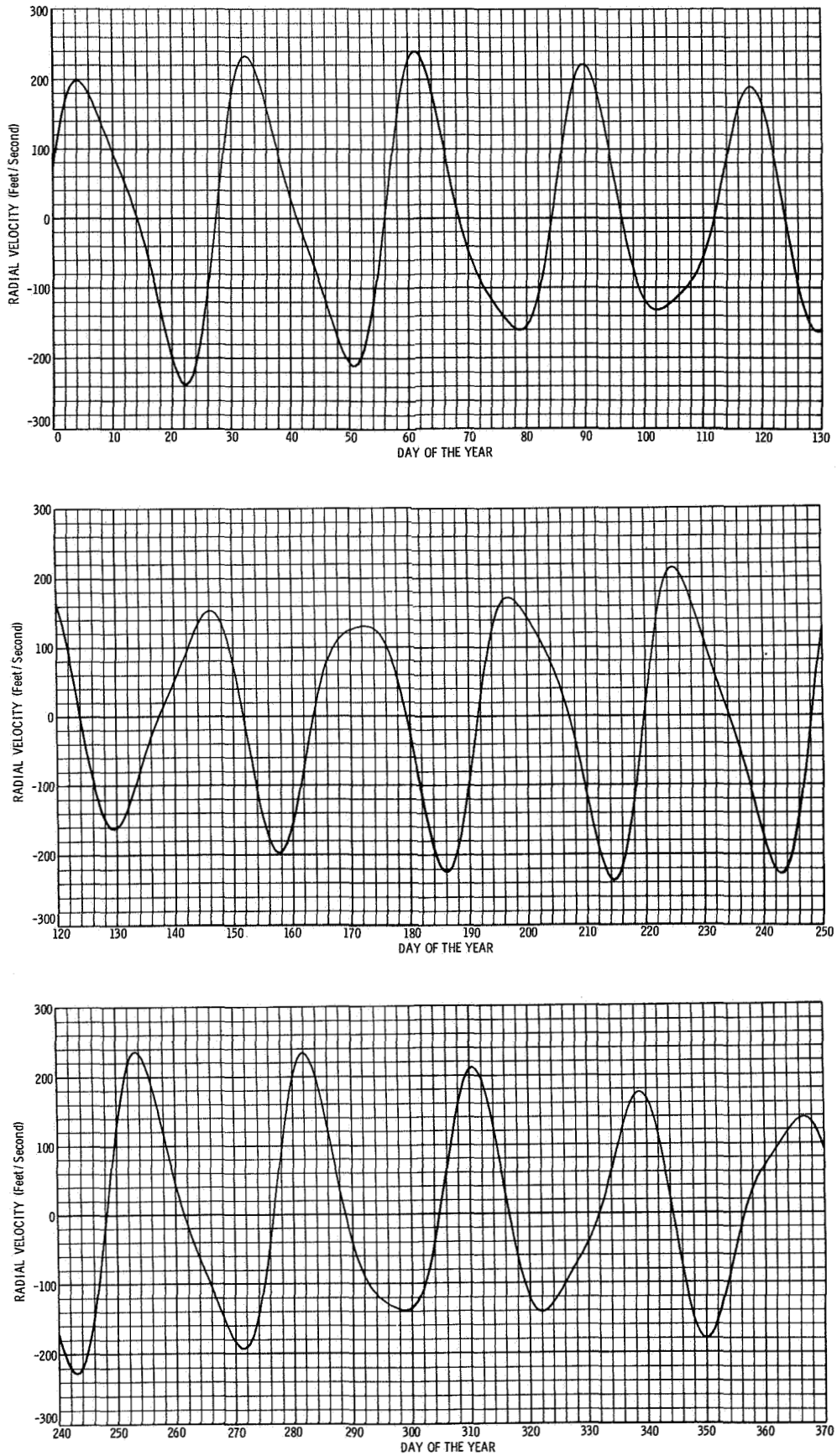
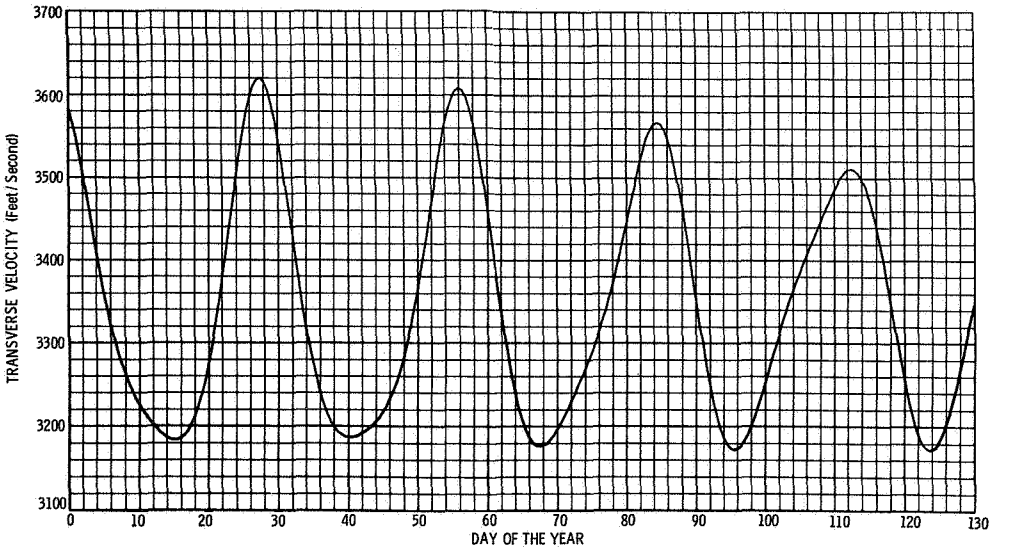
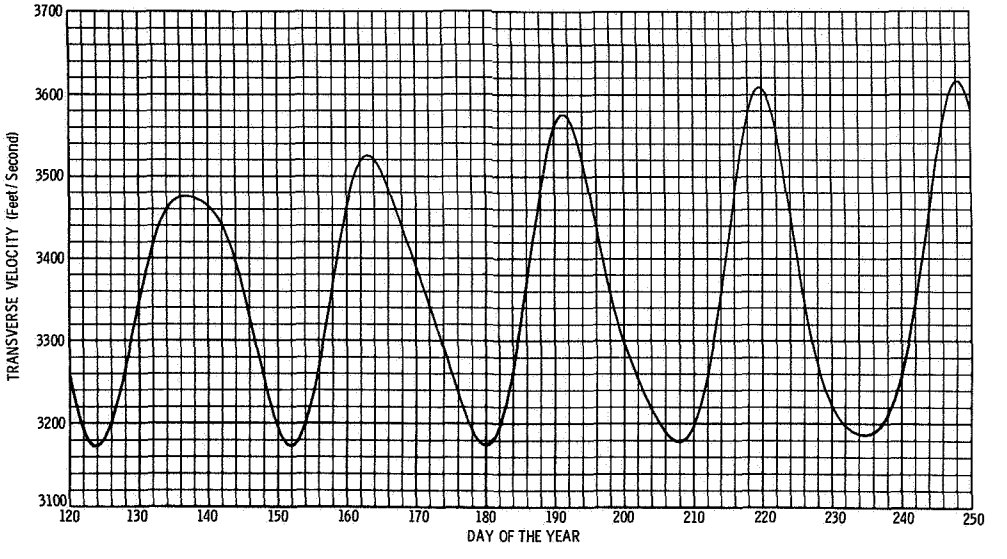


FIGURE B1979-4 RADIAL VELOCITY OF THE MOON





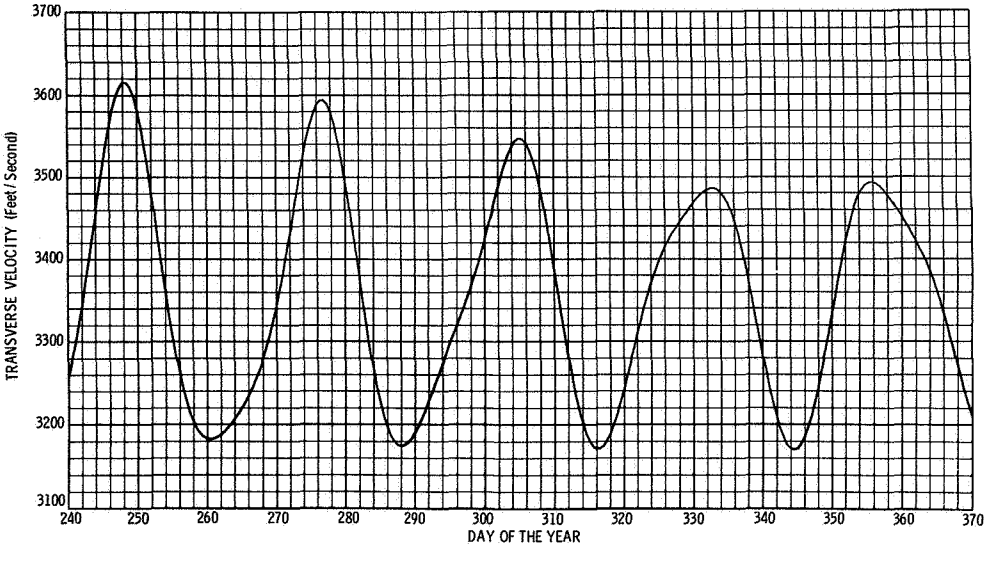


FIGURE B1979-5 TRANSVERSE VELOCITY OF THE MOON

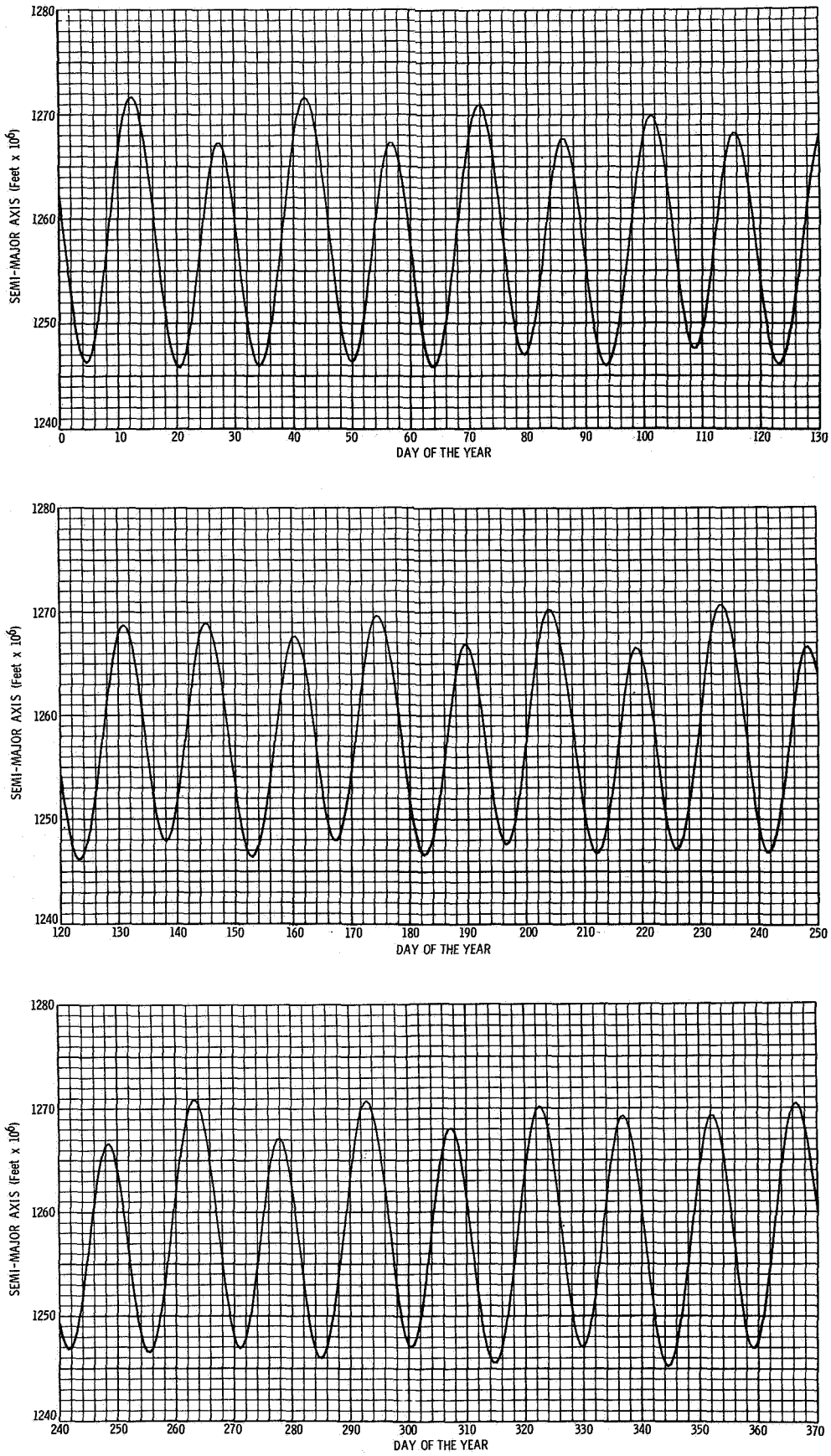


FIGURE B1979-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT

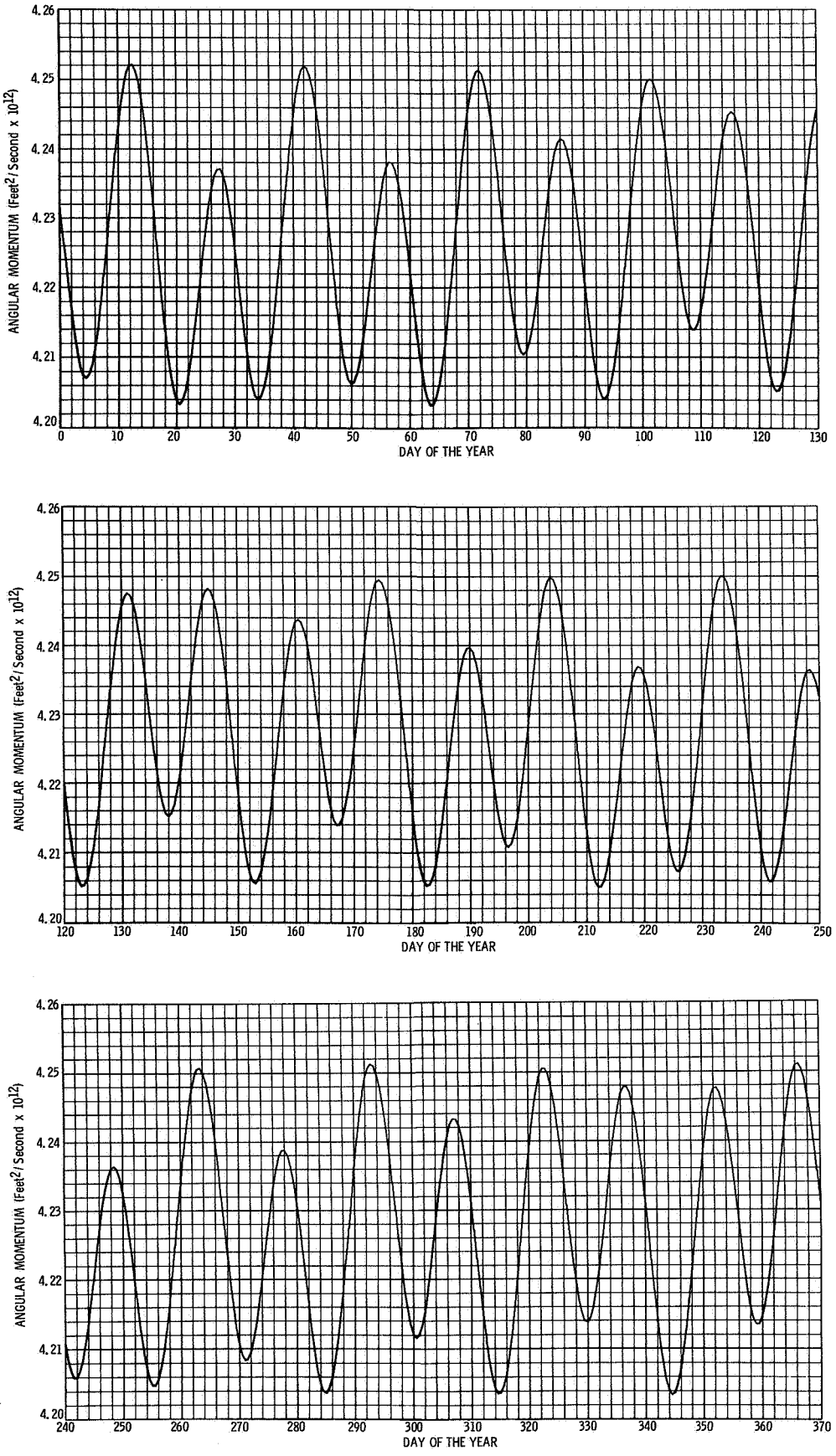
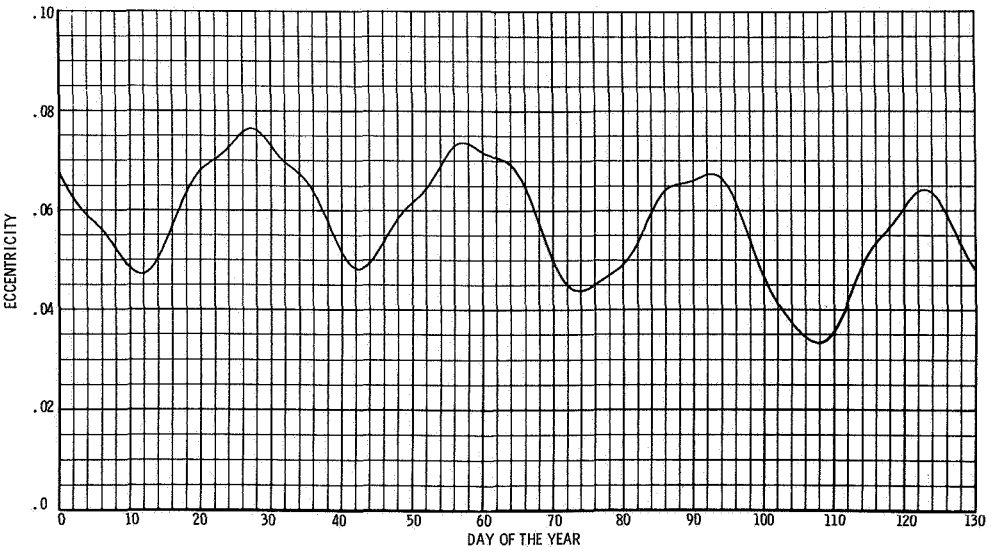
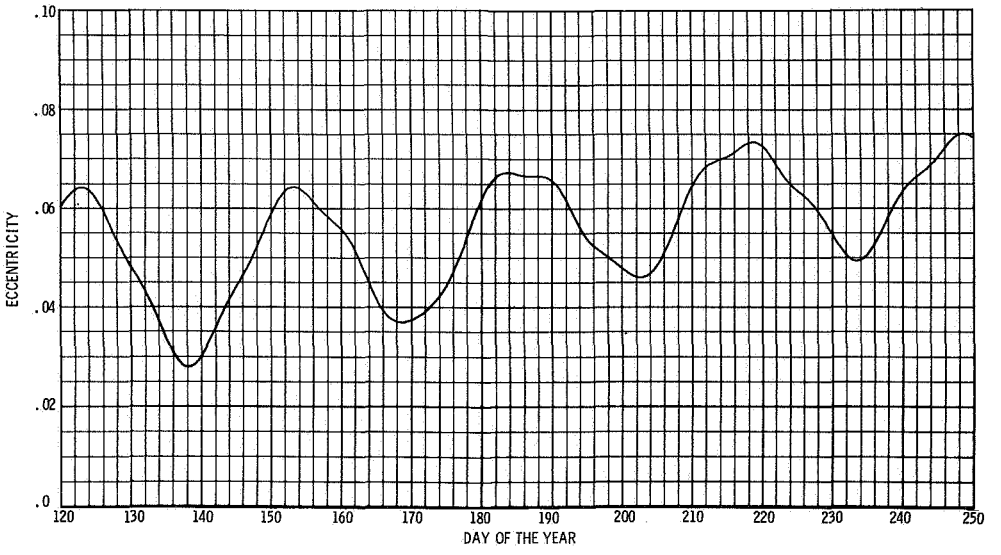


FIGURE B1979-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON





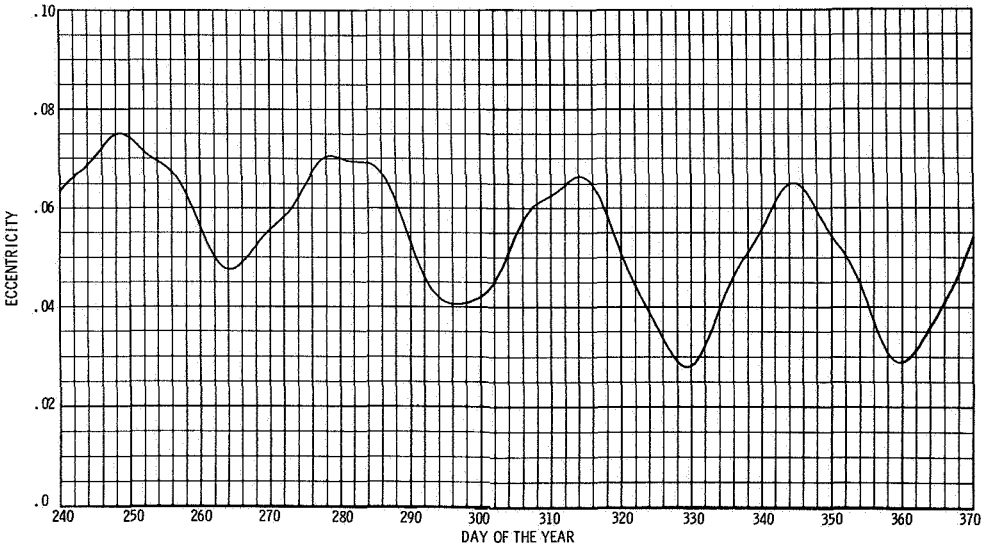


FIGURE B1979-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT

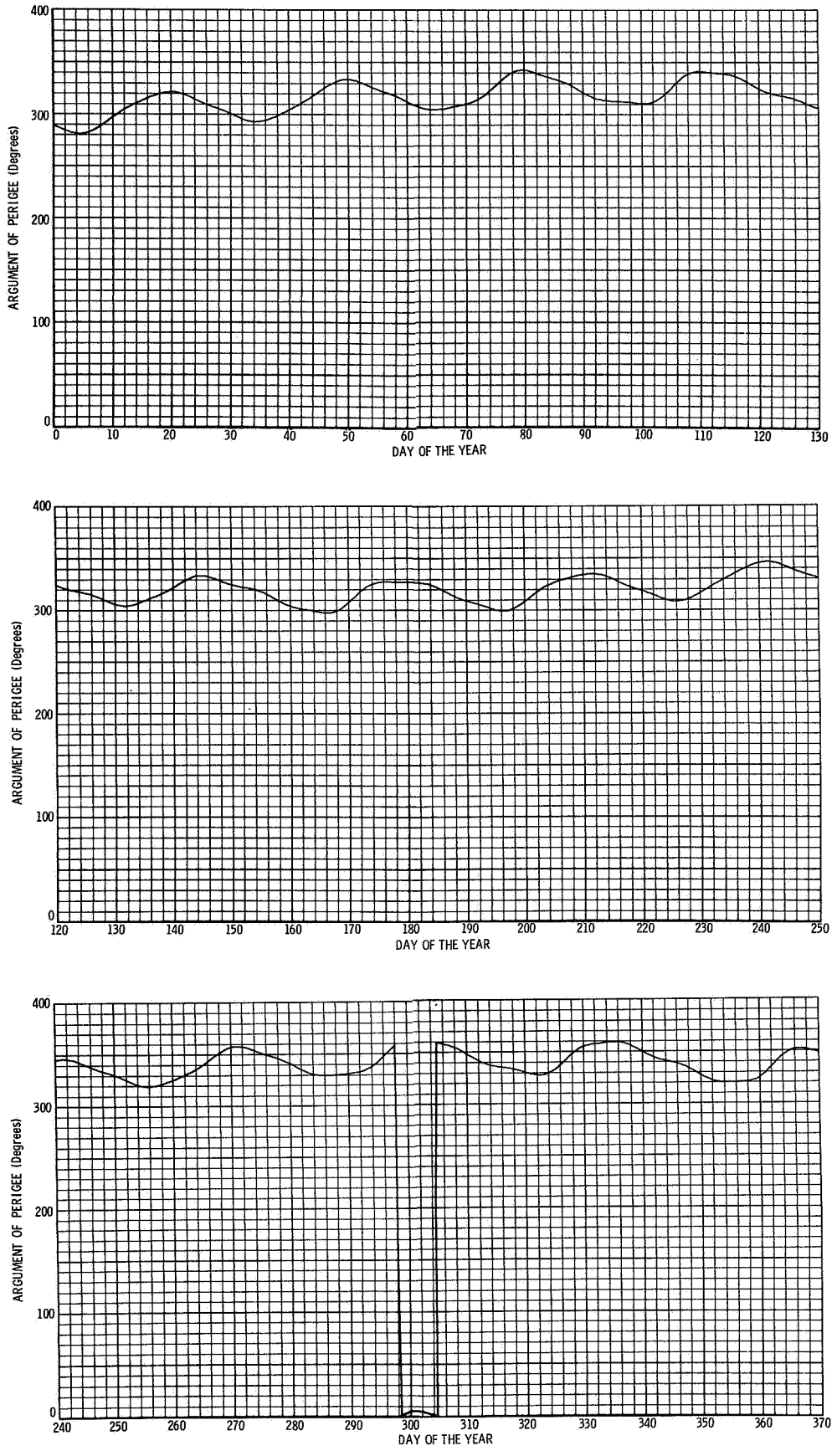
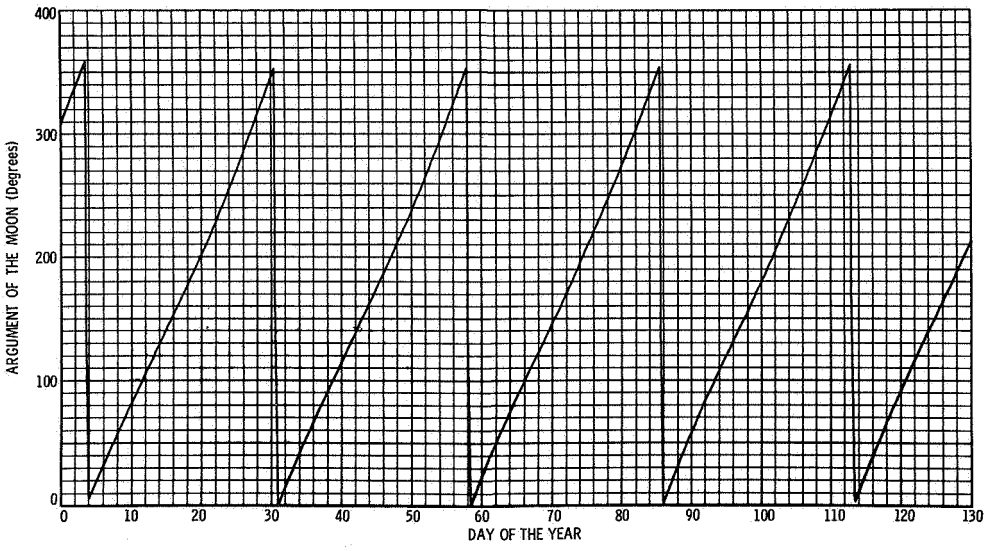
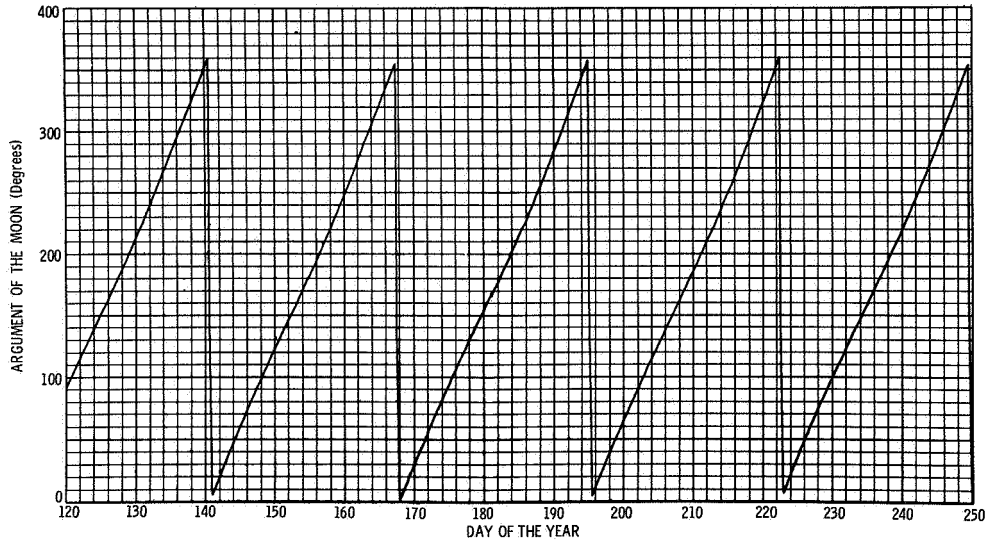


FIGURE B1979-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE





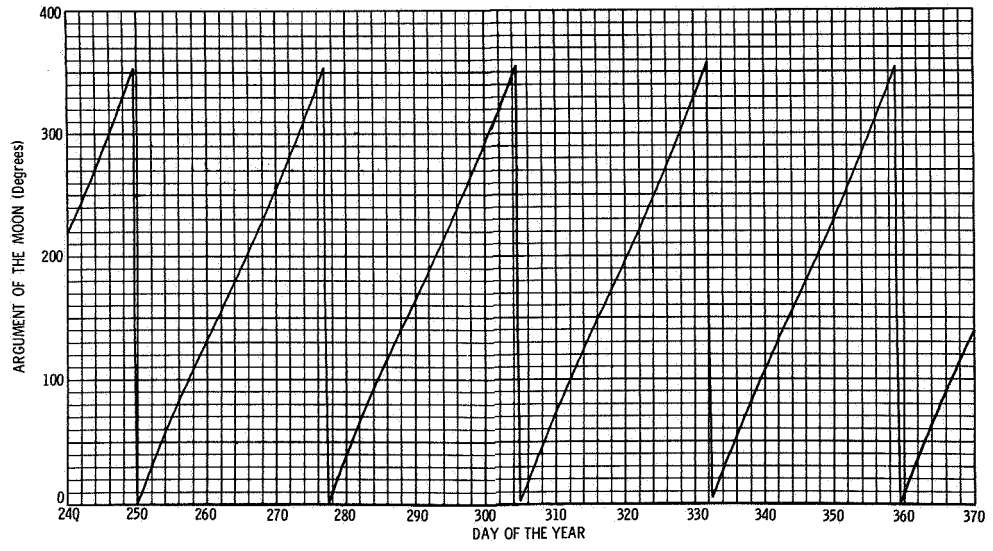
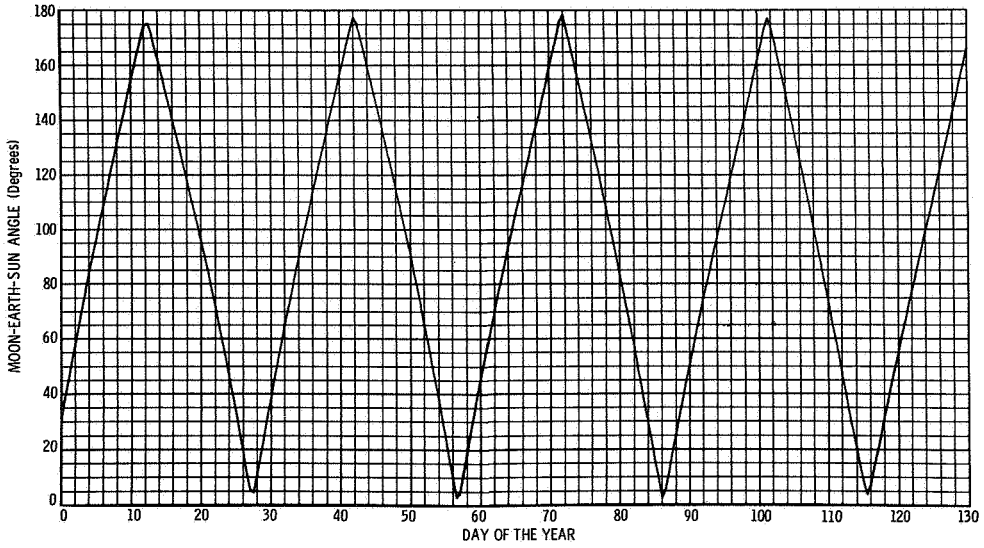
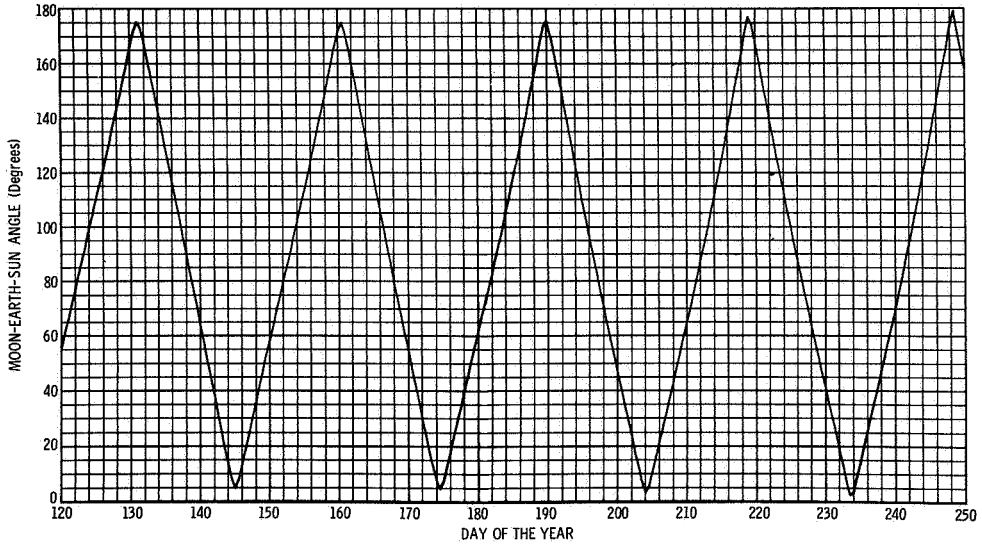


FIGURE B1979-10 ARGUMENT OF THE MOON'S POSITION





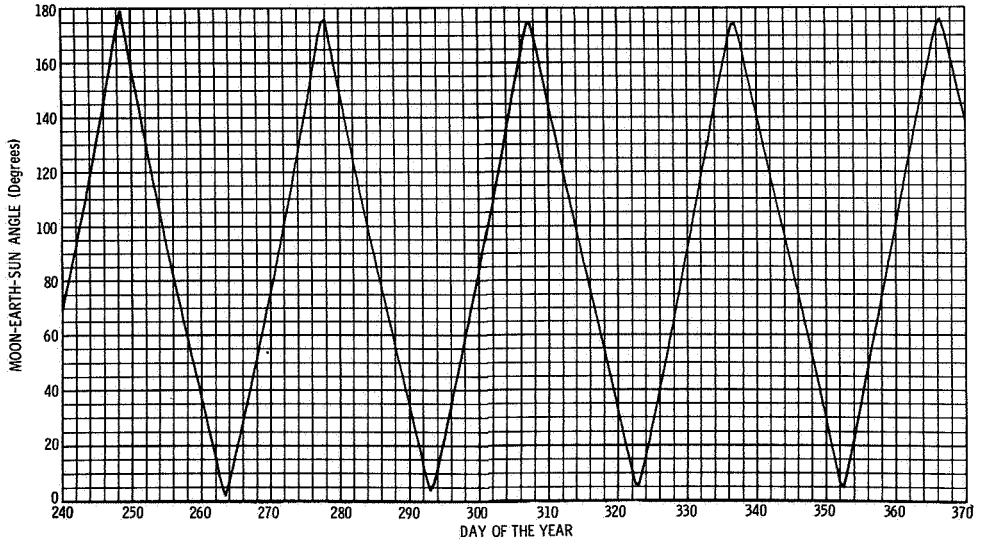
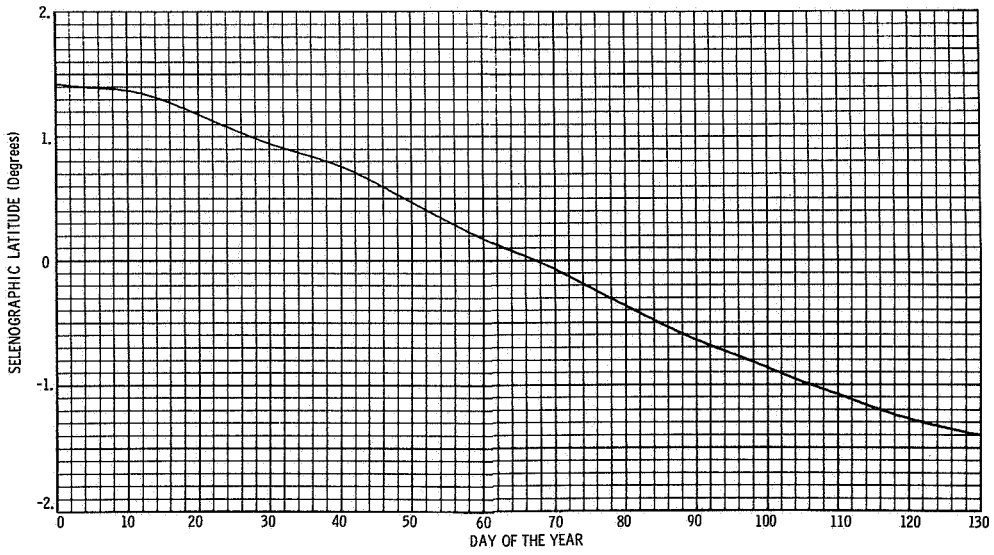
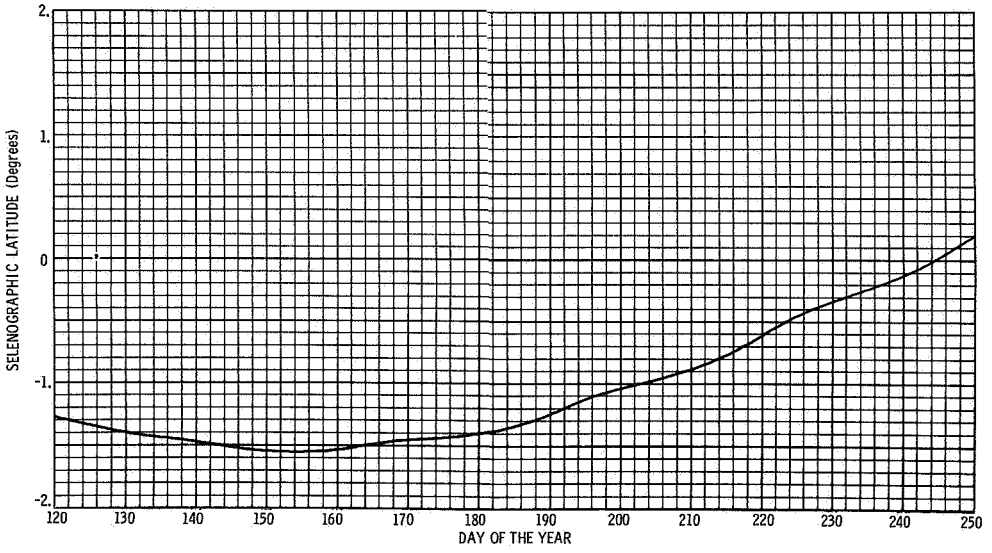


FIGURE B1979-11 MOON-EARTH-SUN ANGLE





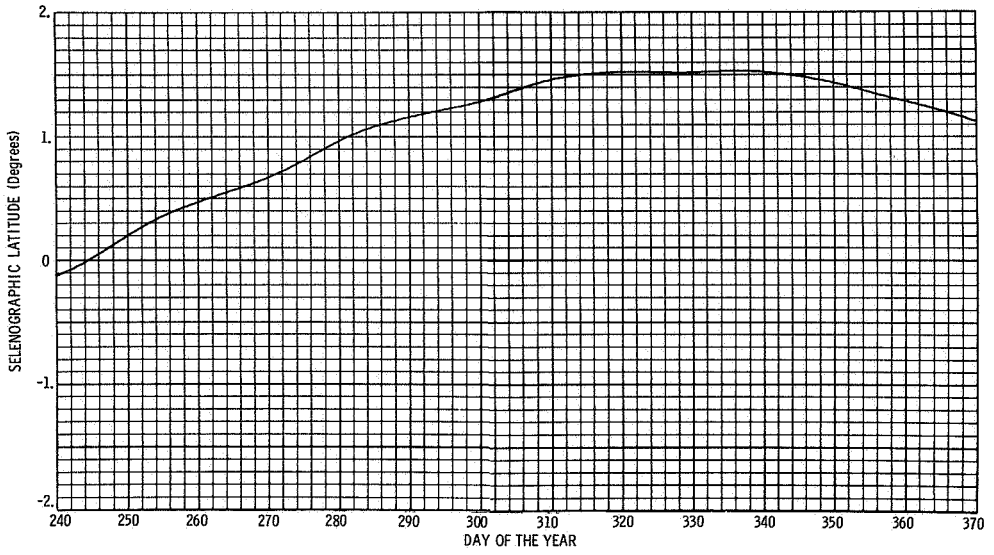
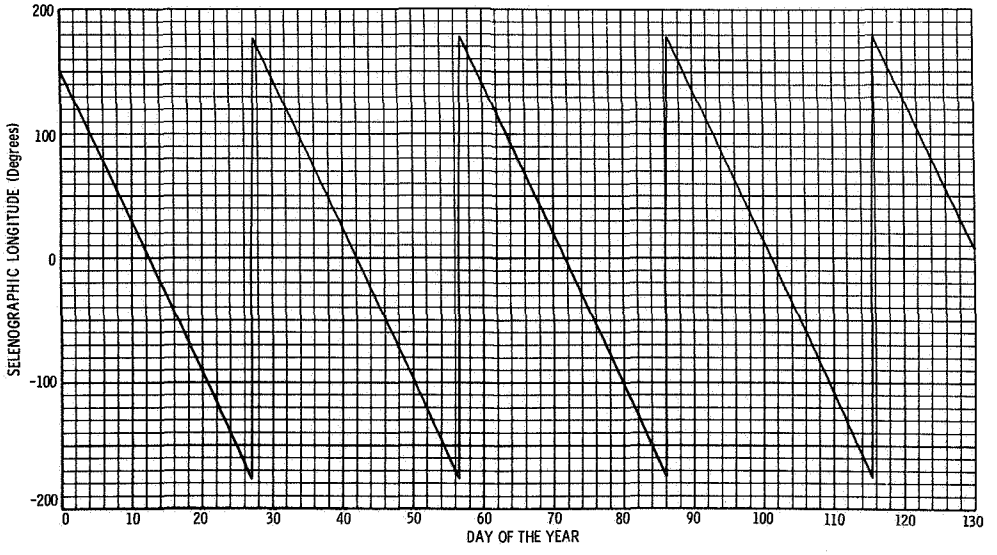
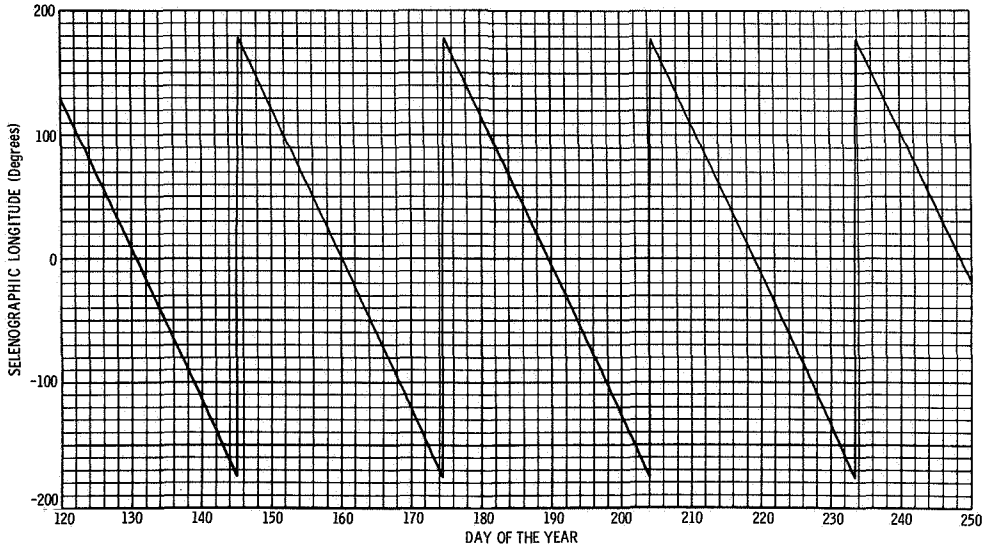


FIGURE B1979-12 SELENOGRAPHIC LATITUDE OF THE SUN





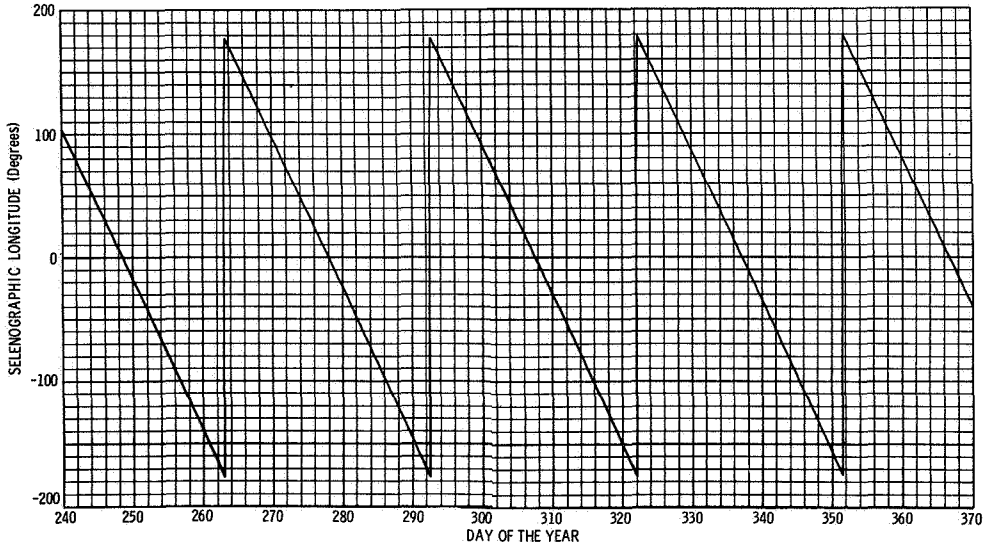
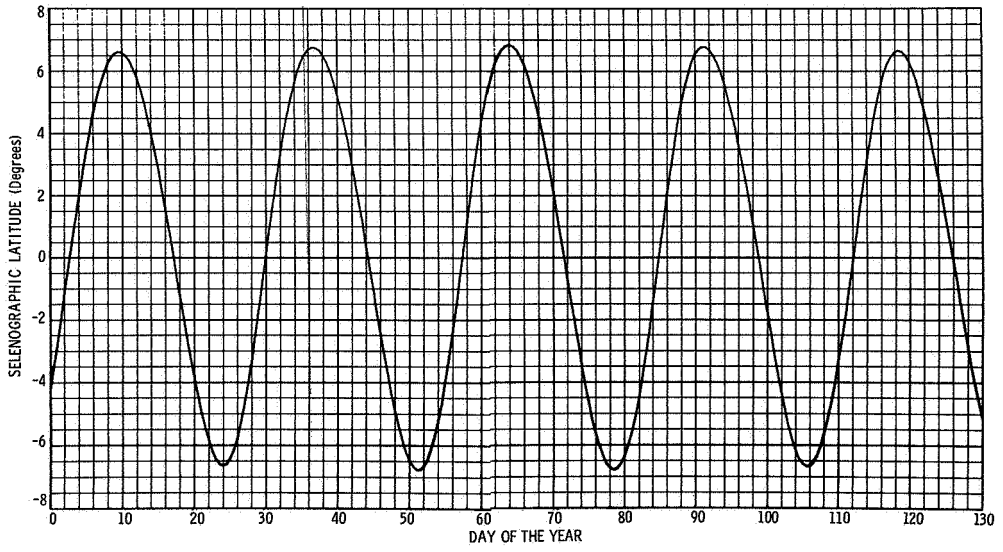
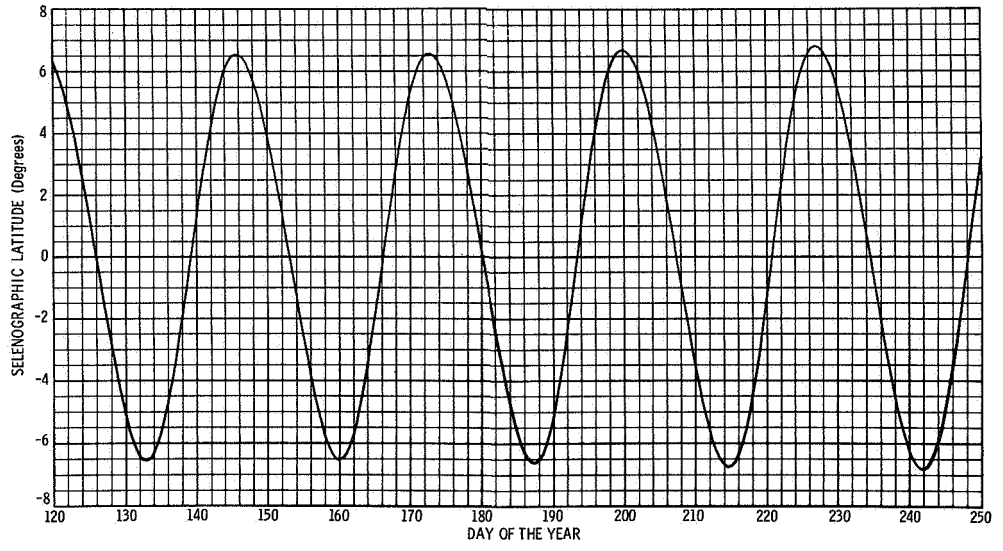


FIGURE B1979-13 SELENOGRAPHIC LONGITUDE OF THE SUN





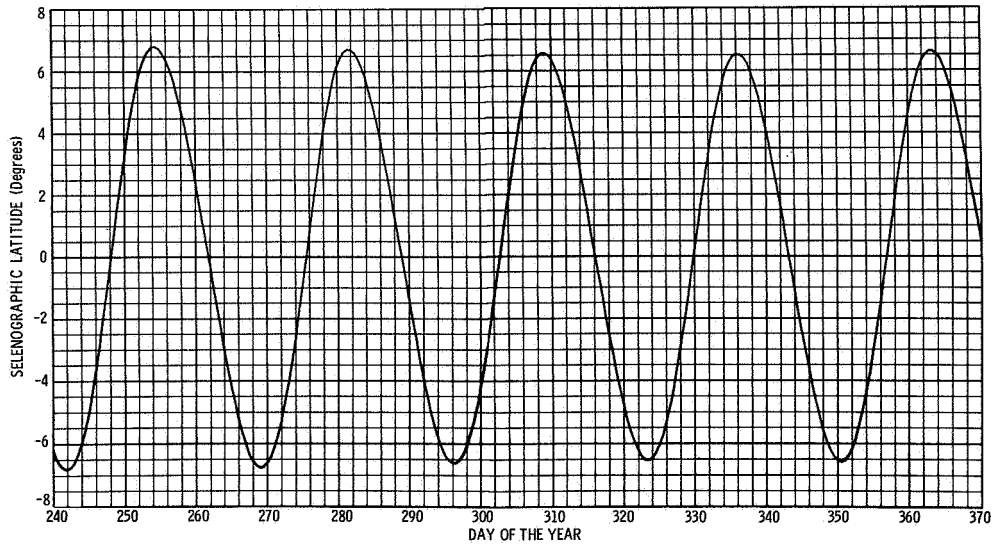
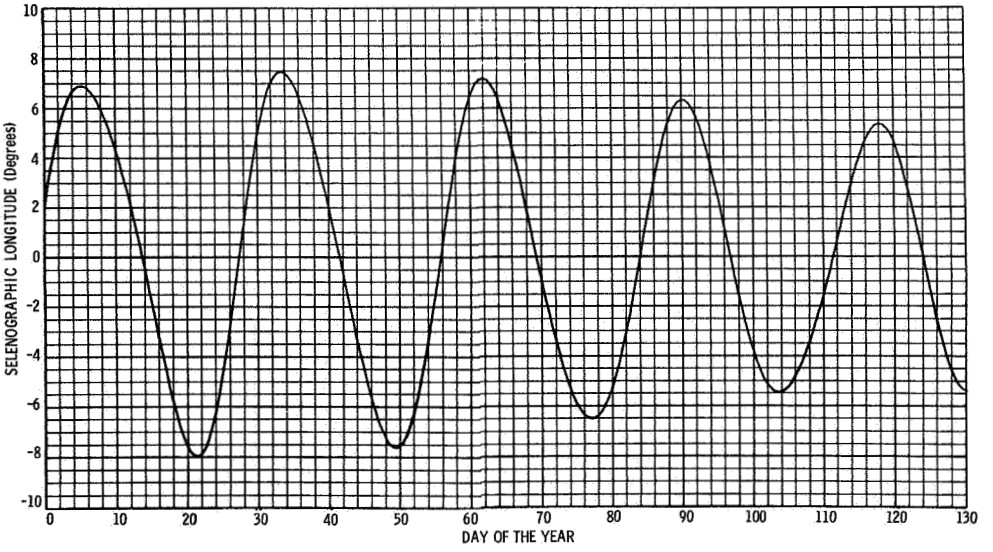
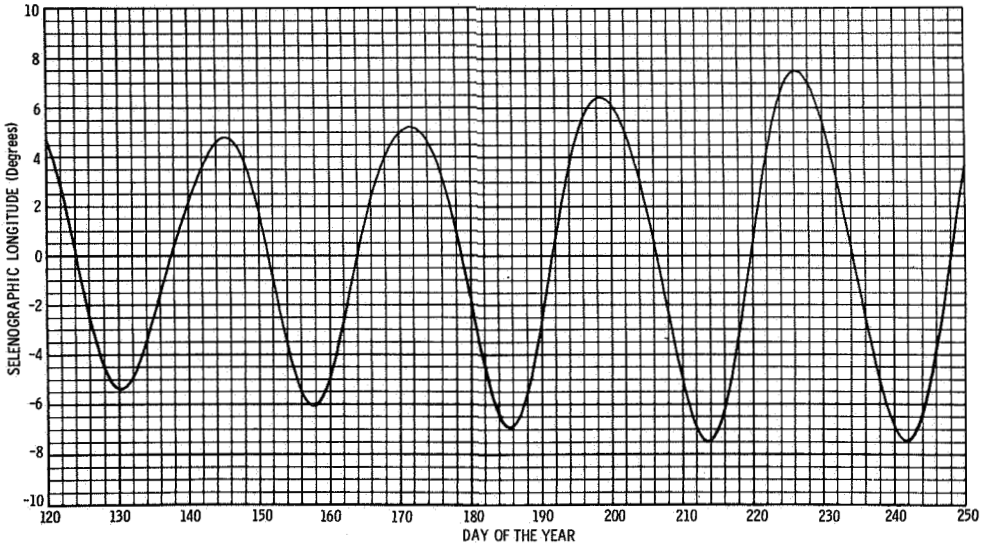


FIGURE B1979-14 SELENOGRAPHIC LATITUDE OF THE EARTH





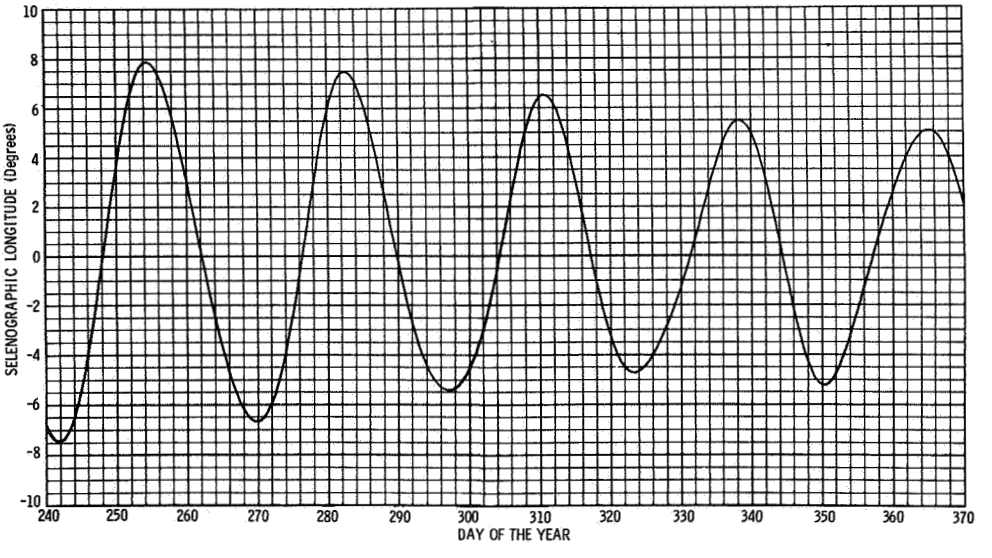


FIGURE B1979-15 SELENOGRAPHIC LONGITUDE OF THE EARTH

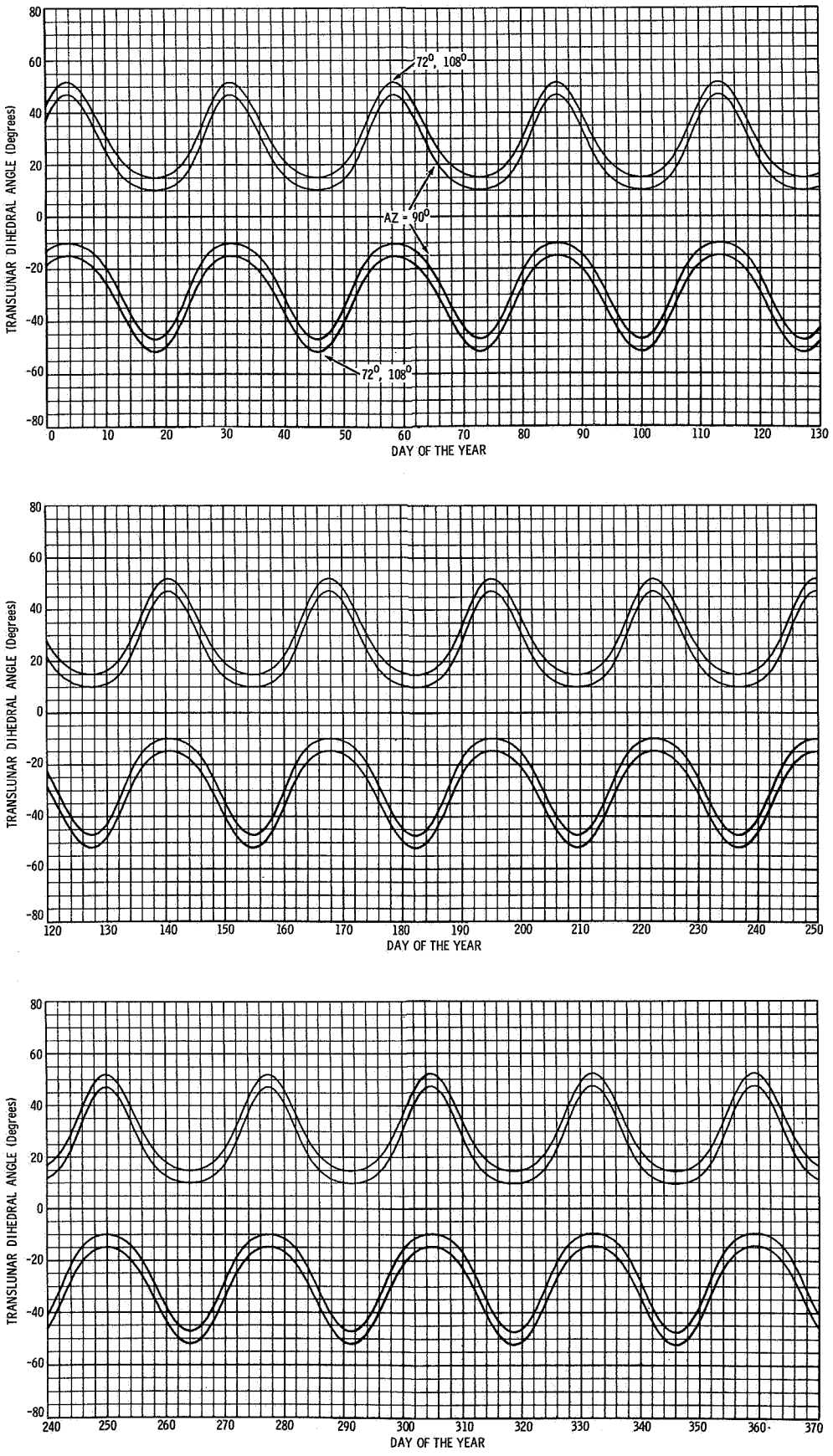


FIGURE B1979-16 TRANSLUNAR DIHEDRAL ANGLES

1980

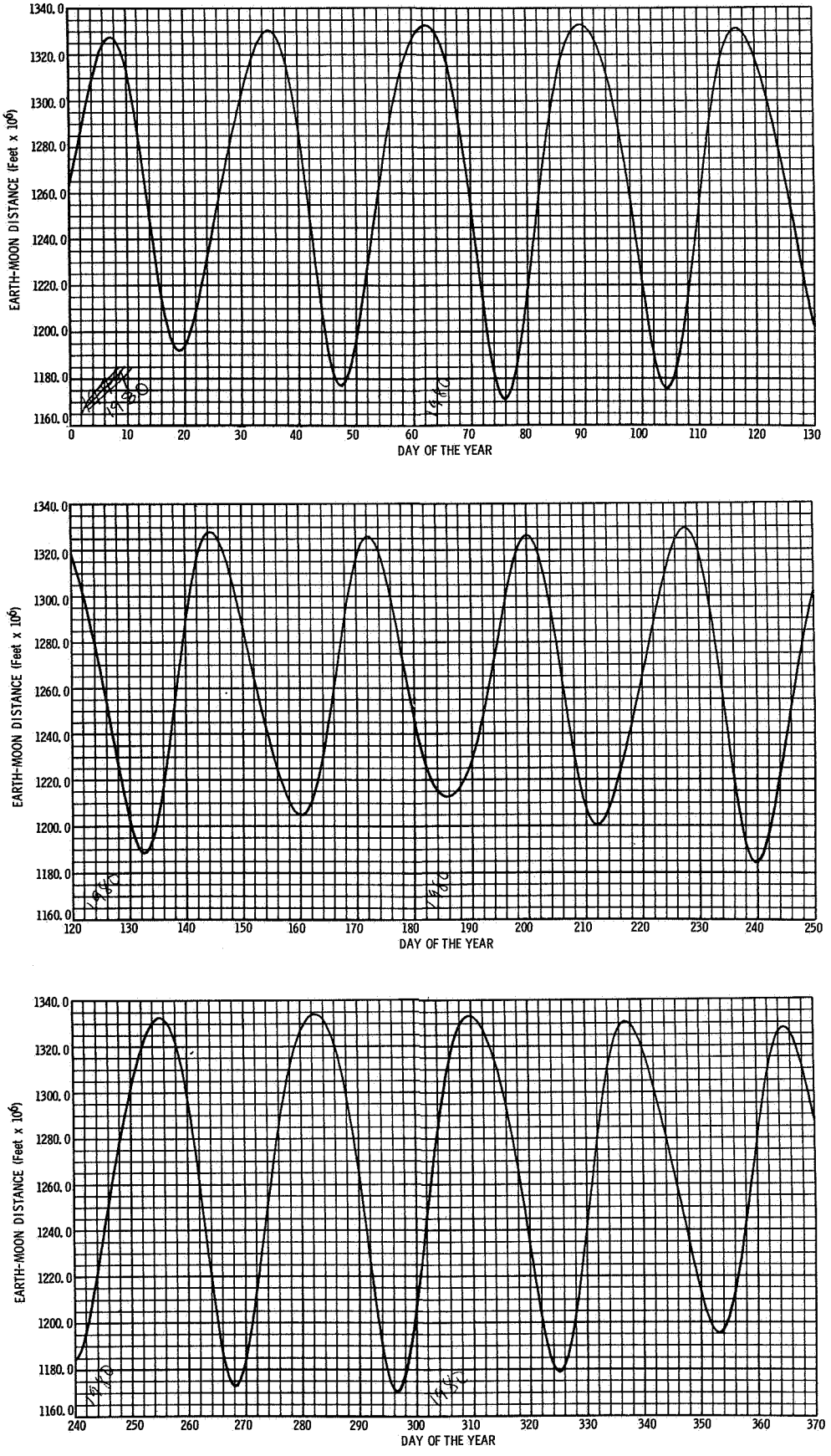
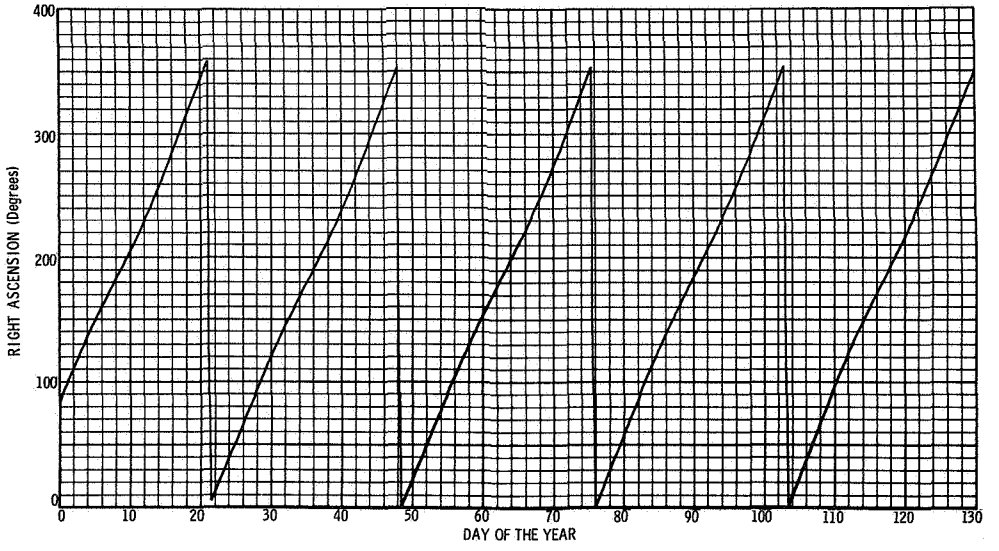
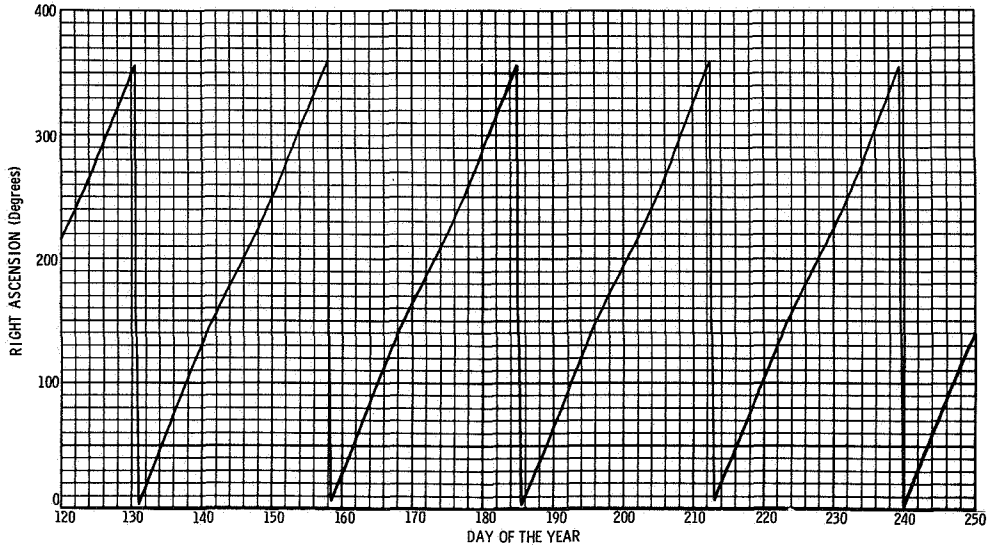


FIGURE B1980-1 EARTH-MOON DISTANCE





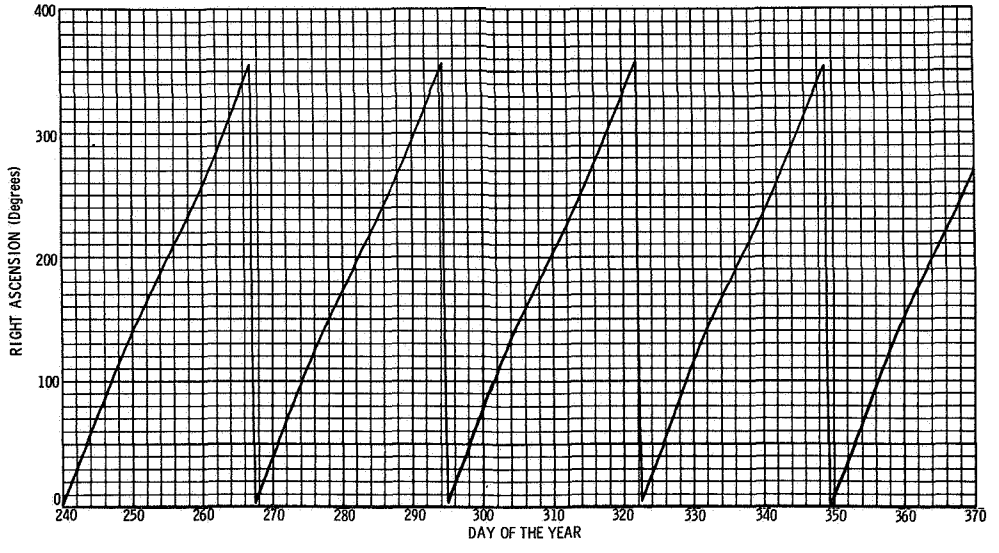


FIGURE B1980-2 RIGHT ASCENSION OF THE MOON

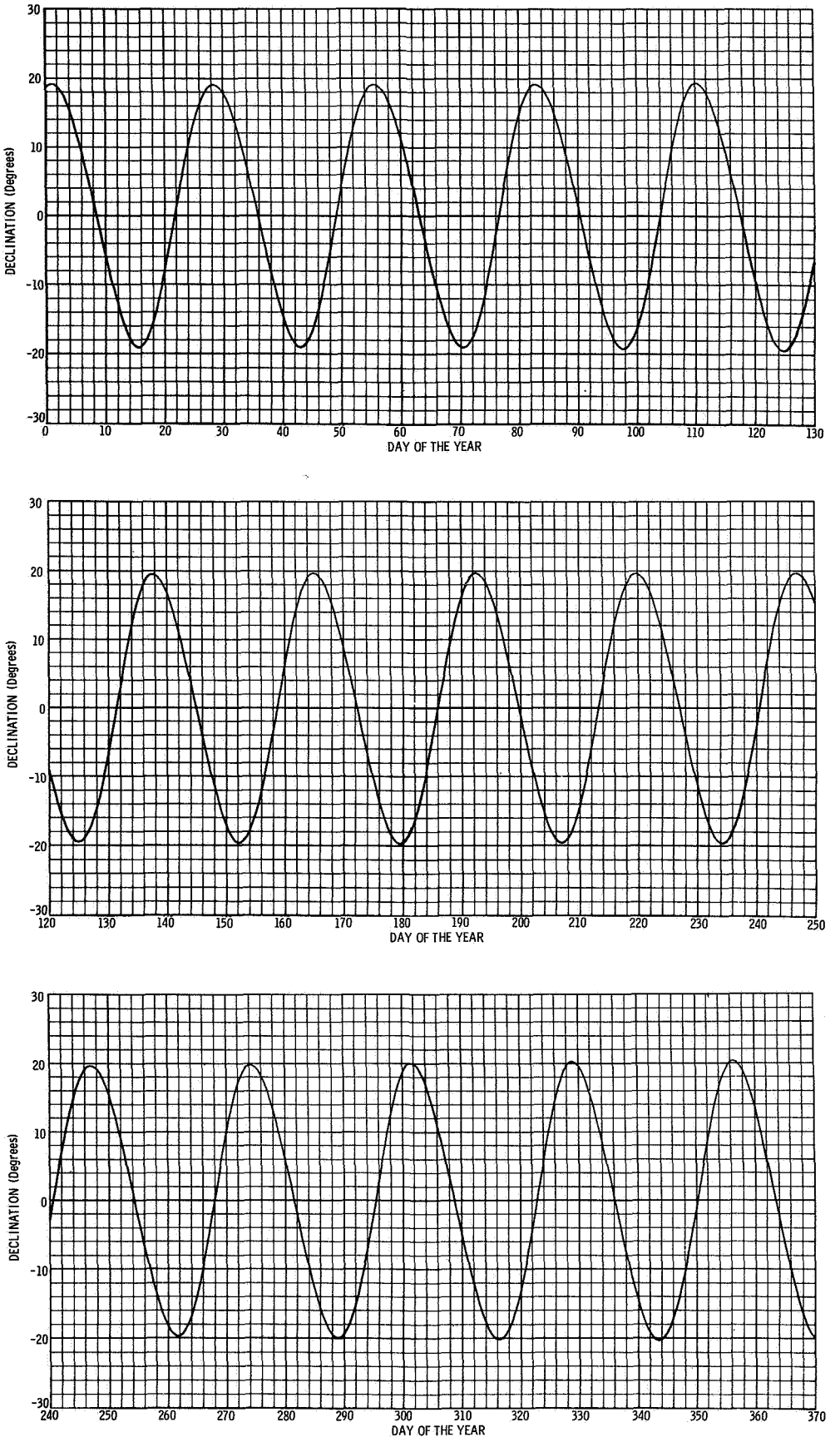


FIGURE B1980-3 DECLINATION OF THE MOON

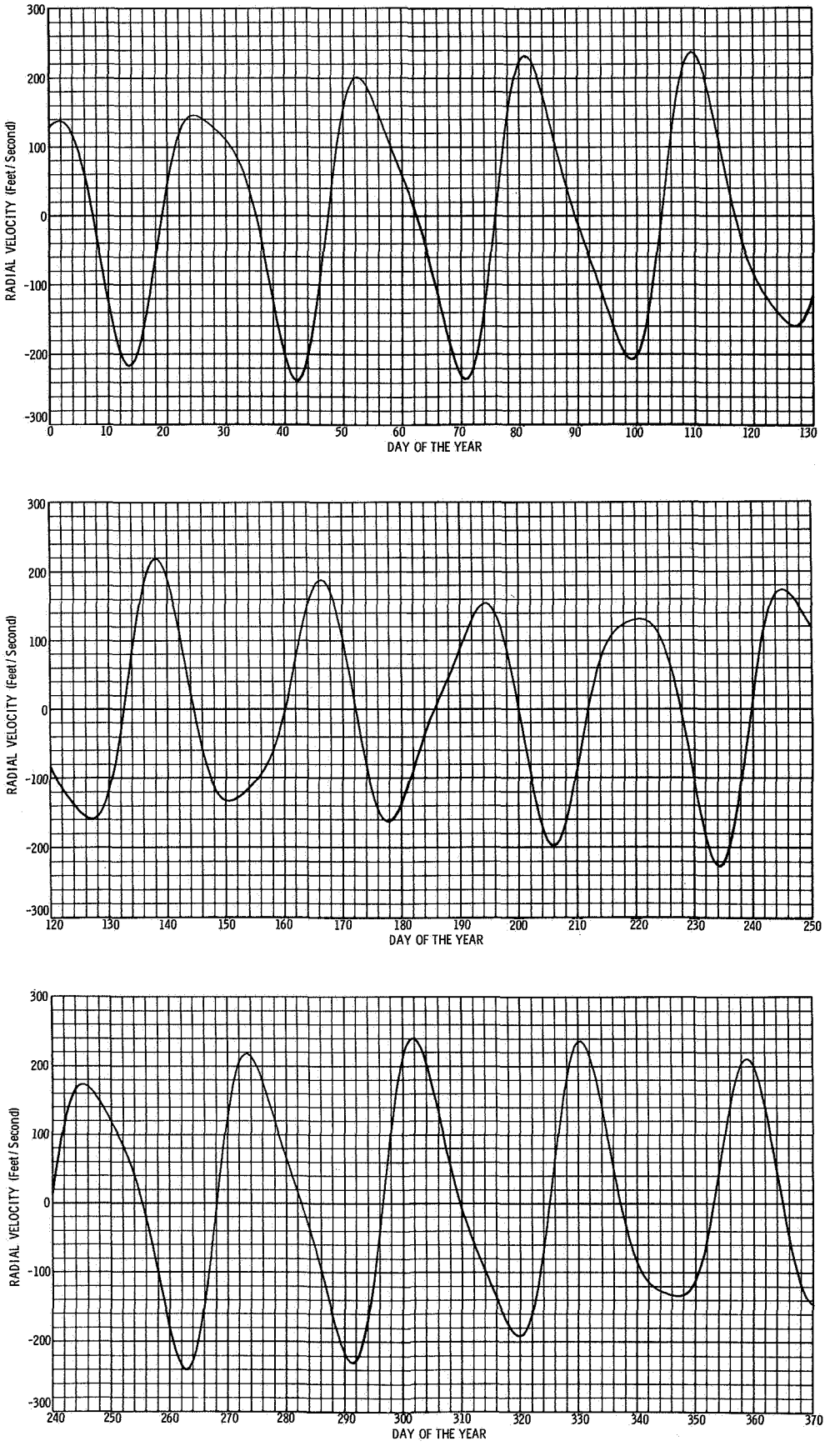
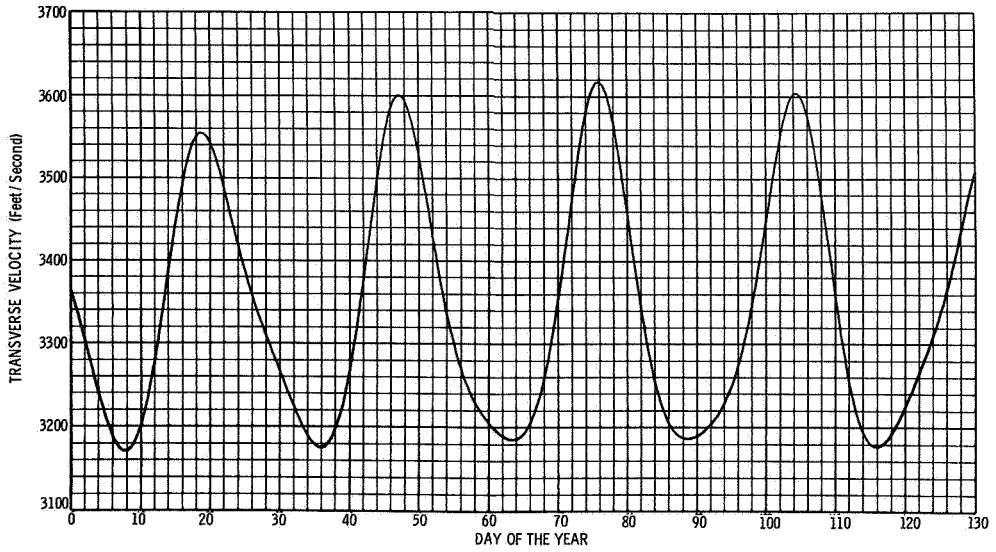
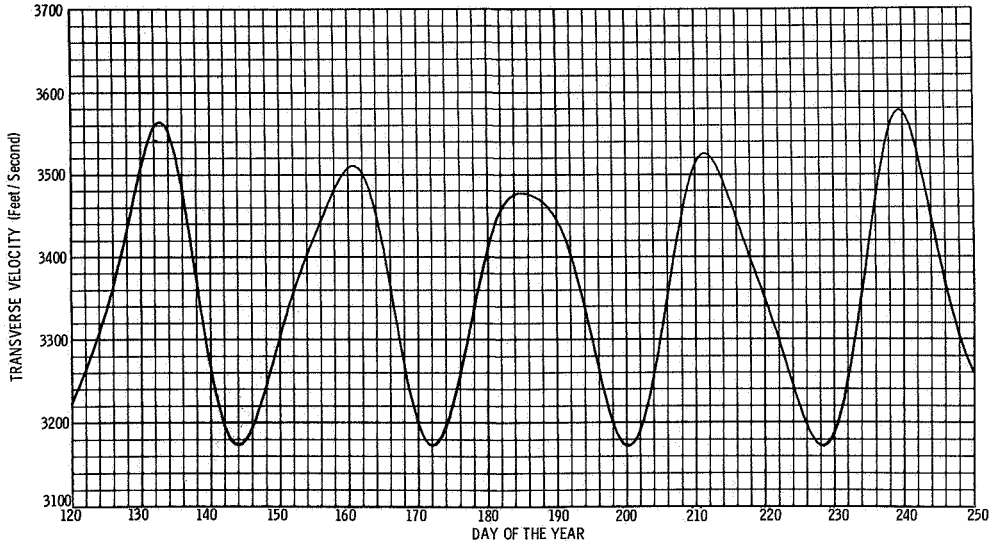


FIGURE B1980-4 RADIAL VELOCITY OF THE MOON





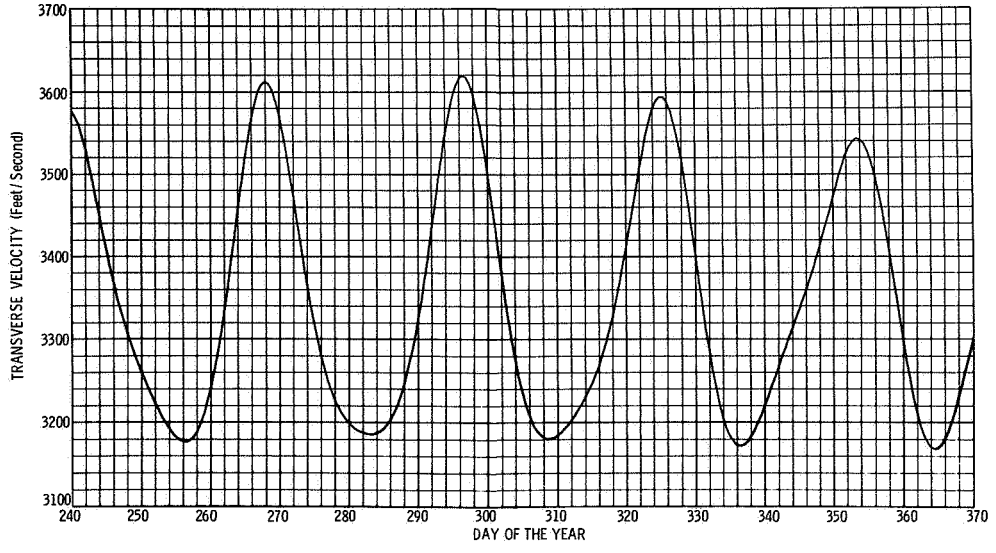


FIGURE B1980-5 TRANSVERSE VELOCITY OF THE MOON

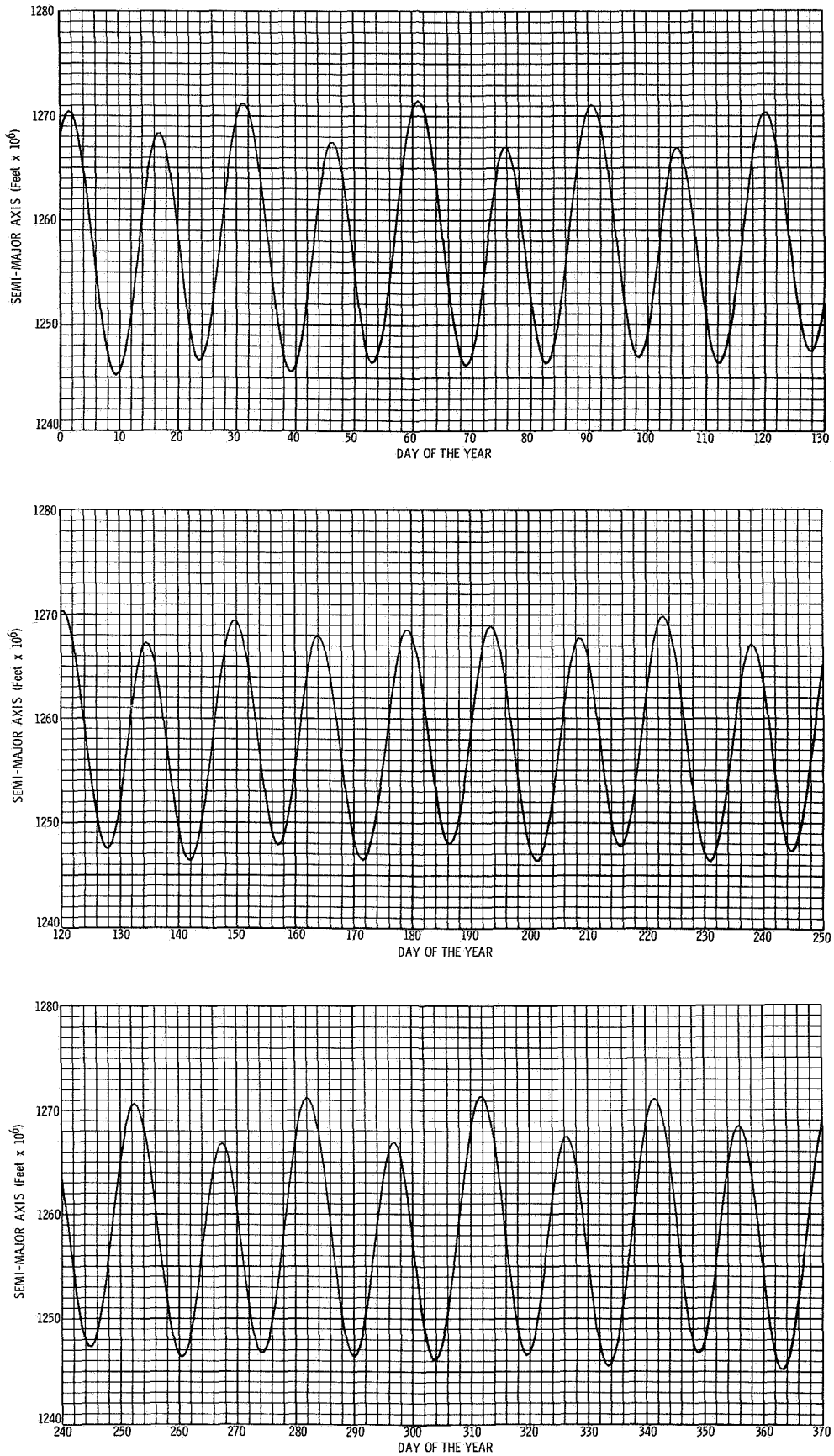


FIGURE B1980-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT

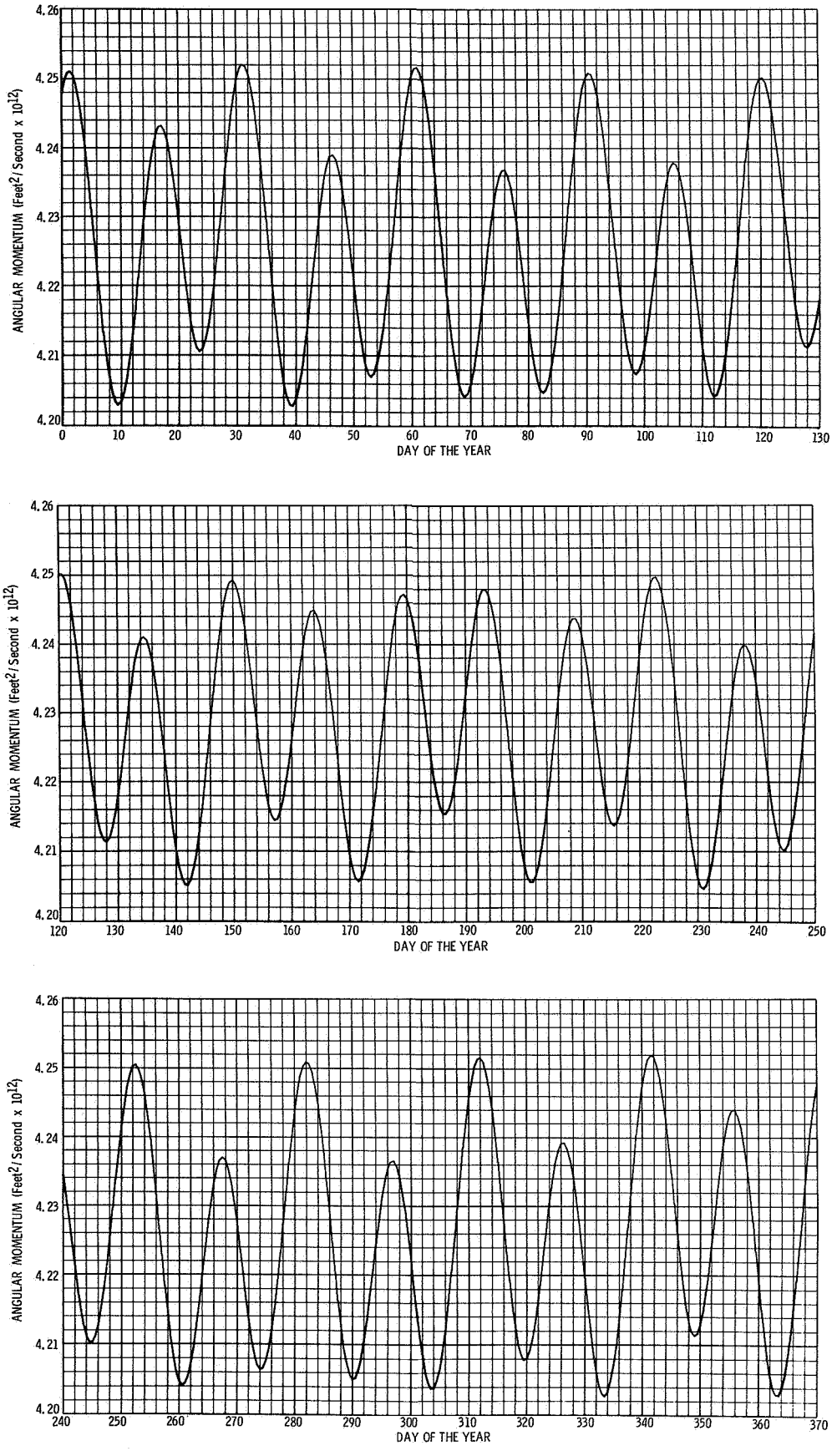


FIGURE B1980-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON

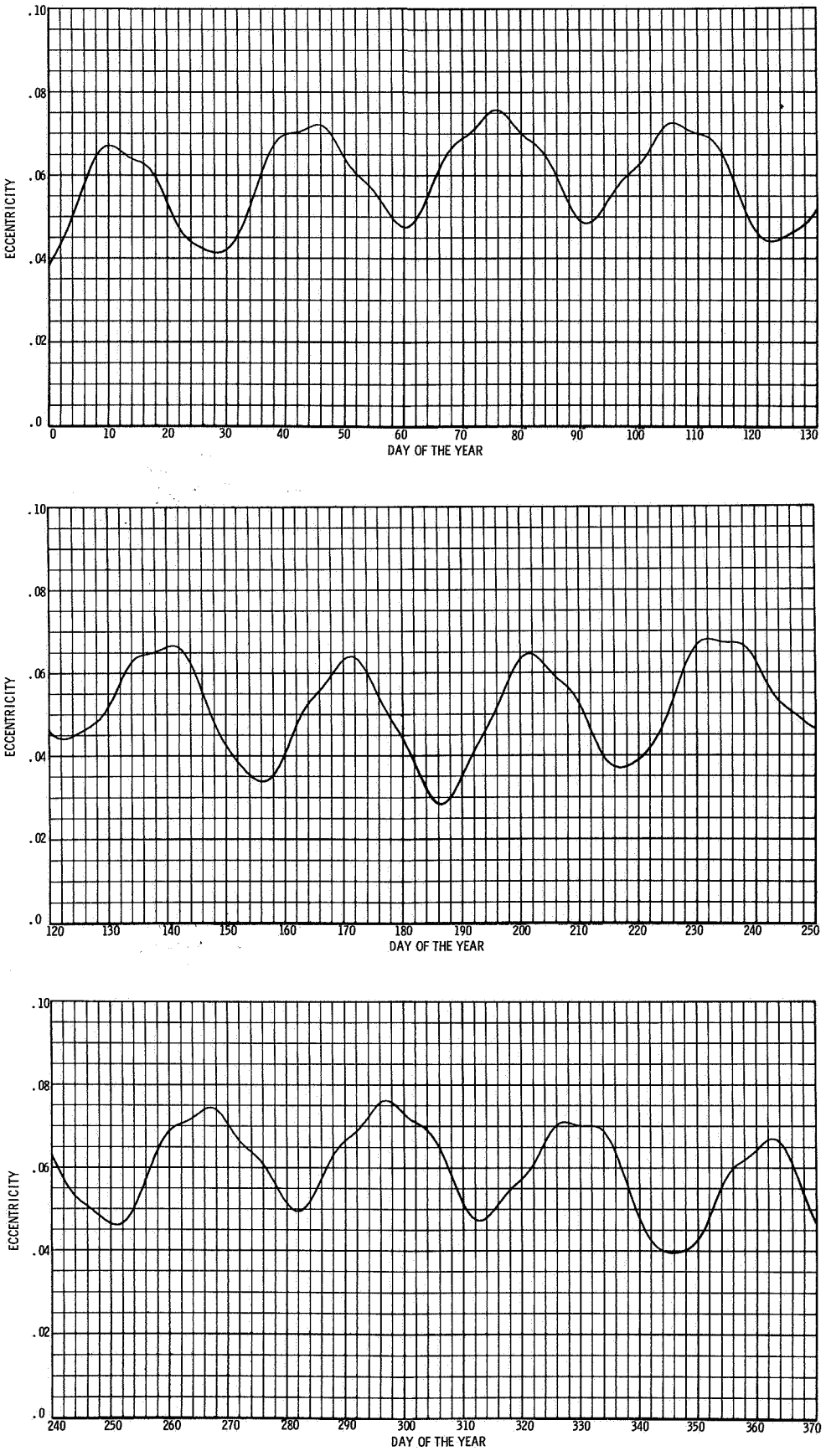


FIGURE B1980-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT

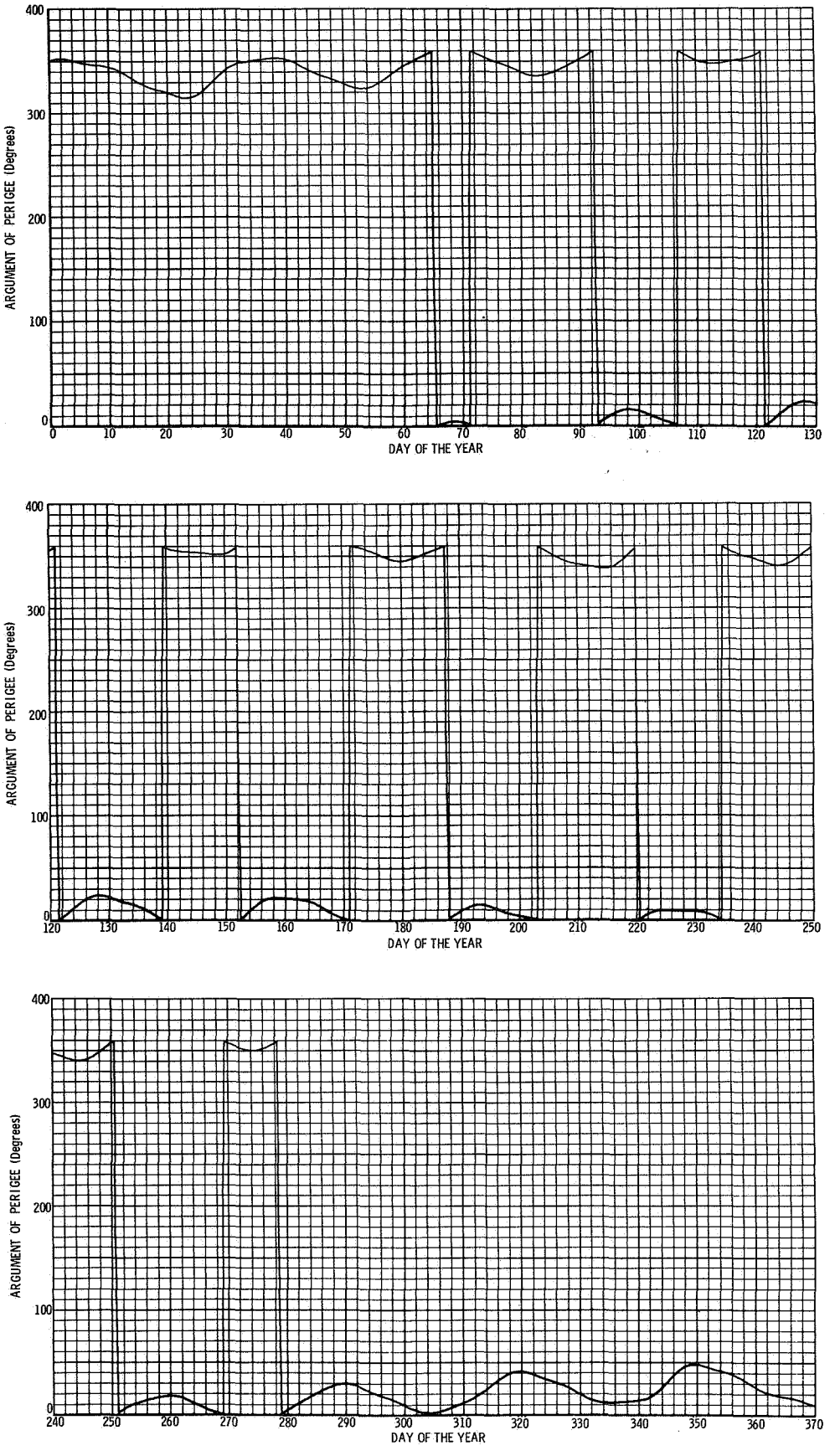
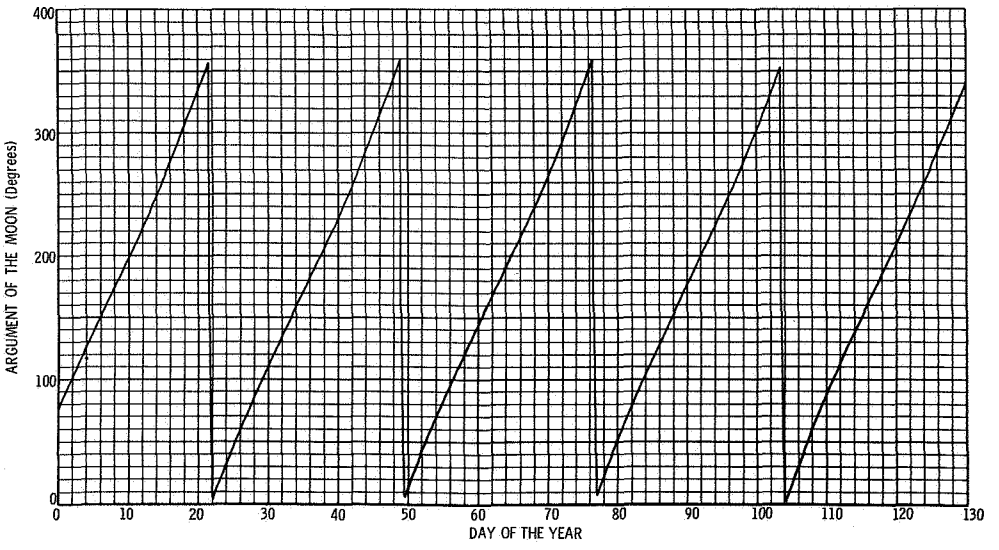
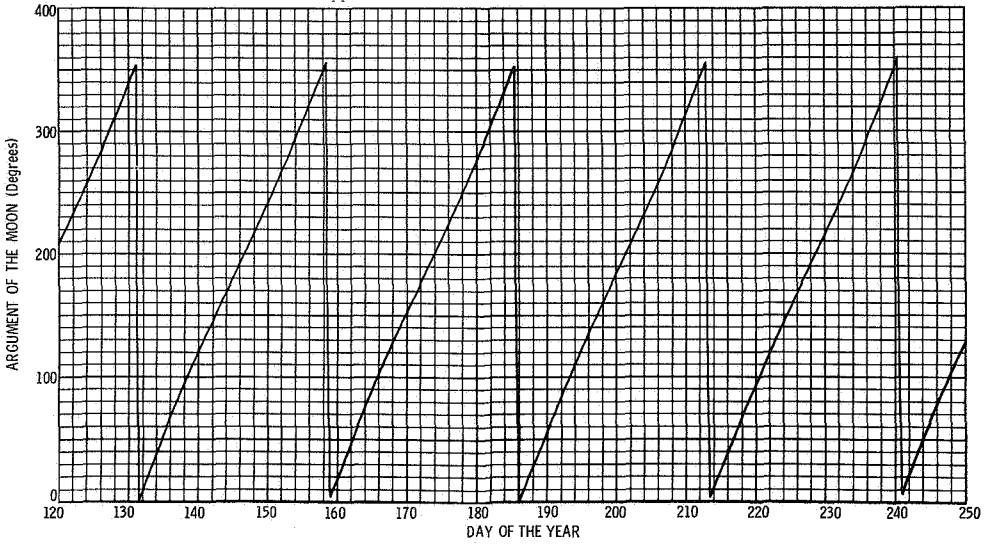


FIGURE B1980-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE





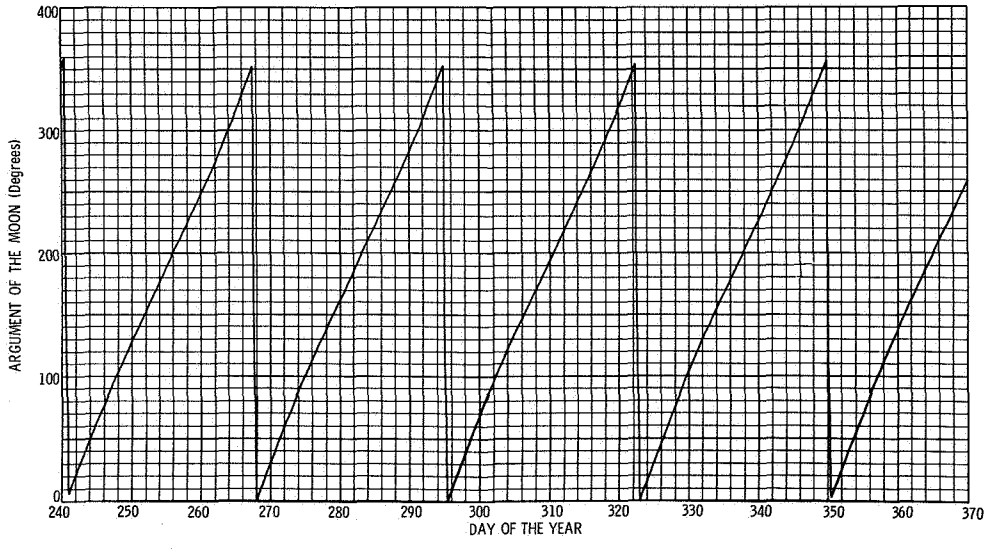


FIGURE B1980-10 ARGUMENT OF THE MOON'S POSITION

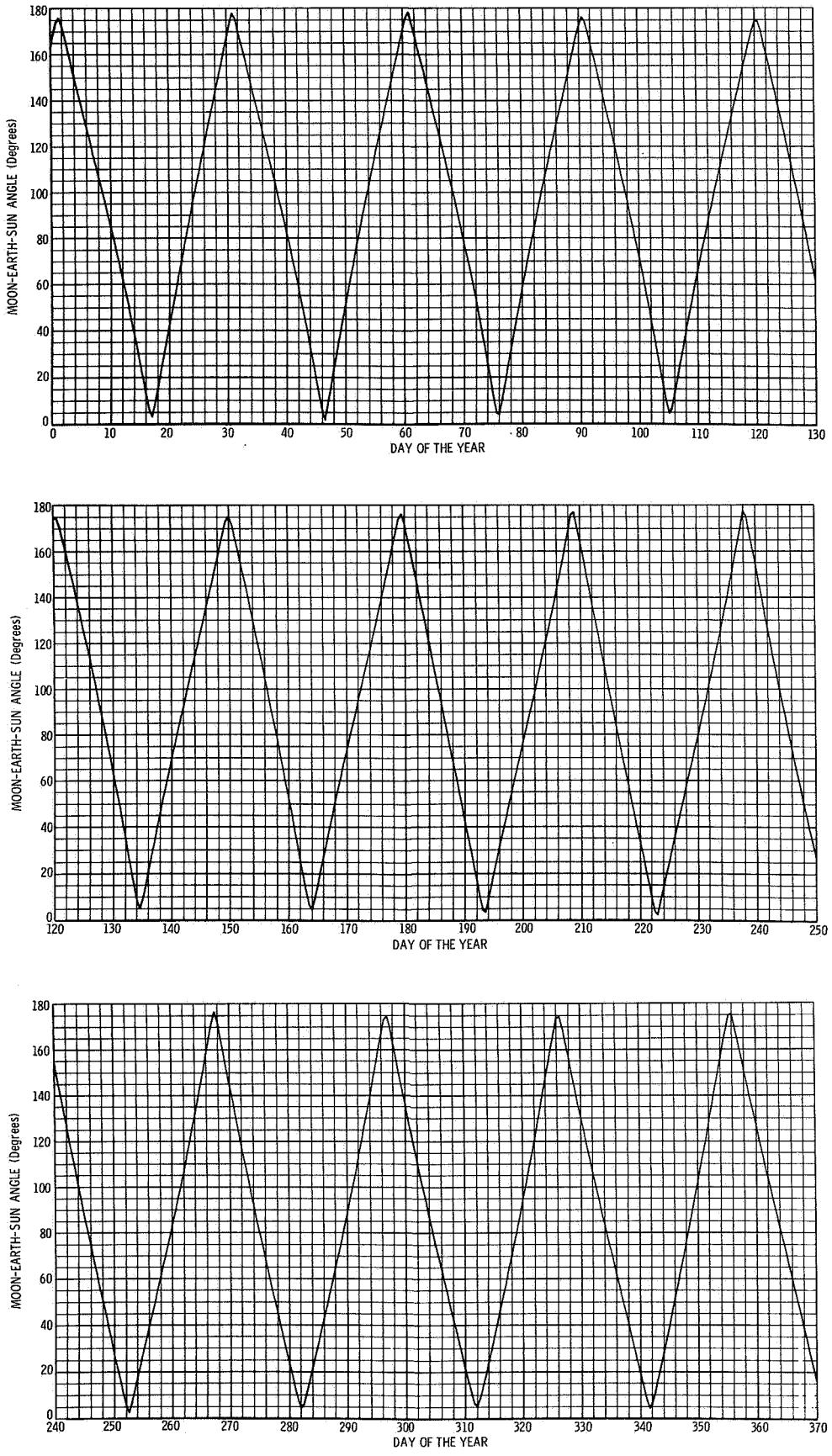
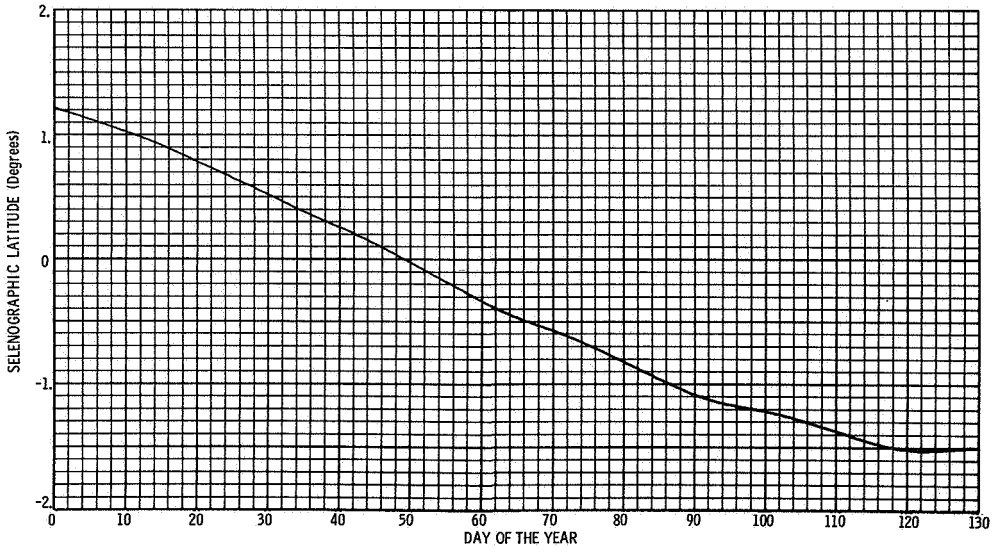
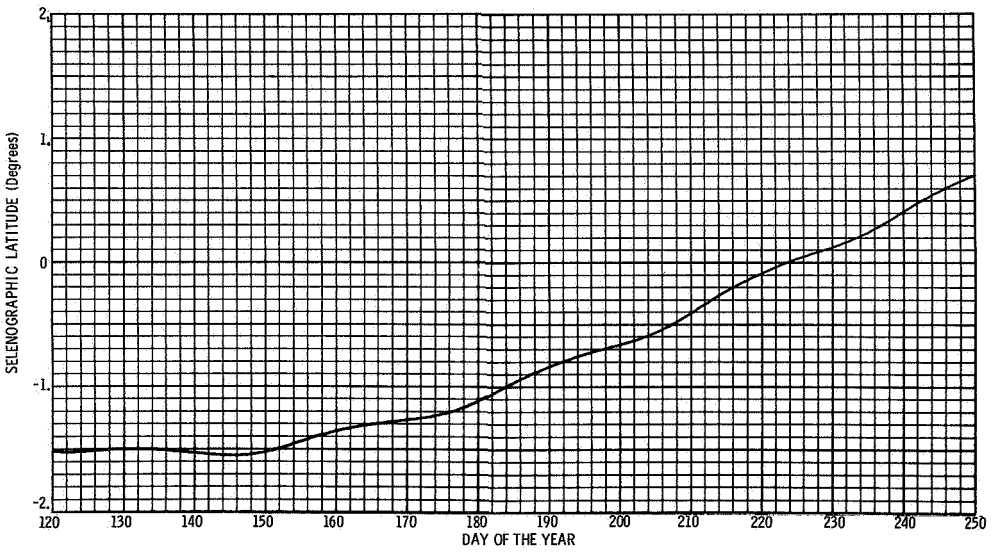


FIGURE B1980-11 MOON-EARTH-SUN ANGLE





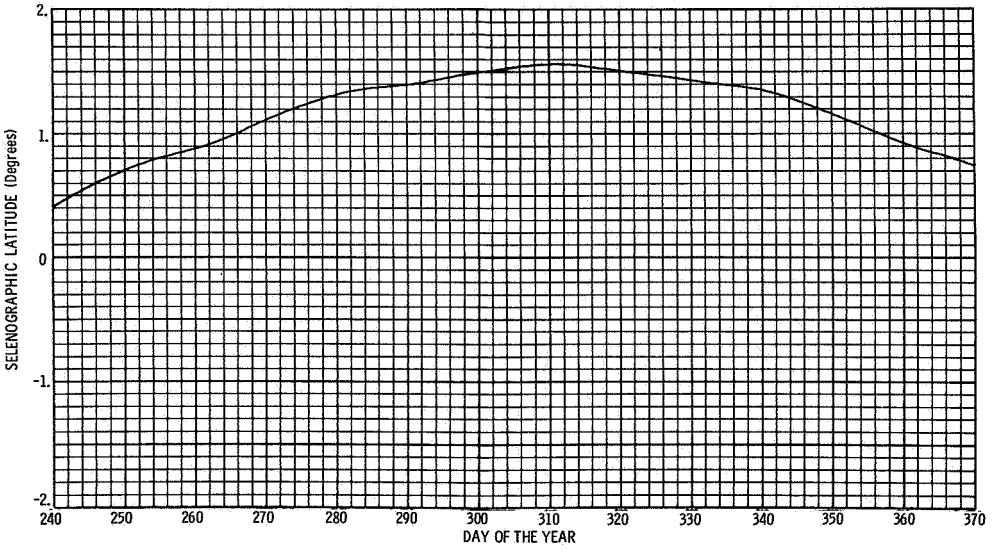
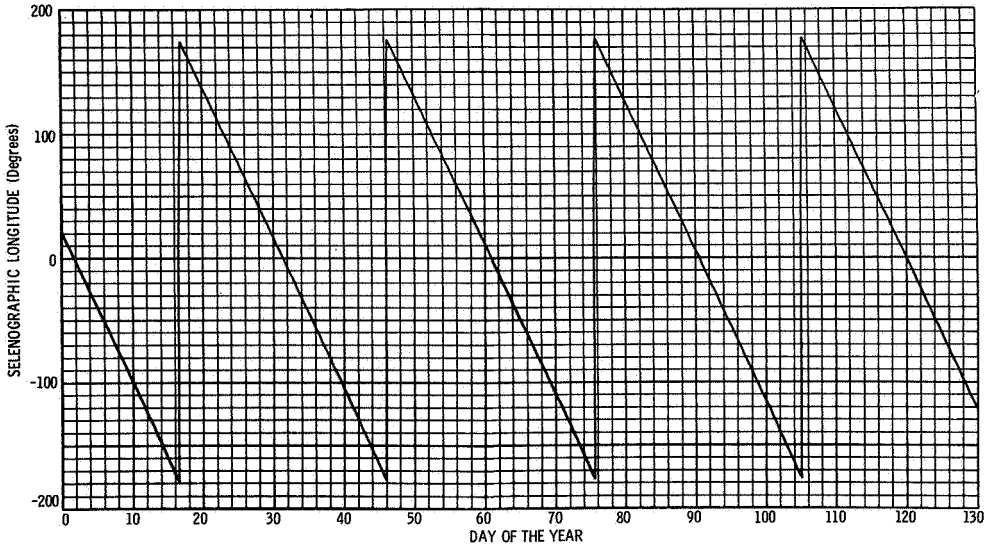
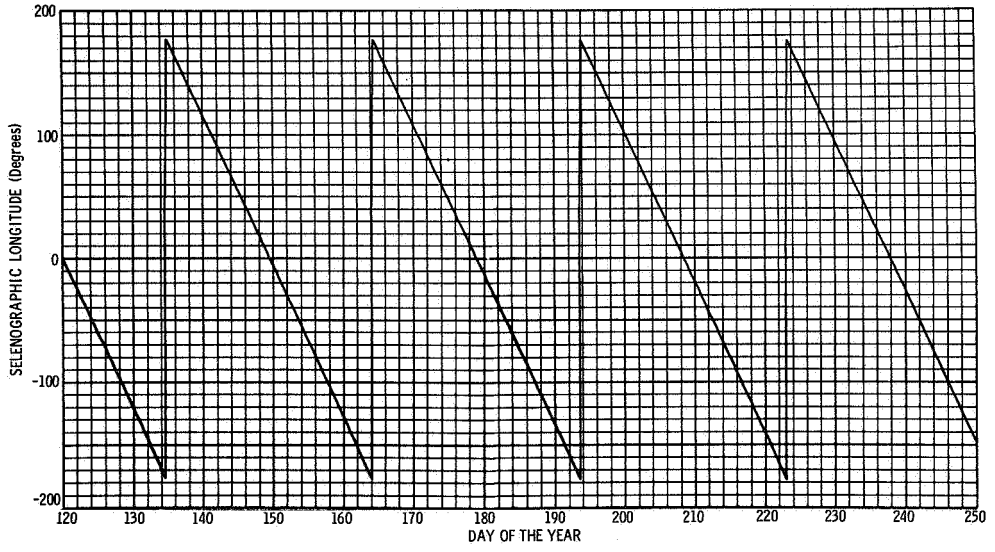


FIGURE B1980-12 SELENOGRAPHIC LATITUDE OF THE SUN





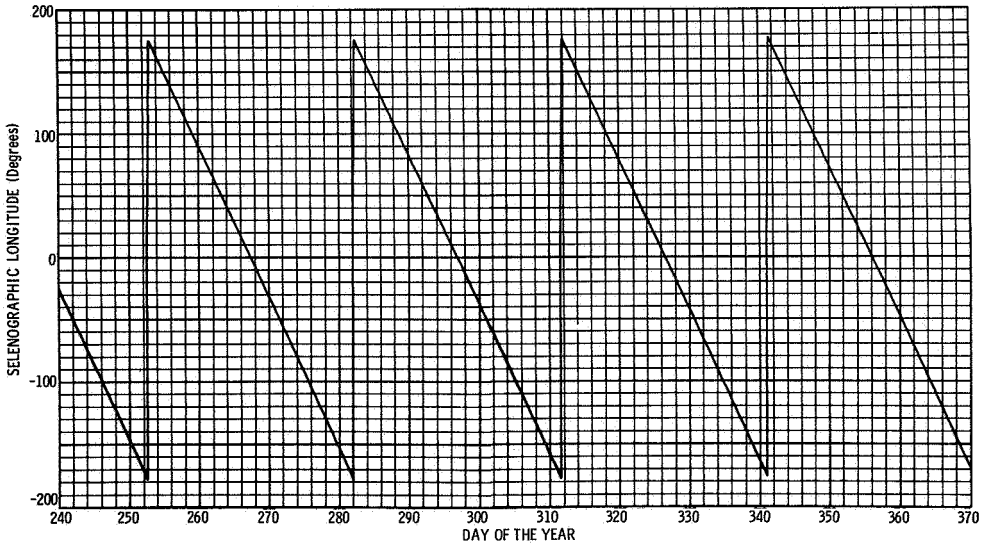
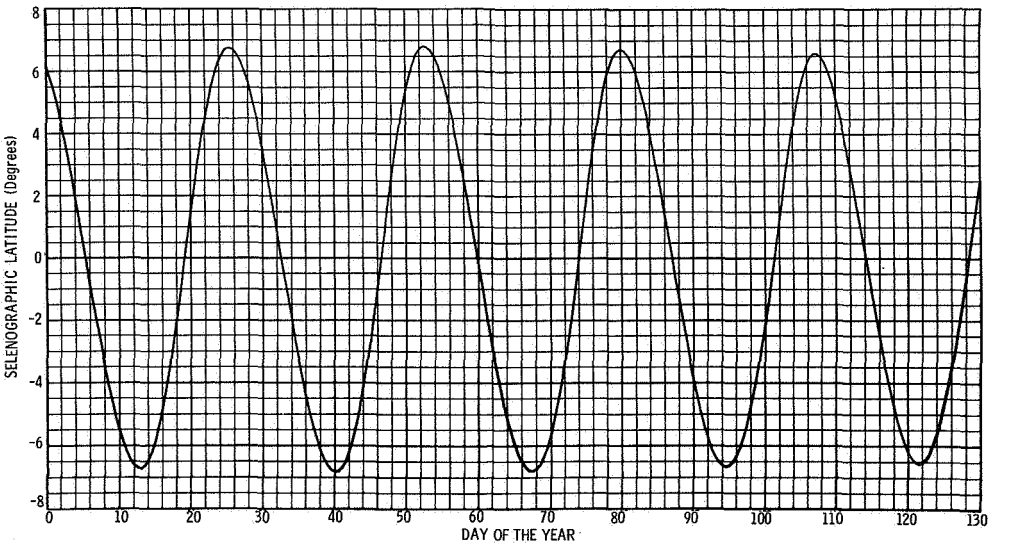
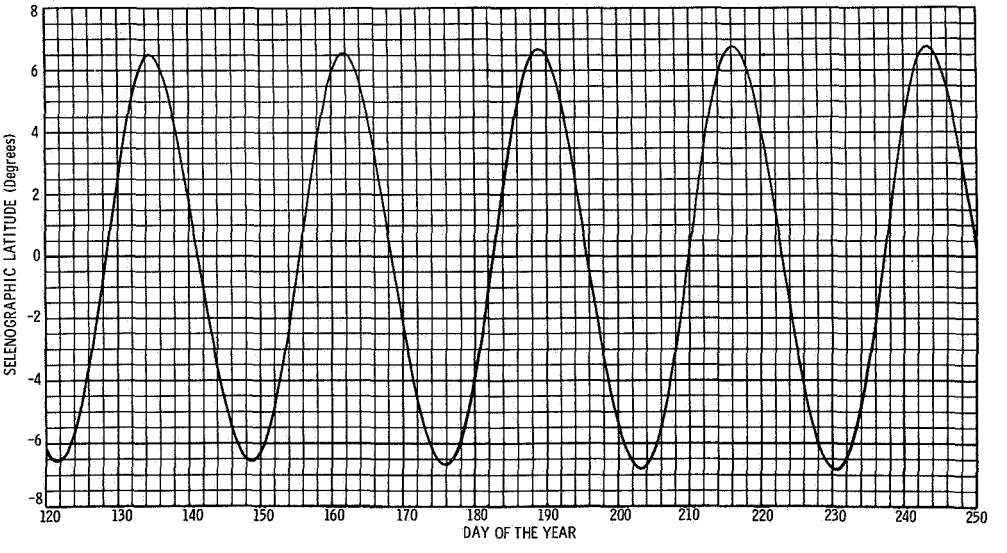


FIGURE B1980-13 SELENOGRAPHIC LONGITUDE OF THE SUN





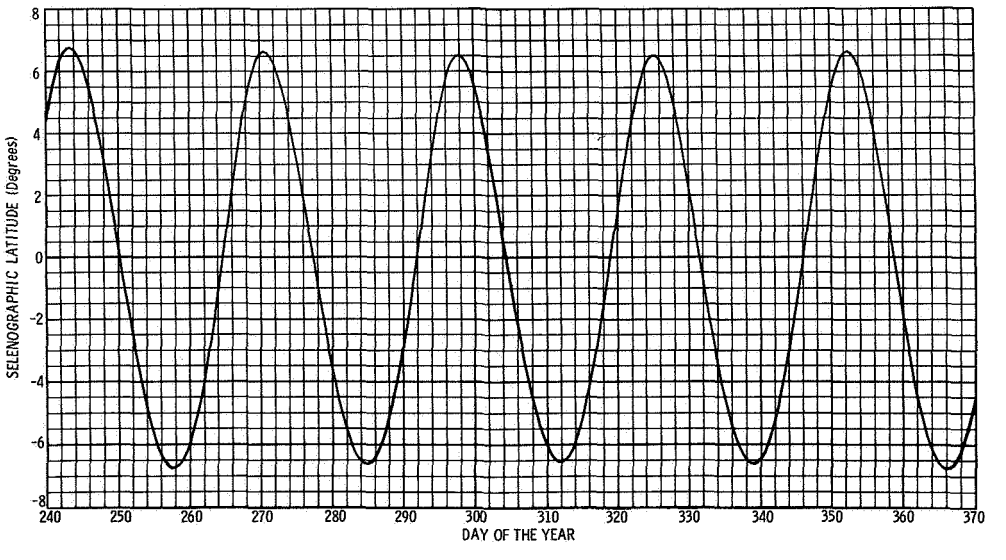
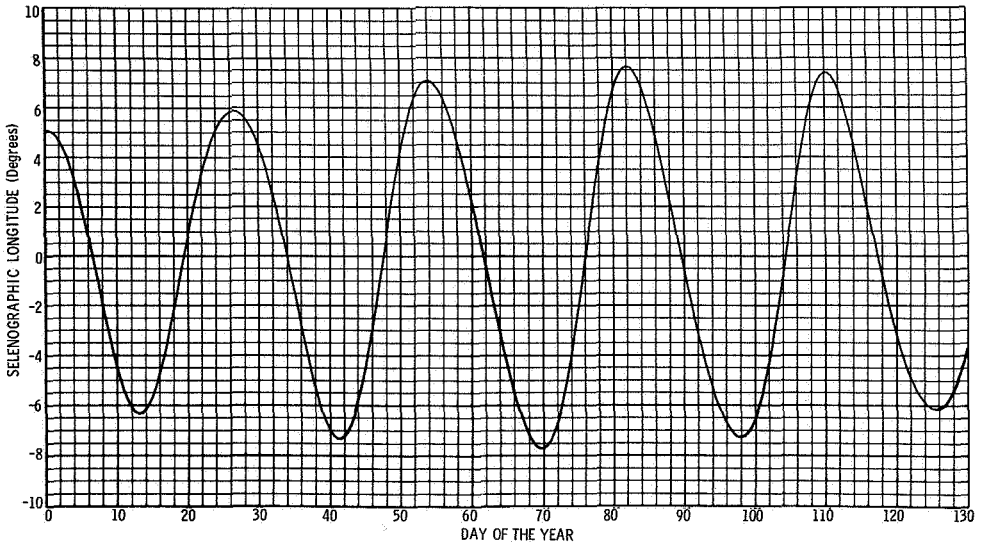
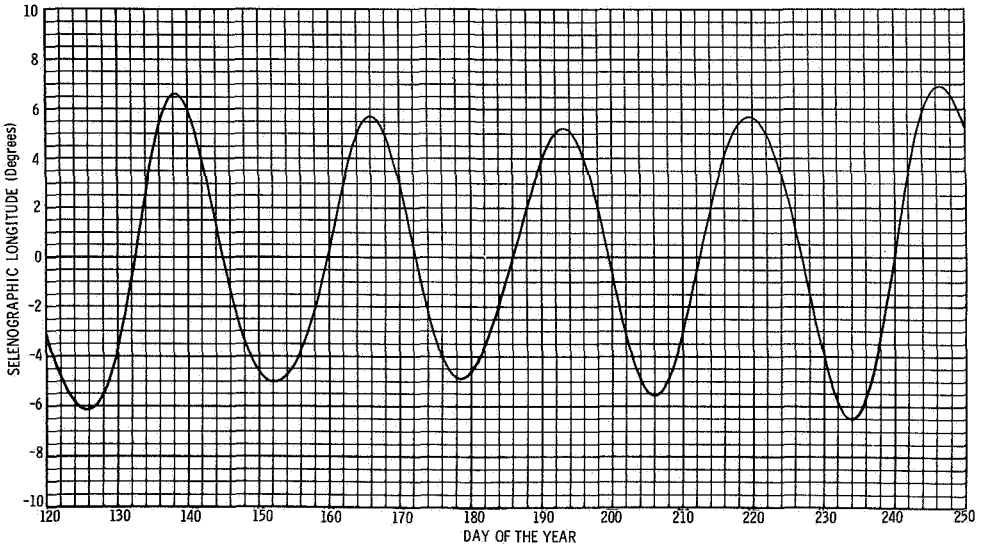


FIGURE B1980-14 SELENOGRAPHIC LATITUDE OF THE EARTH





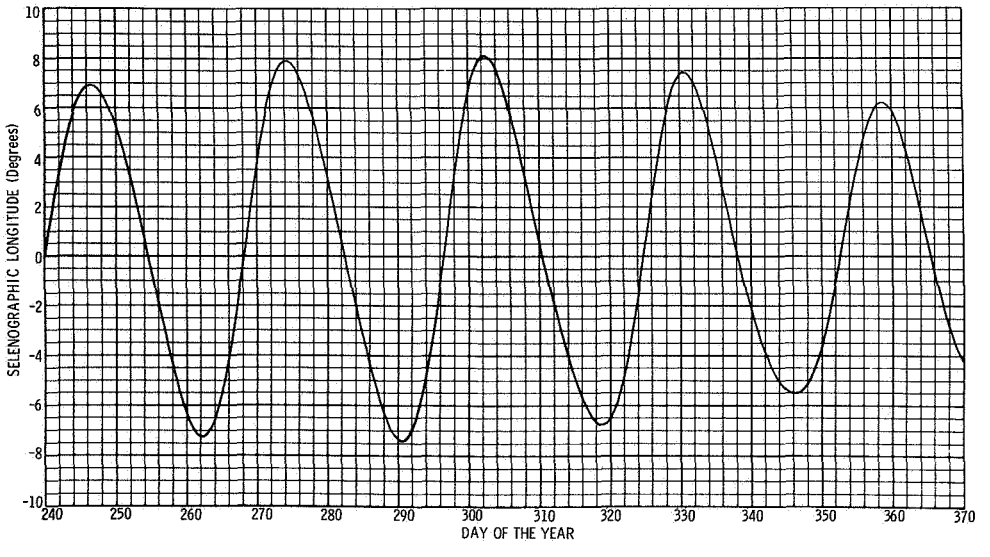


FIGURE B1980-15 SELENOGRAPHIC LONGITUDE OF THE EARTH

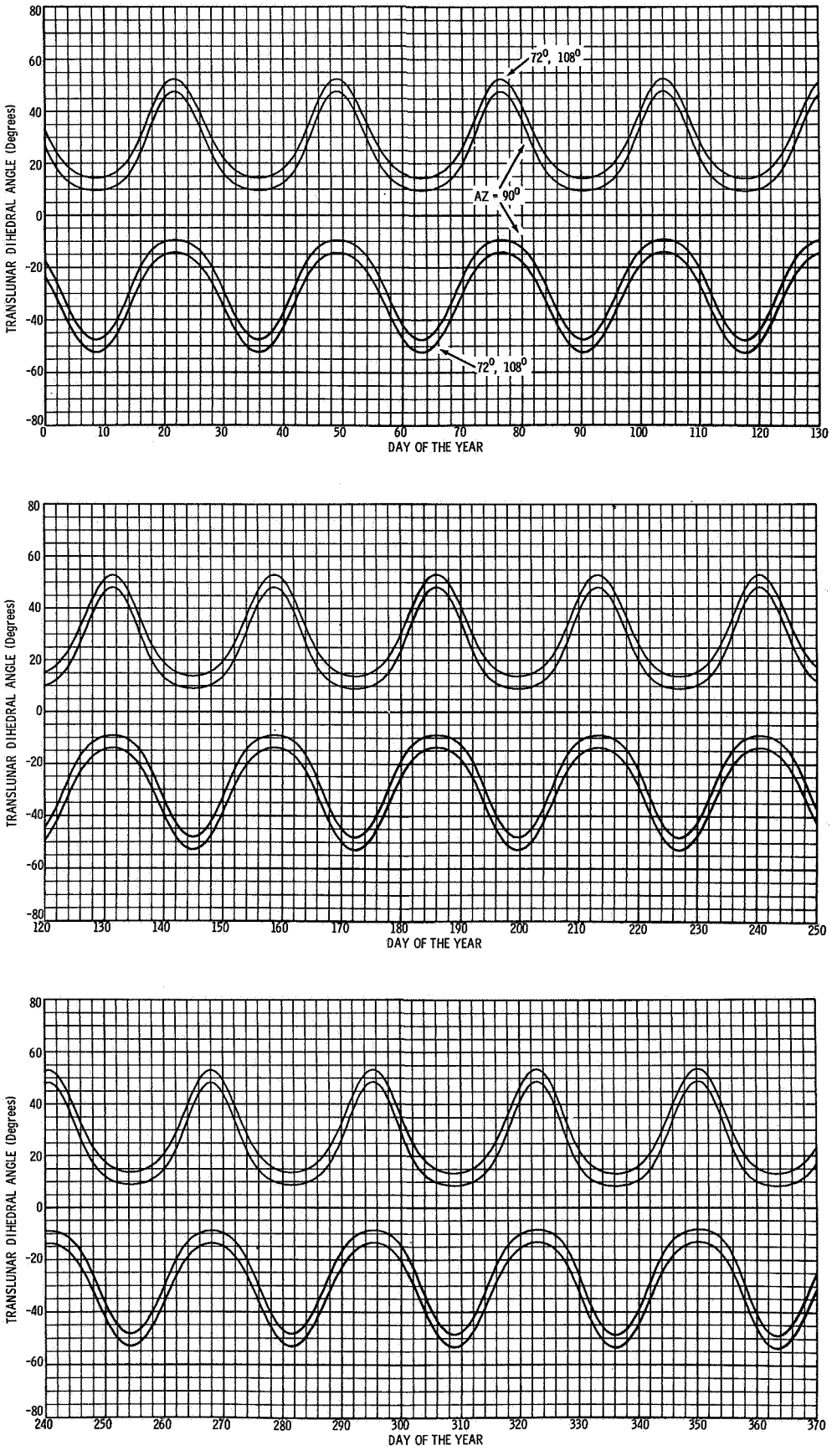


FIGURE B1980-16 TRANSLUNAR DIHEDRAL ANGLES

1981

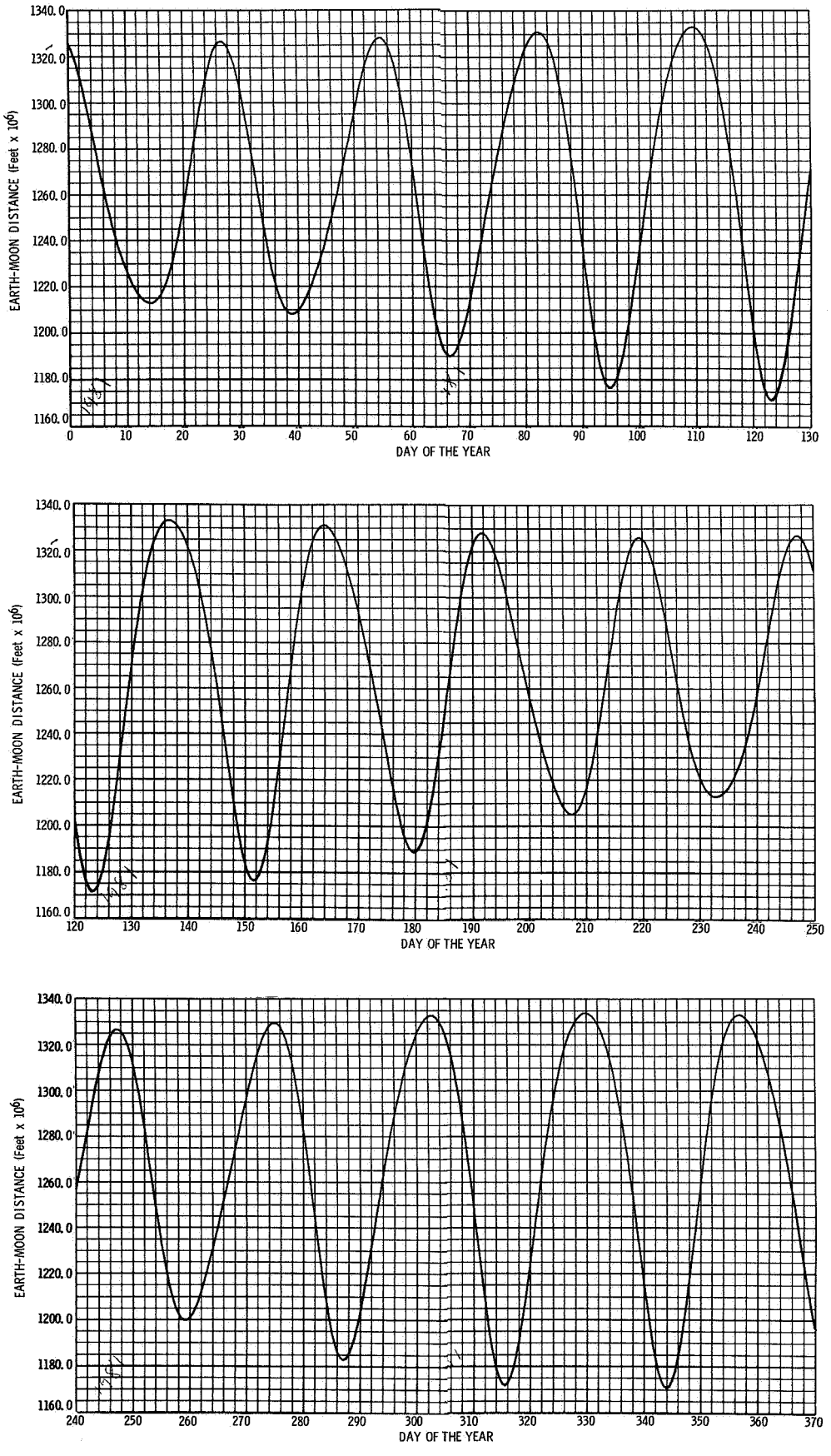
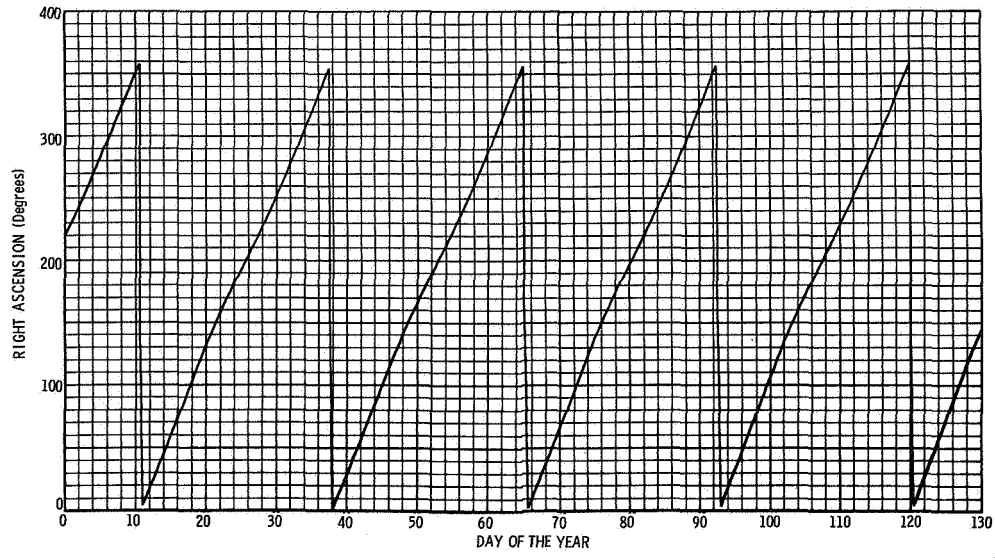
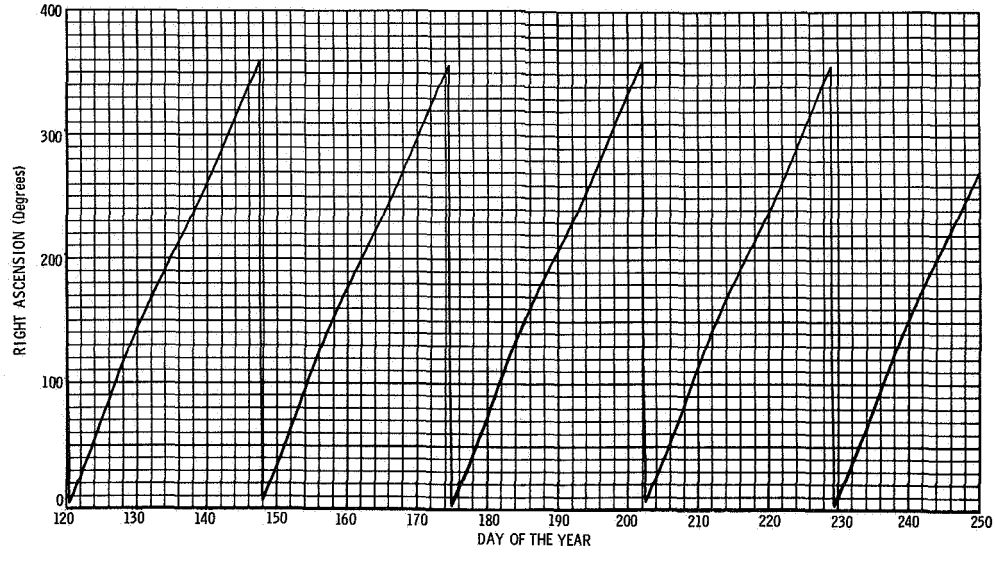


FIGURE B1981-1 EARTH-MOON DISTANCE





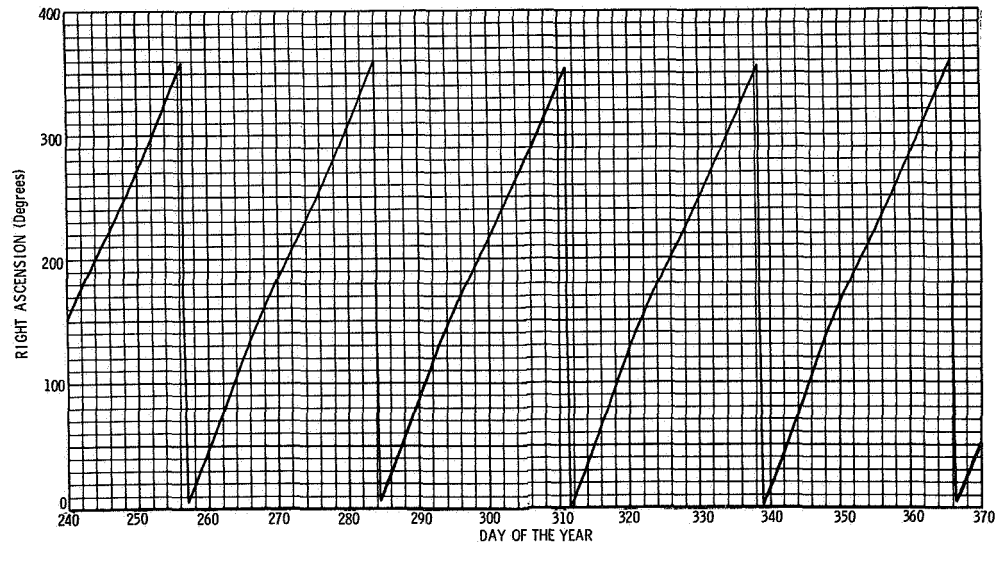


FIGURE B1981-2 RIGHT ASCENSION OF THE MOON

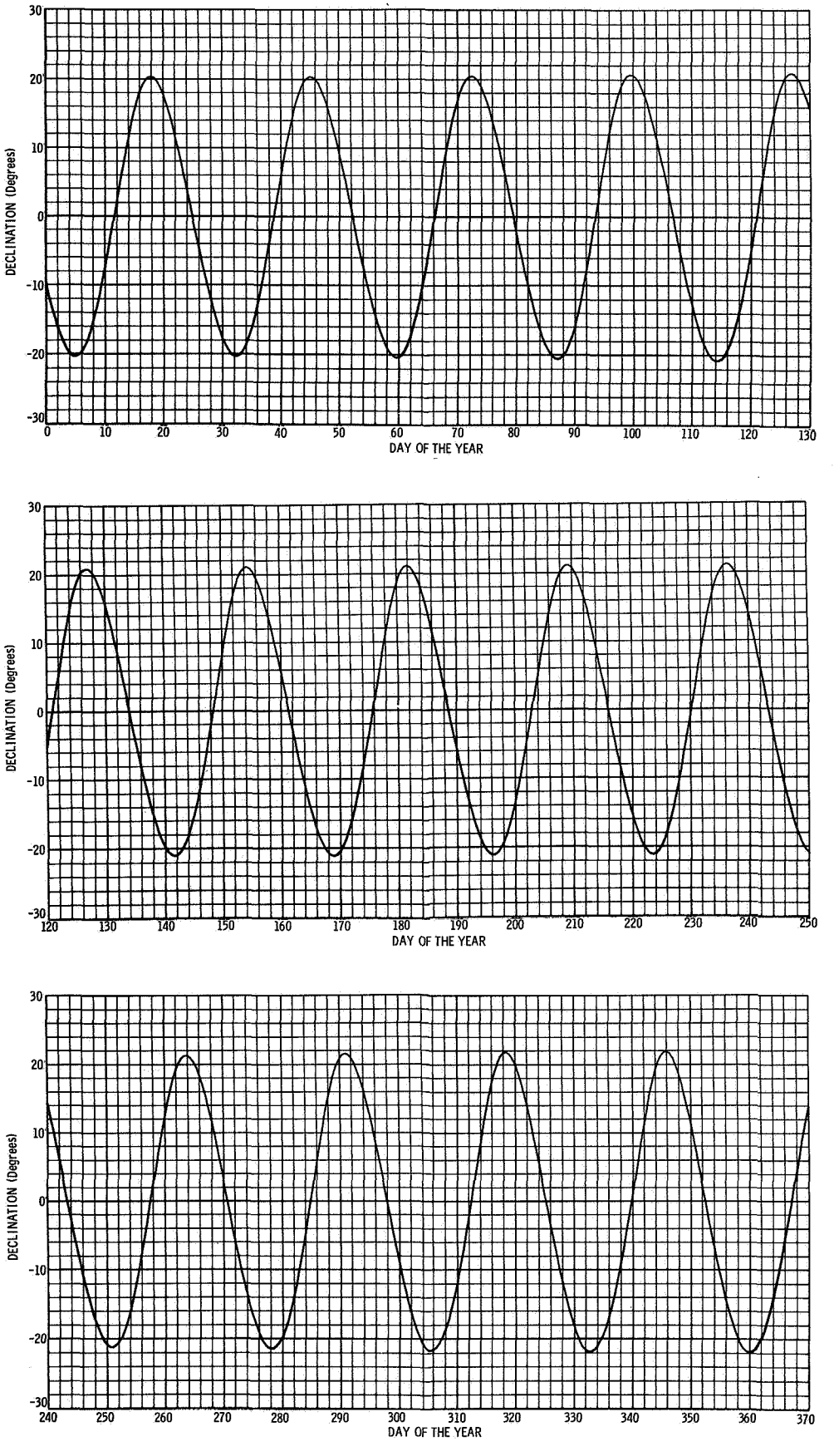
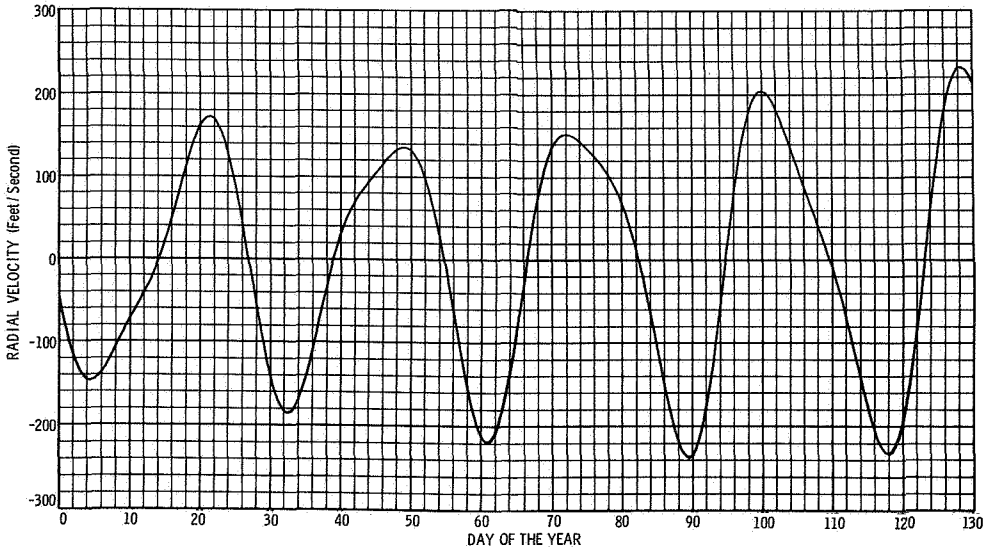
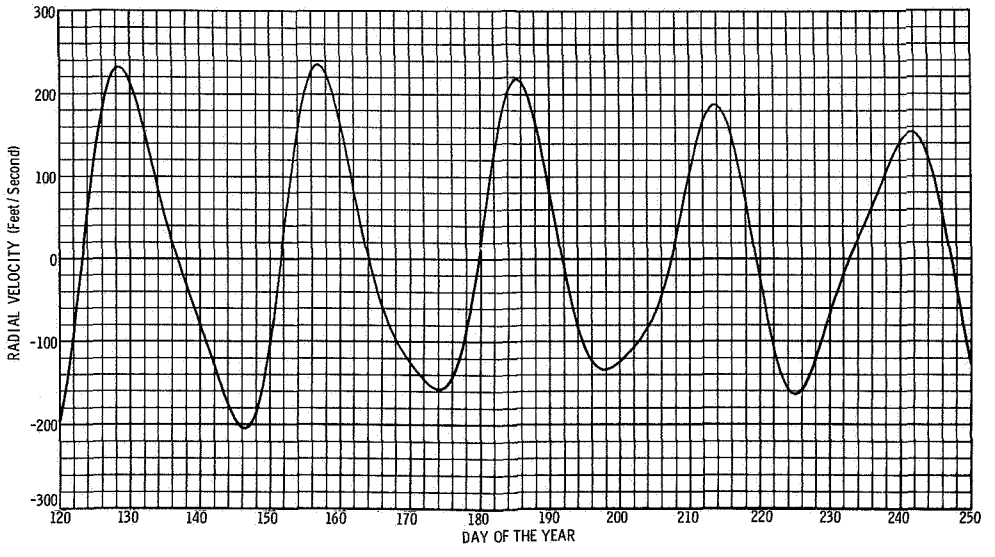


FIGURE B1981-3 DECLINATION OF THE MOON





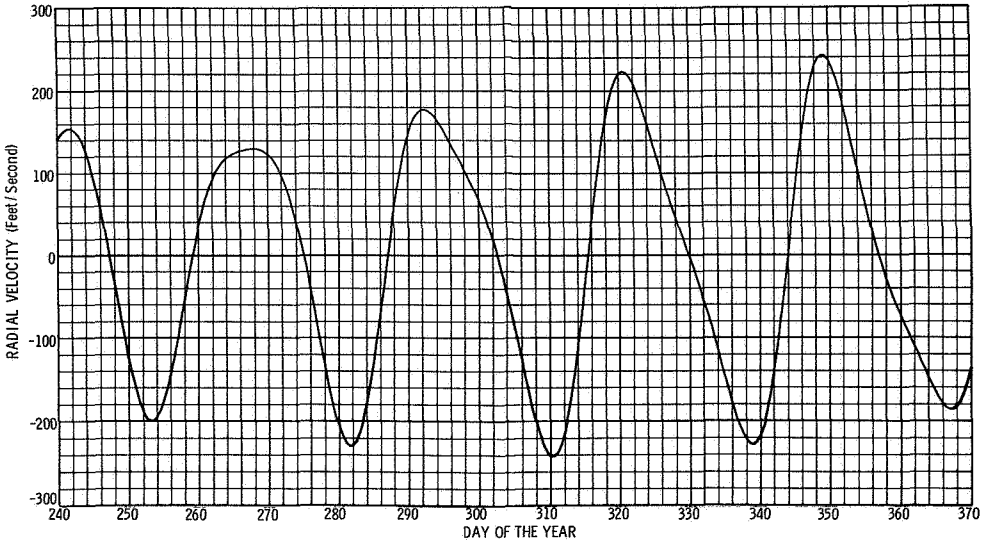


FIGURE B1981-4 RADIAL VELOCITY OF THE MOON

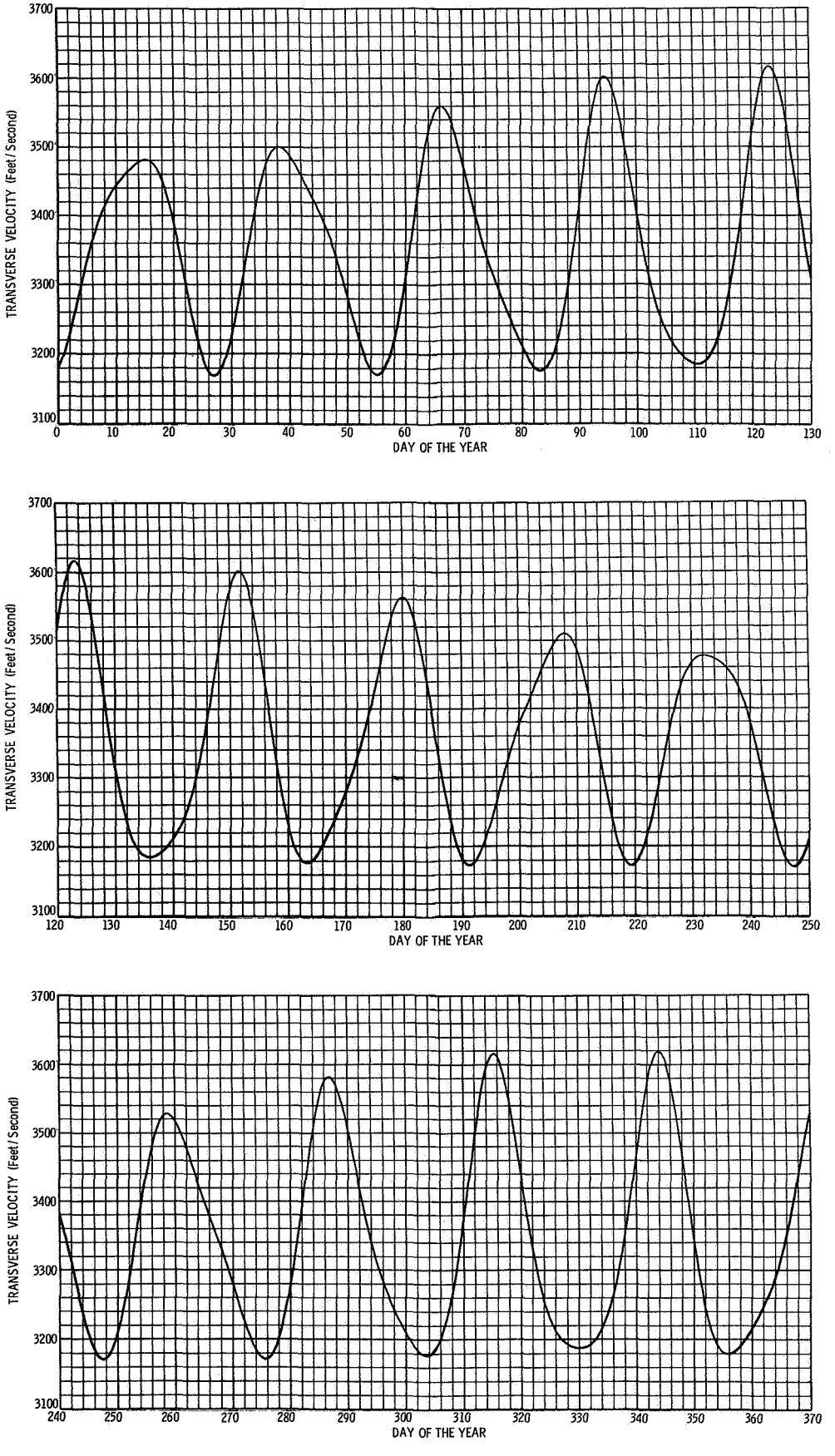


FIGURE B1981-5 TRANSVERSE VELOCITY OF THE MOON

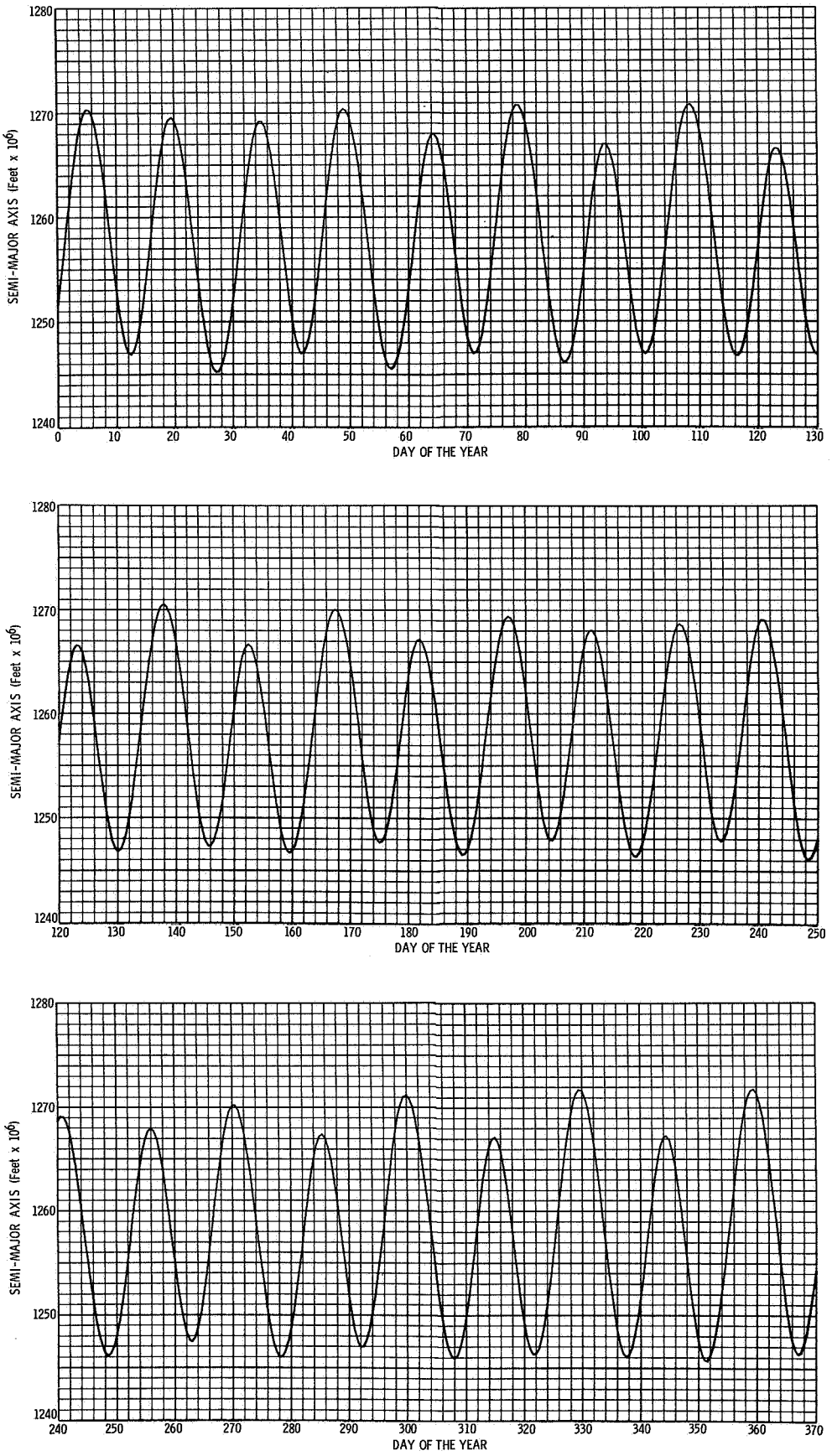
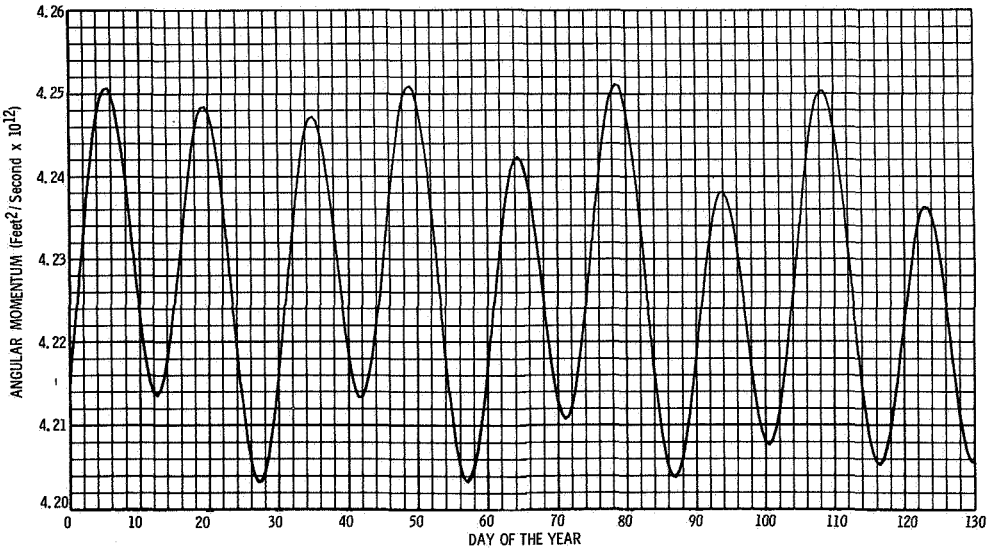
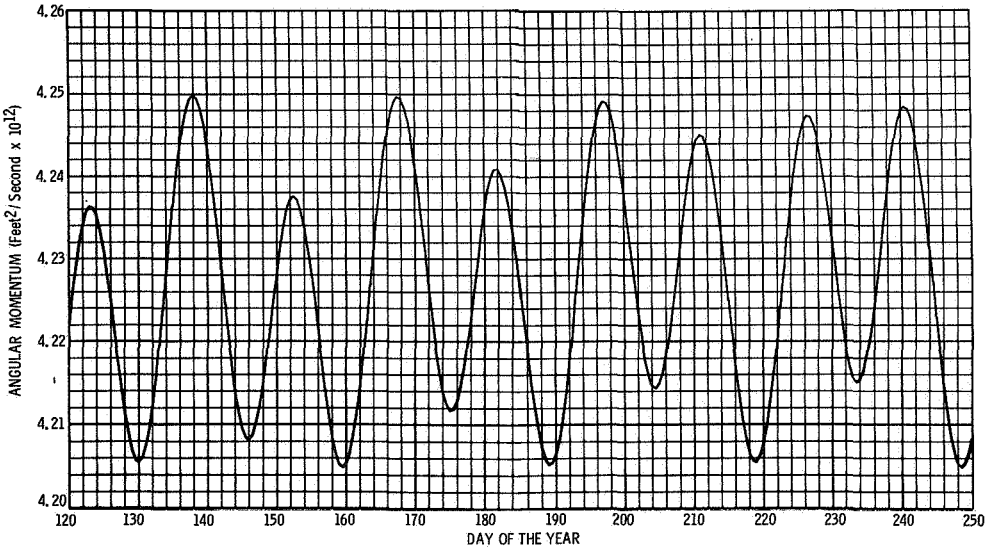


FIGURE B1981-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT





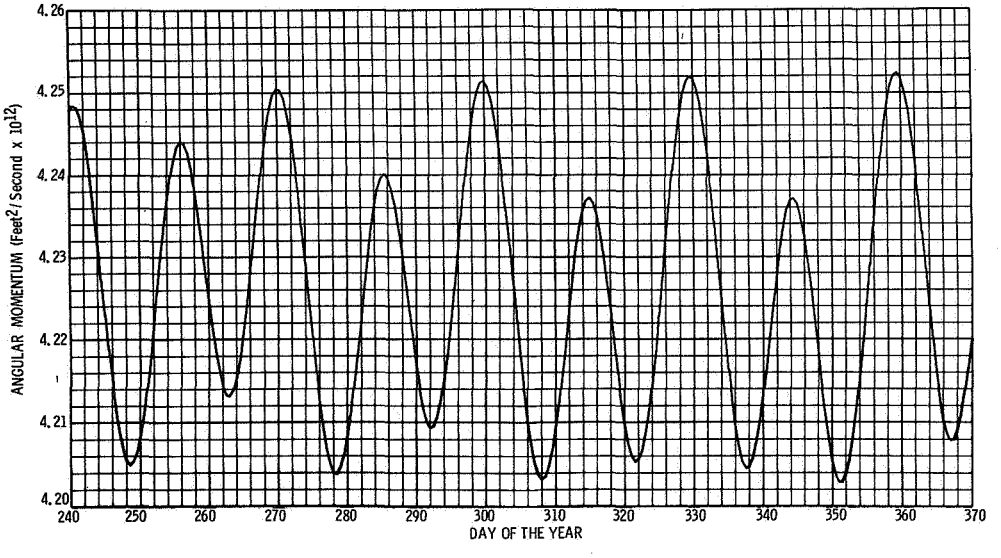


FIGURE B1981-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON

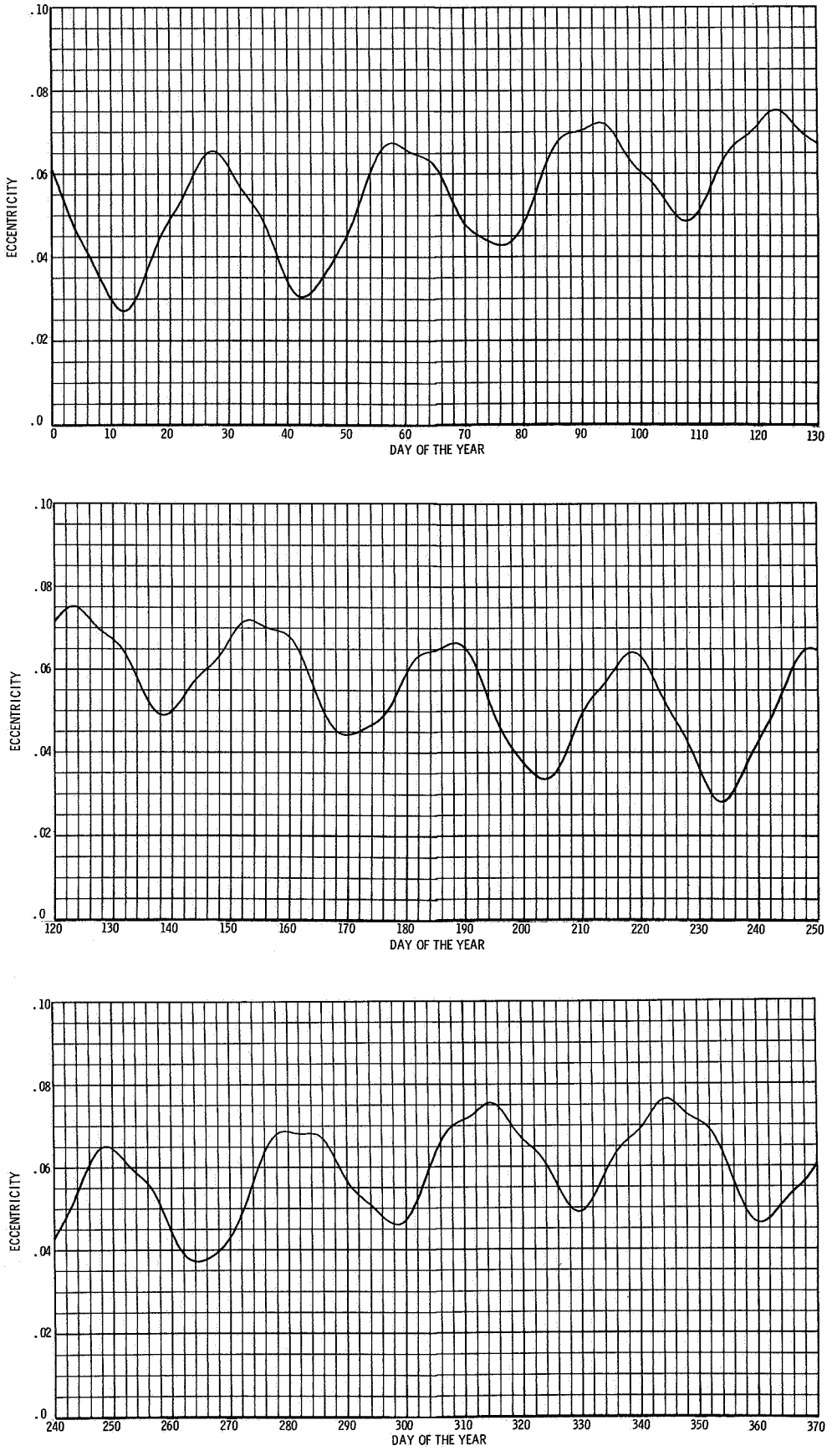


FIGURE B1981-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT

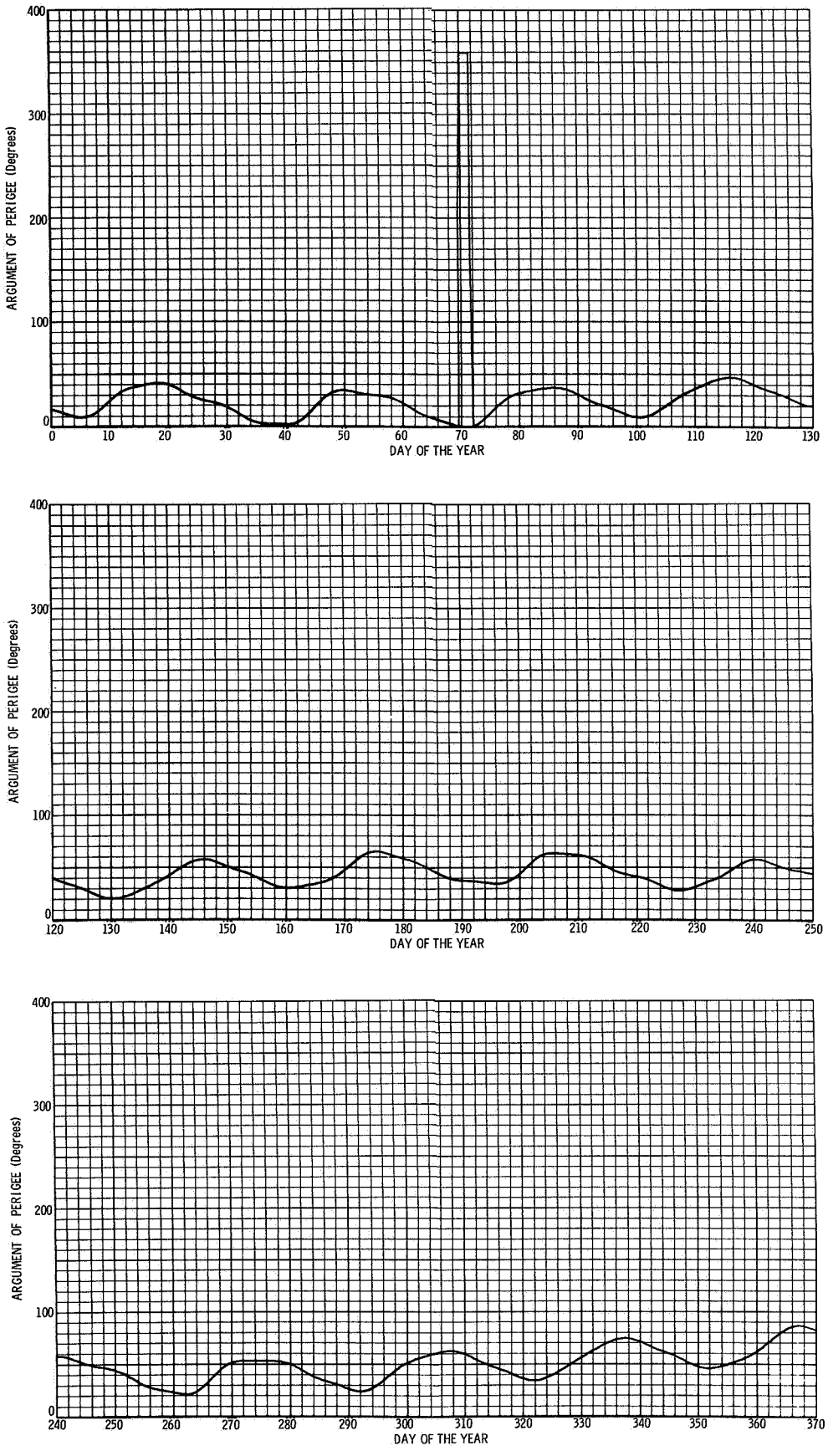


FIGURE B1981-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE

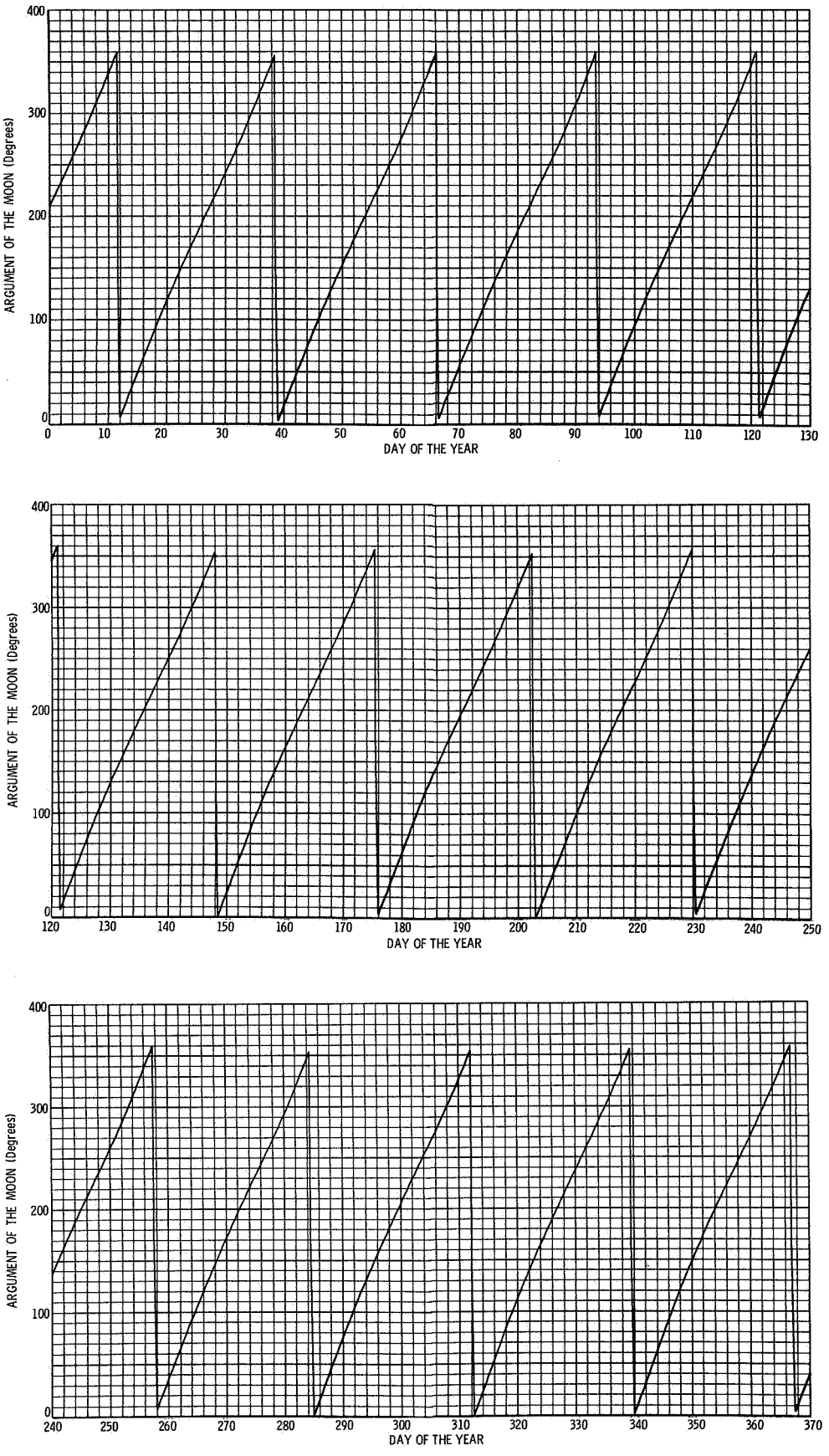
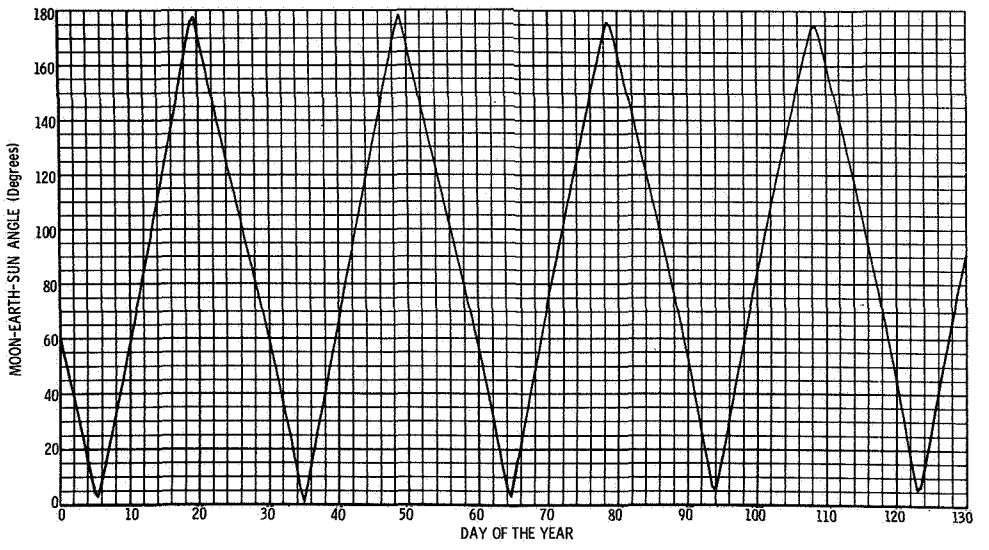
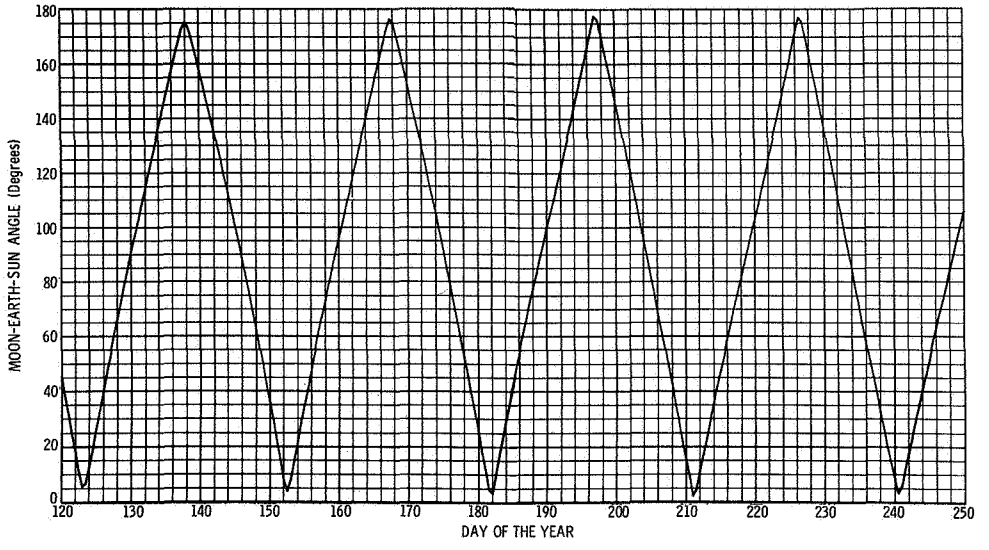


FIGURE B1981-10 ARGUMENT OF THE MOON'S POSITION





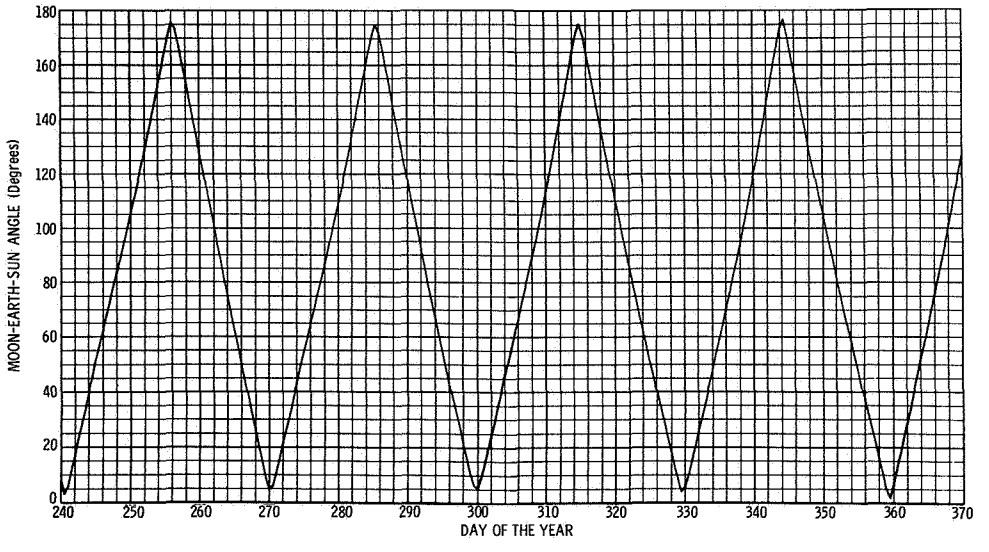


FIGURE B1981-11 MOON-EARTH-SUN ANGLE

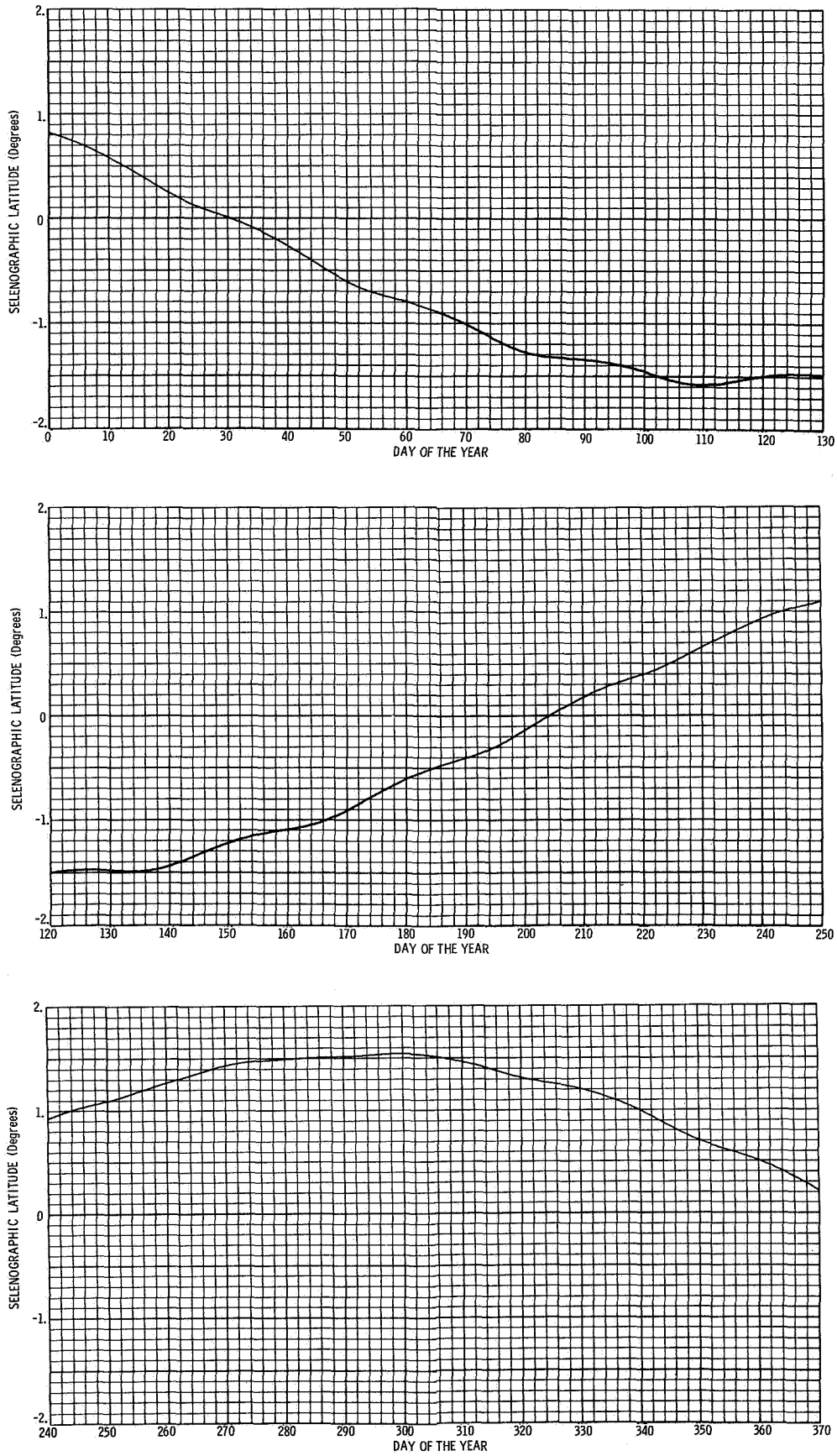
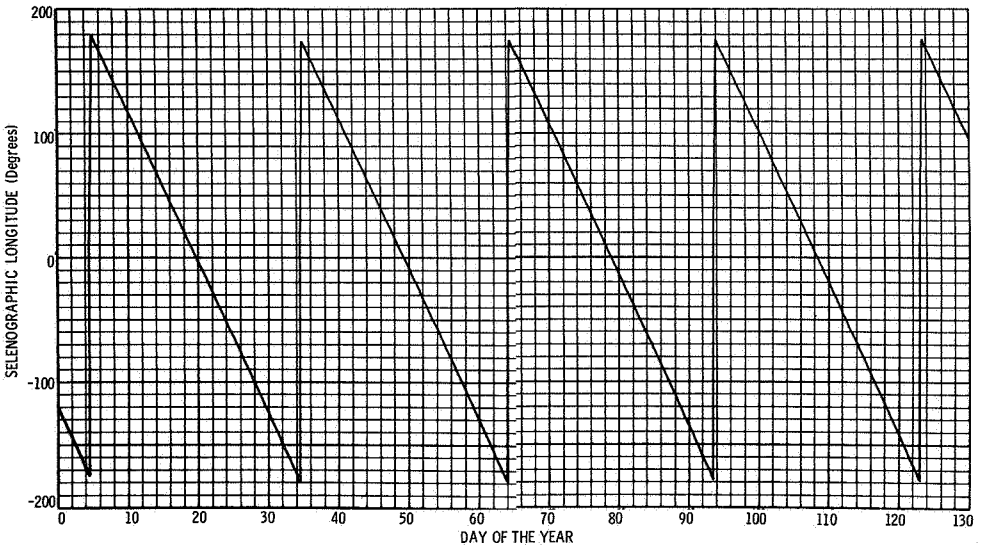
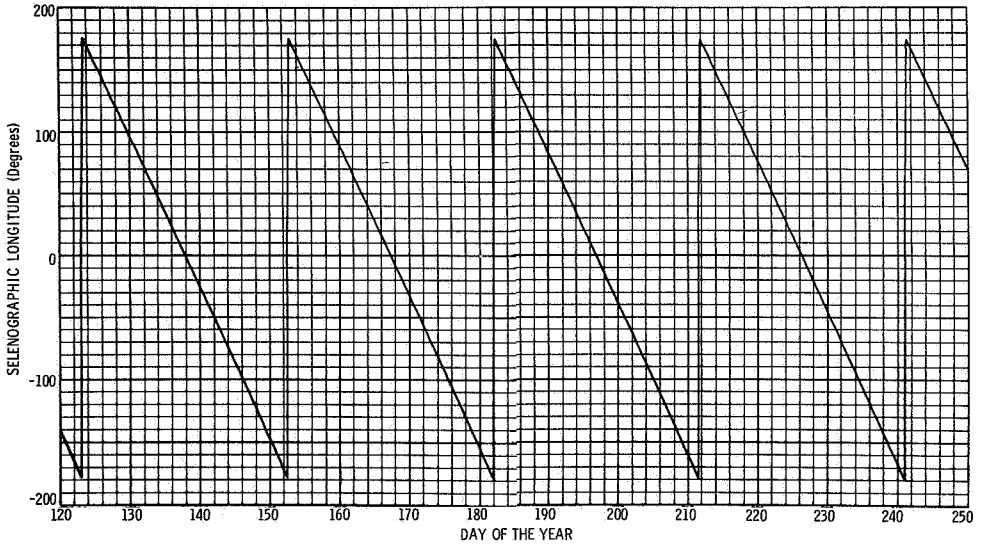


FIGURE B1981-12 SELENOGRAPHIC LATITUDE OF THE SUN





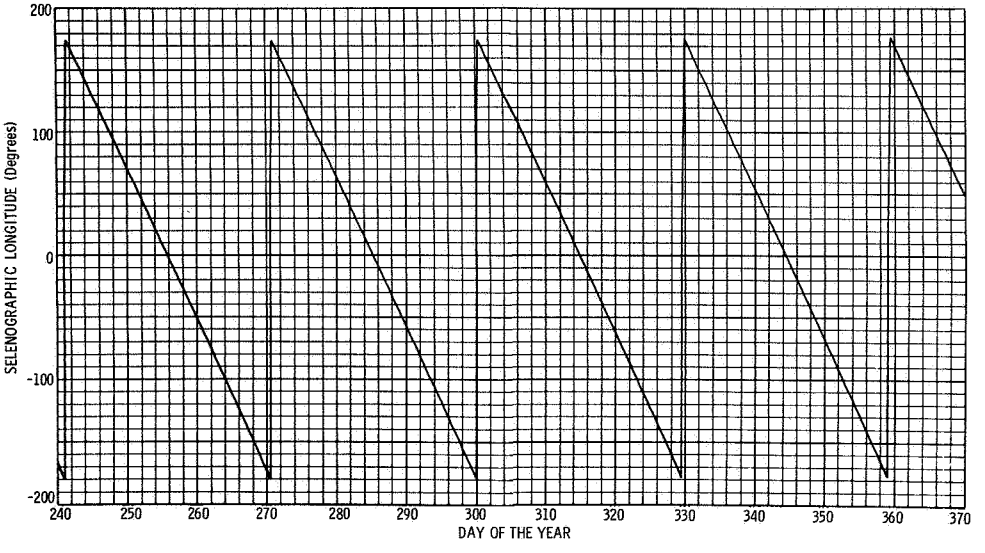
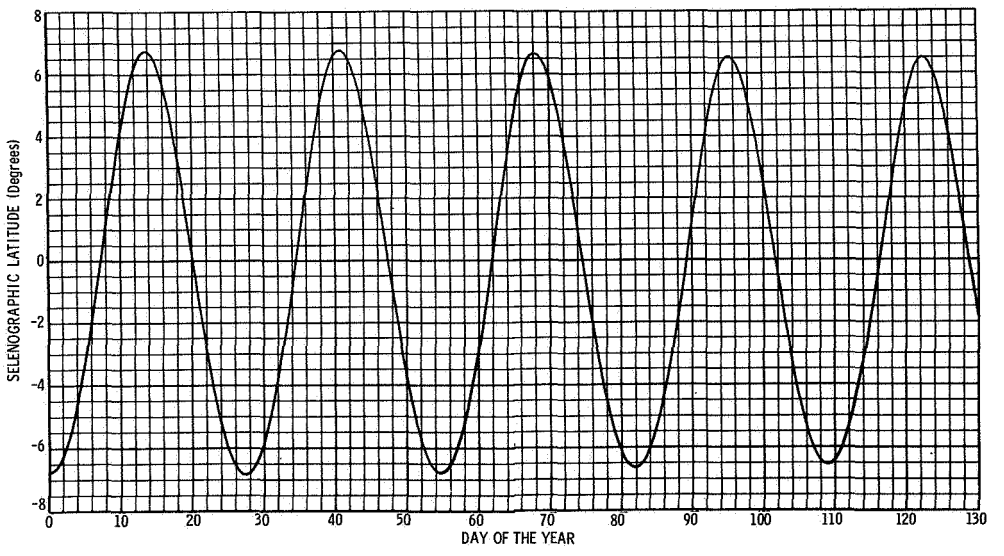
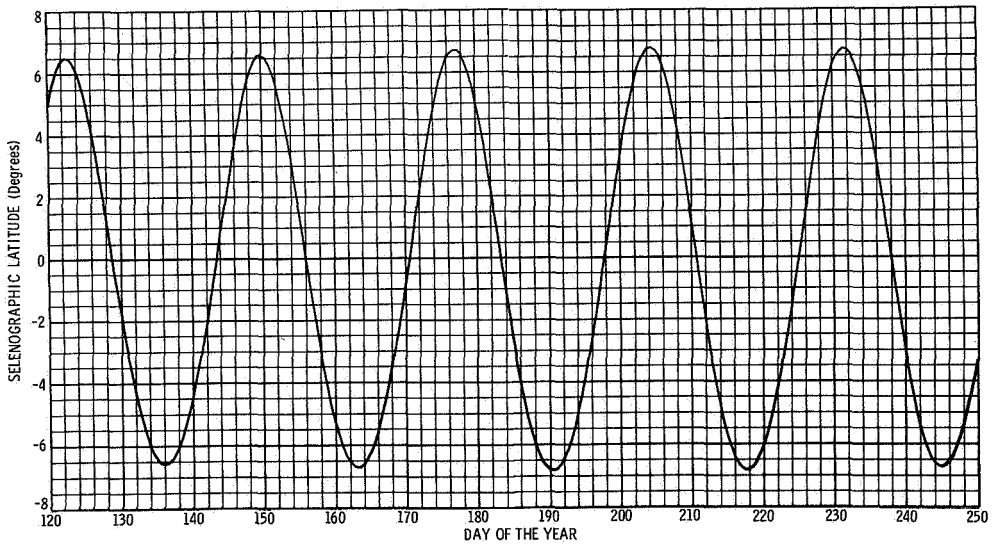


FIGURE B1981-13 SELENOGRAPHIC LONGITUDE OF THE SUN





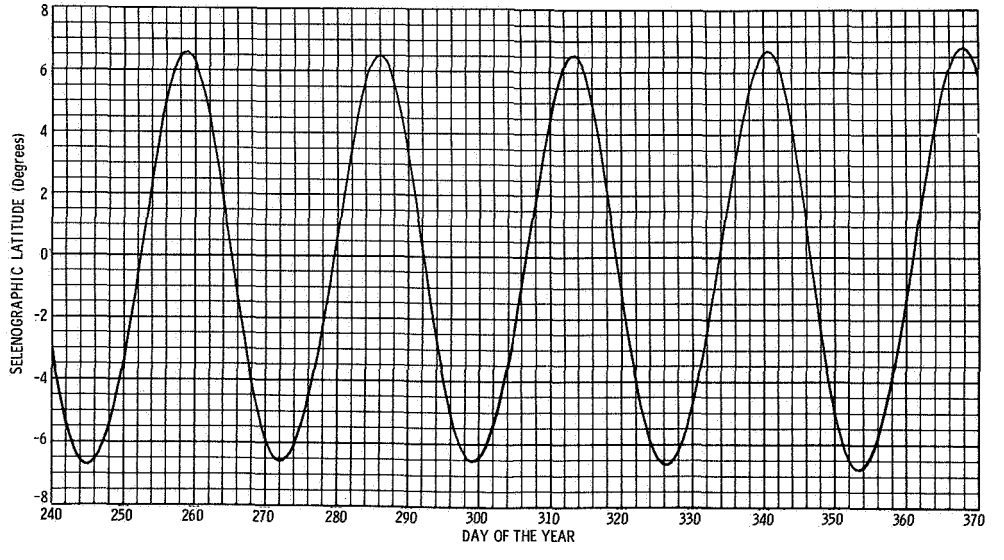


FIGURE B1981-14 SELENOGRAPHIC LATITUDE OF THE EARTH

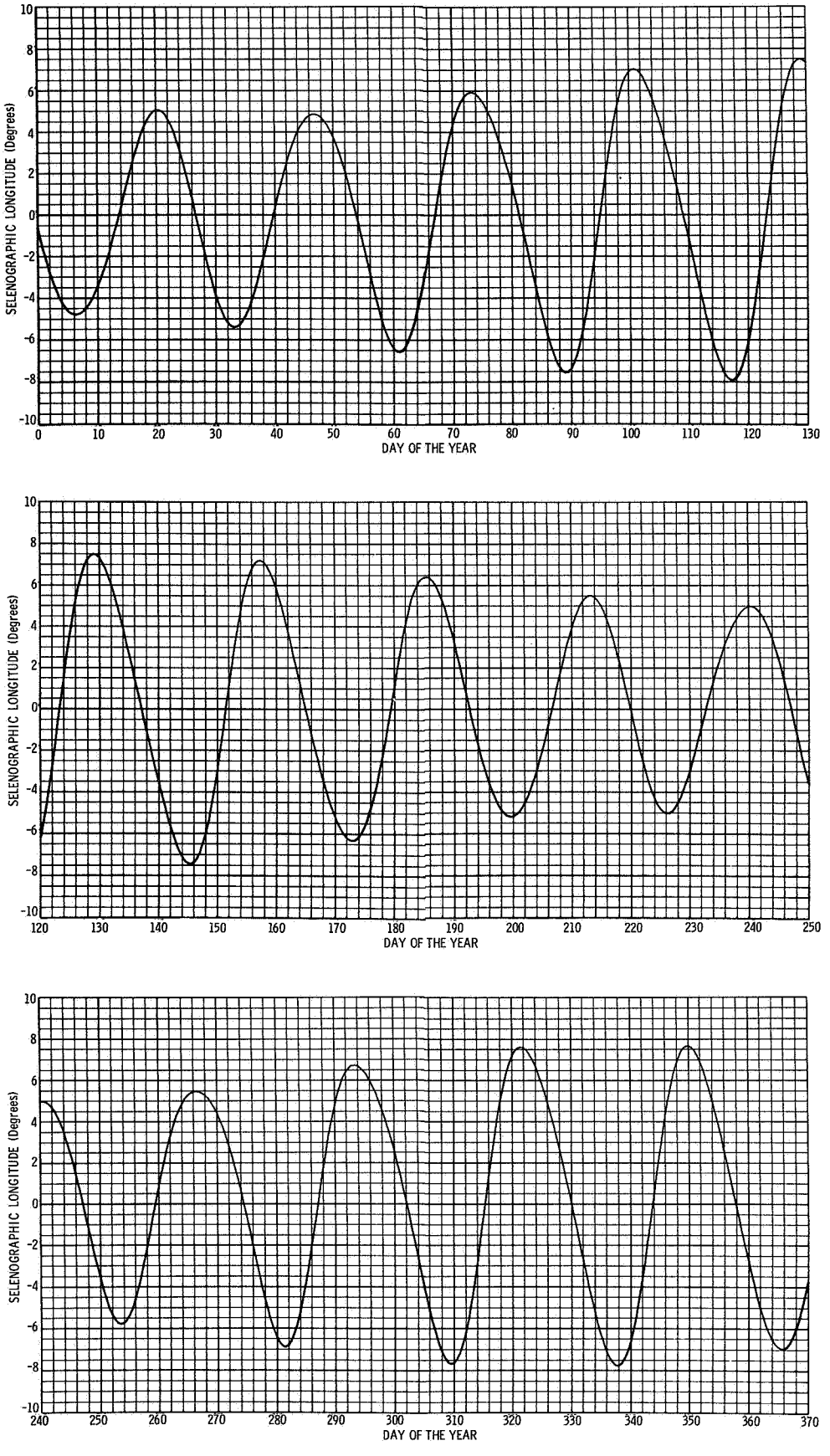


FIGURE B1981-15 SELENOGRAPHIC LONGITUDE OF THE EARTH

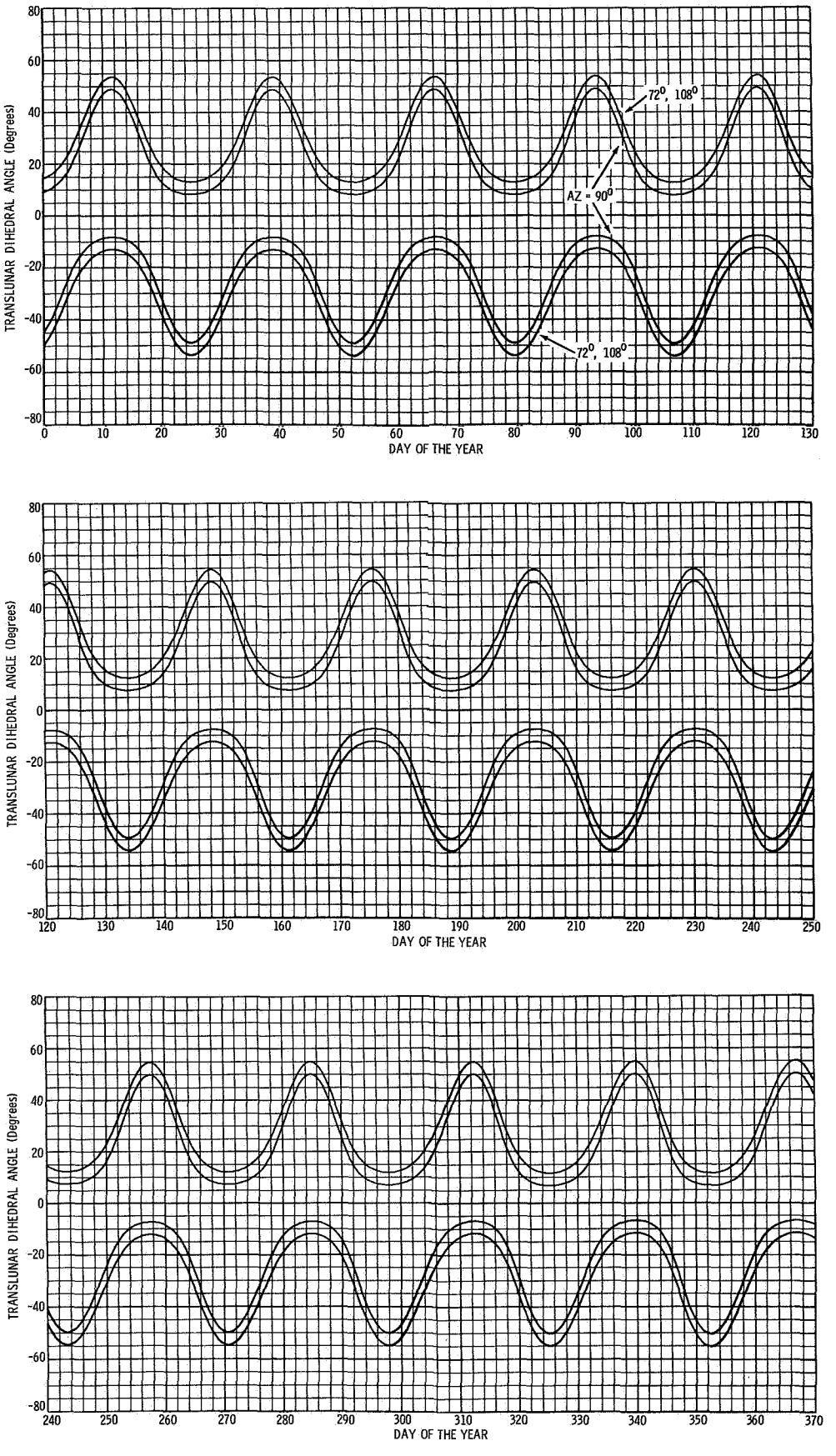
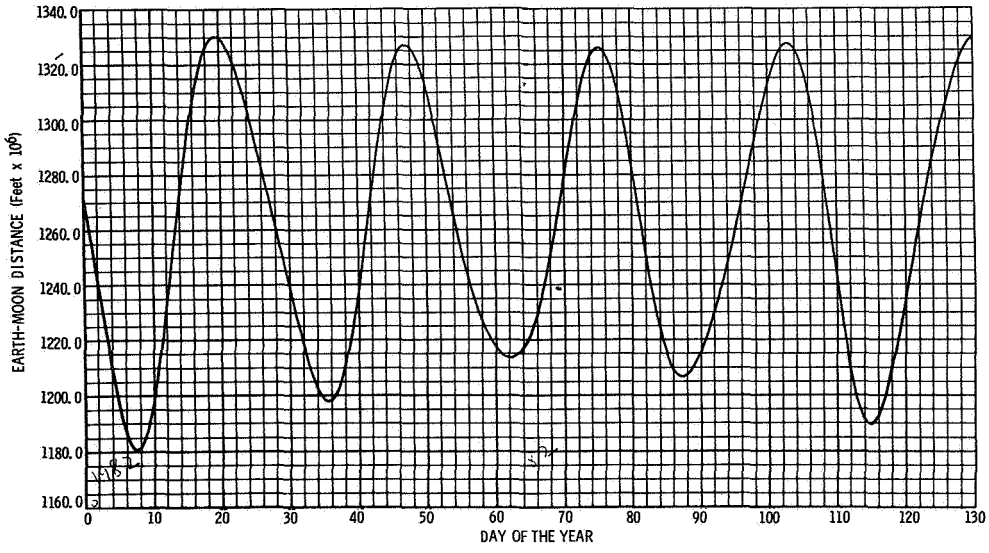
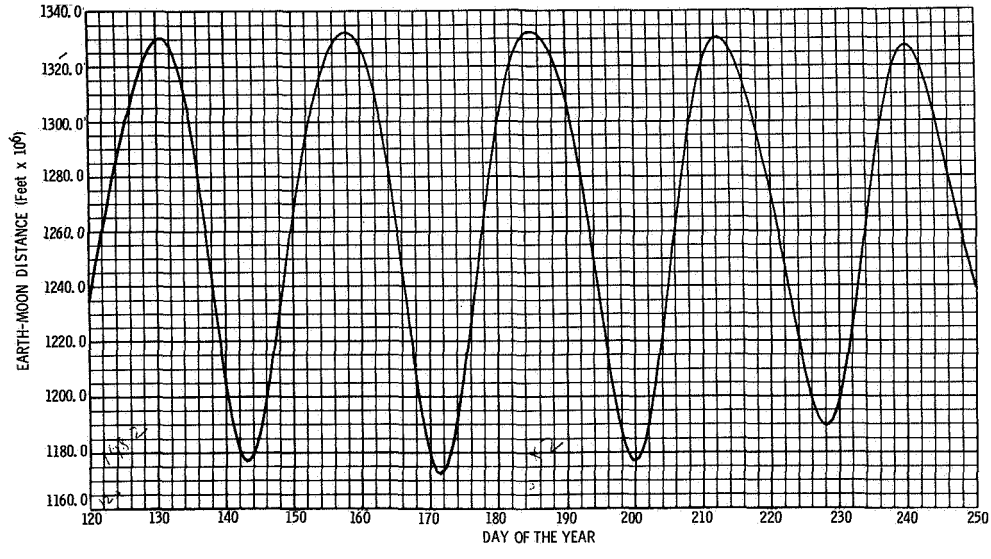


FIGURE B1981-16 TRANSLUNAR DIHEDRAL ANGLES

1982





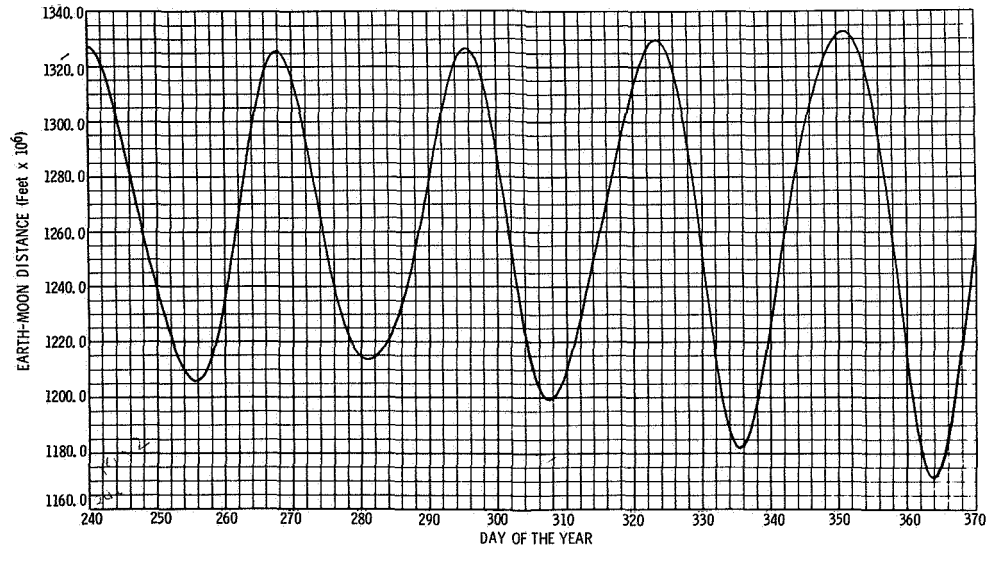


FIGURE B1982-1 EARTH-MOON DISTANCE

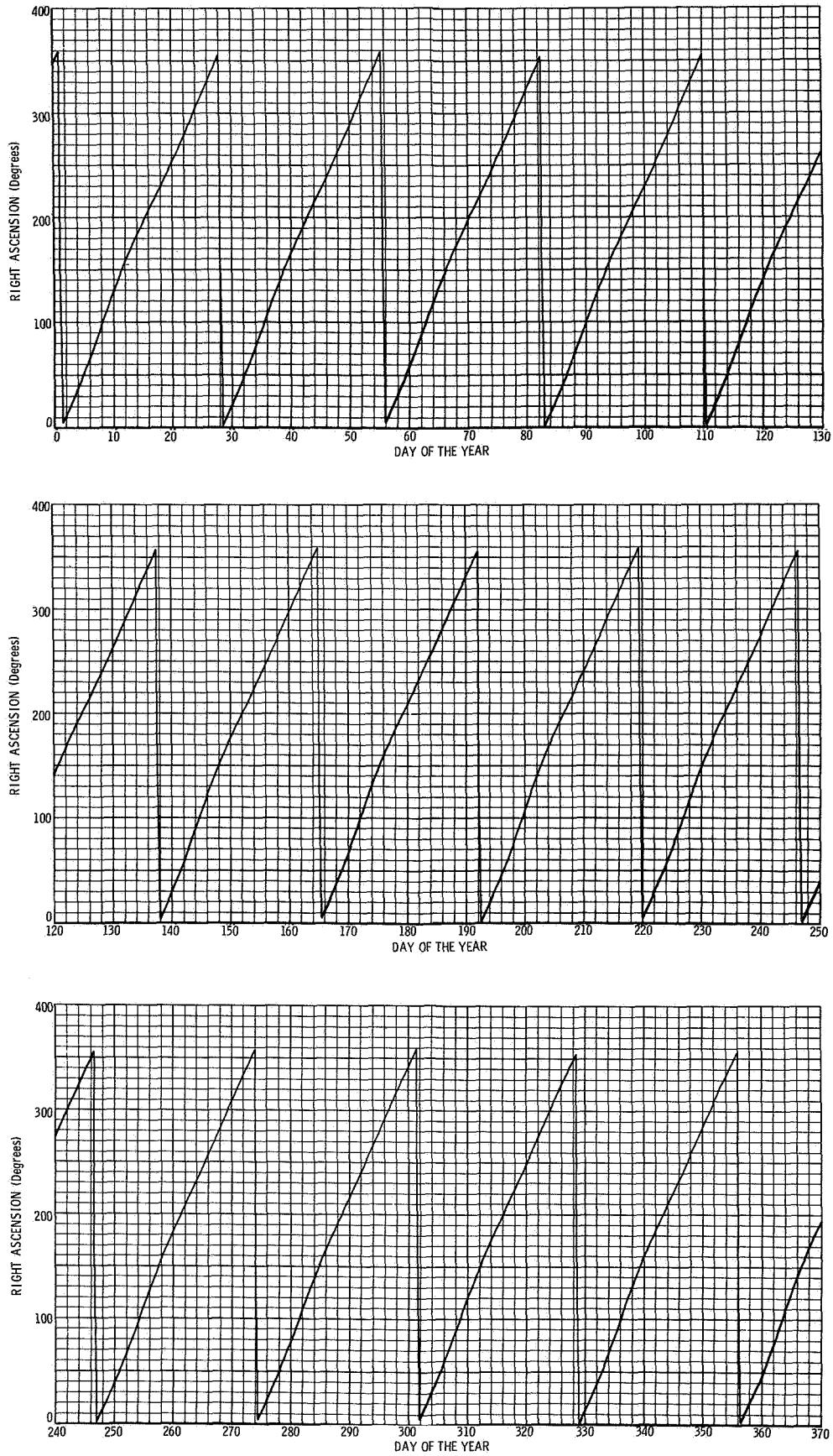


FIGURE B1982-2 RIGHT ASCENSION OF THE MOON

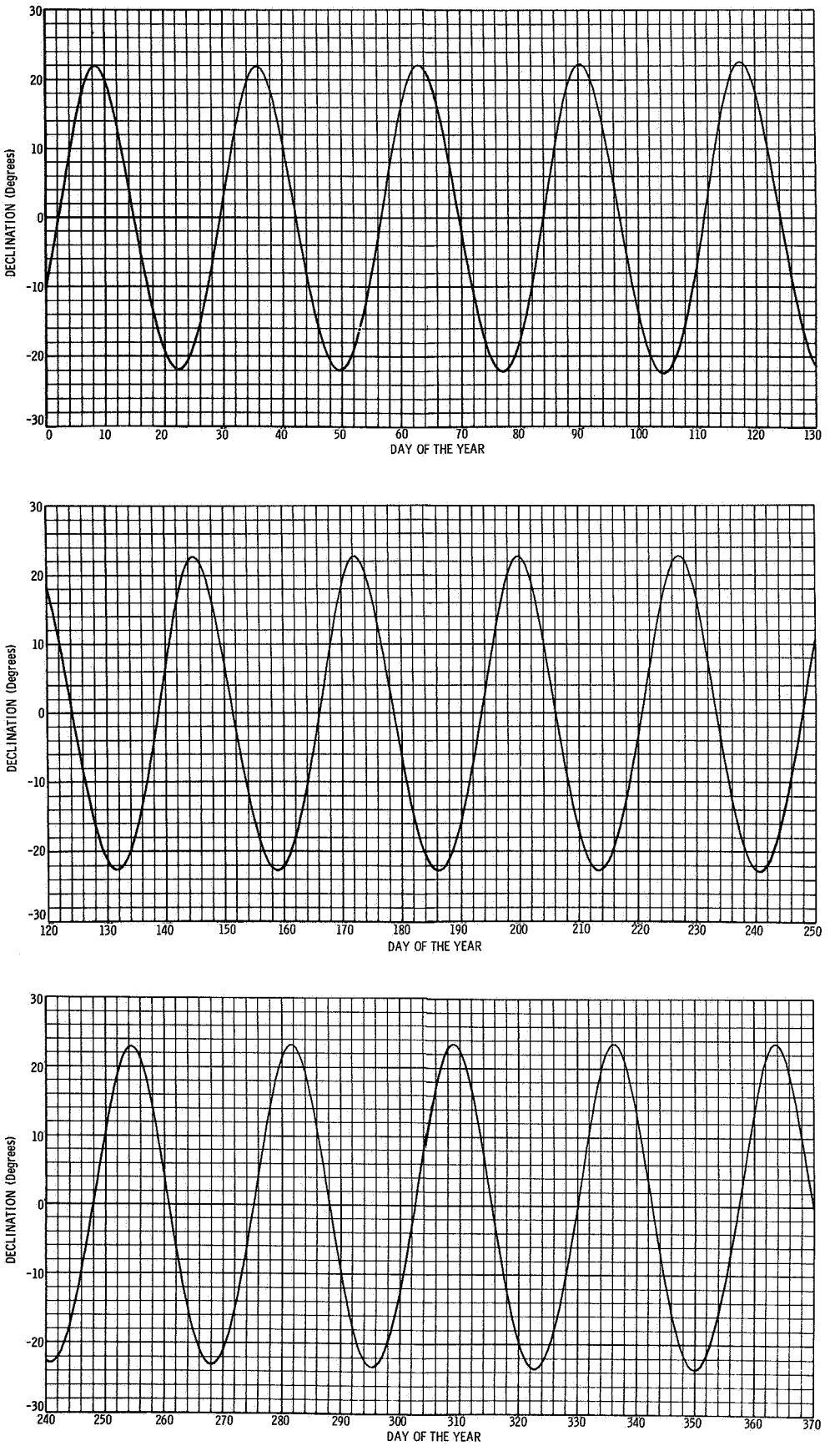


FIGURE B1982-3 DECLINATION OF THE MOON

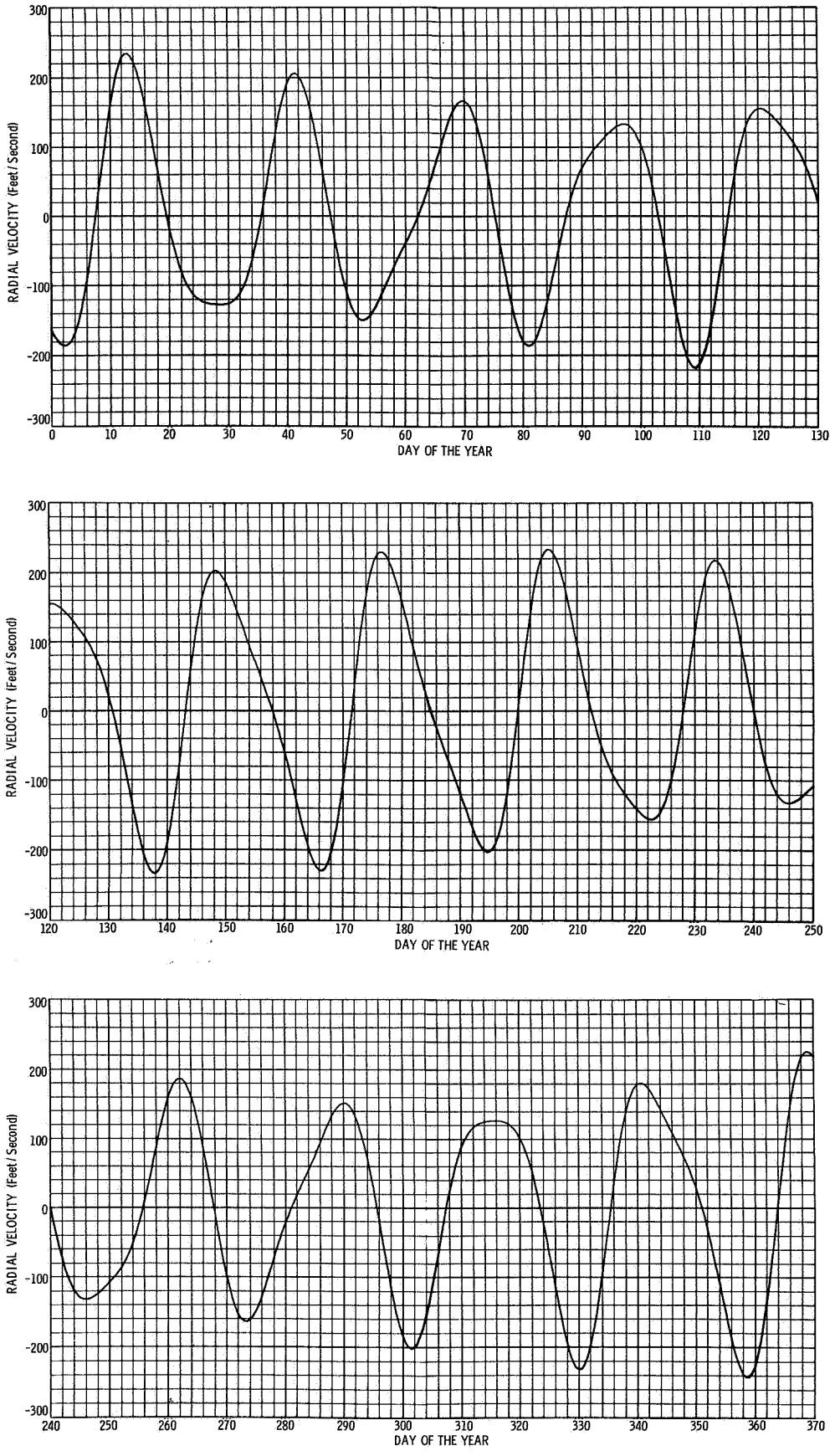


FIGURE B1982-4 RADIAL VELOCITY OF THE MOON

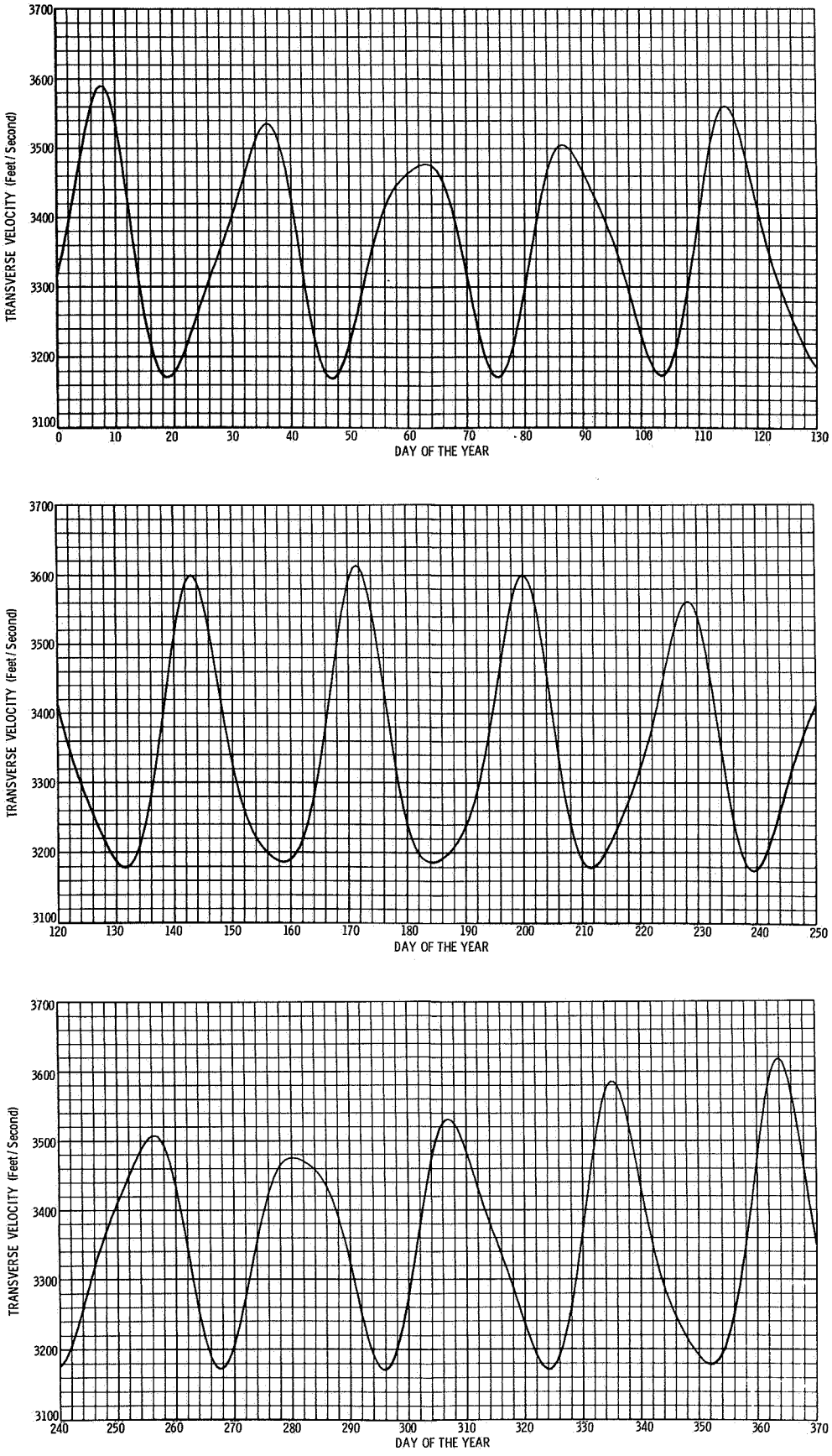
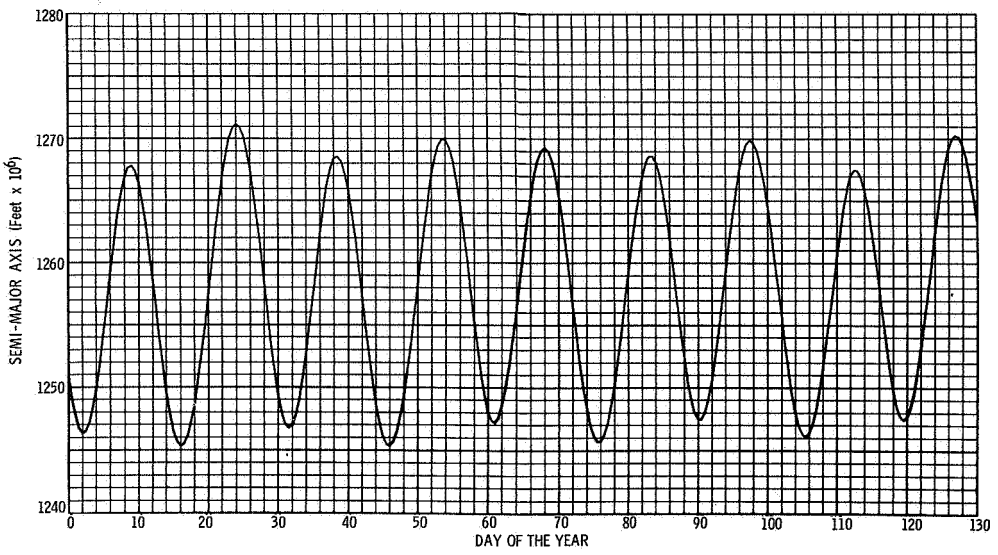
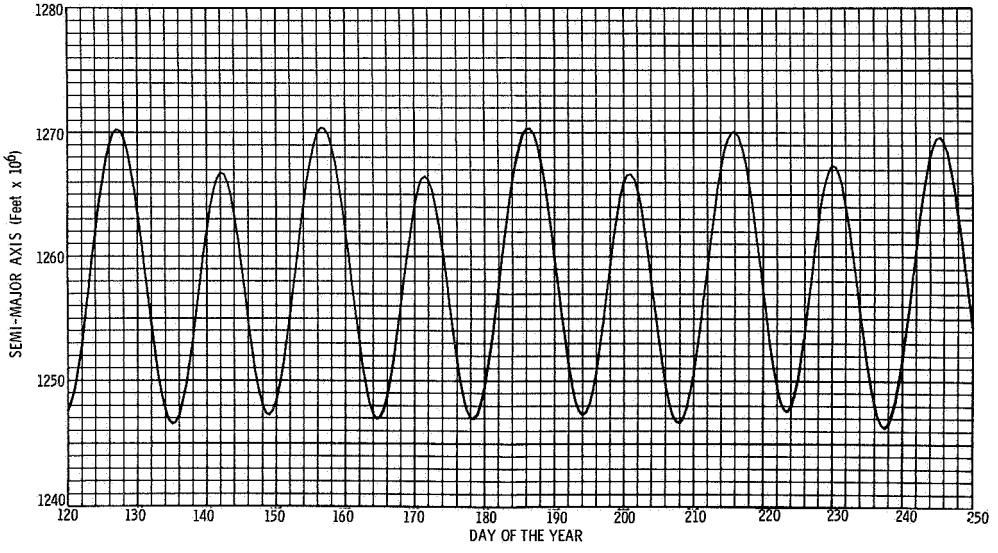


FIGURE B1982-5 TRANSVERSE VELOCITY OF THE MOON





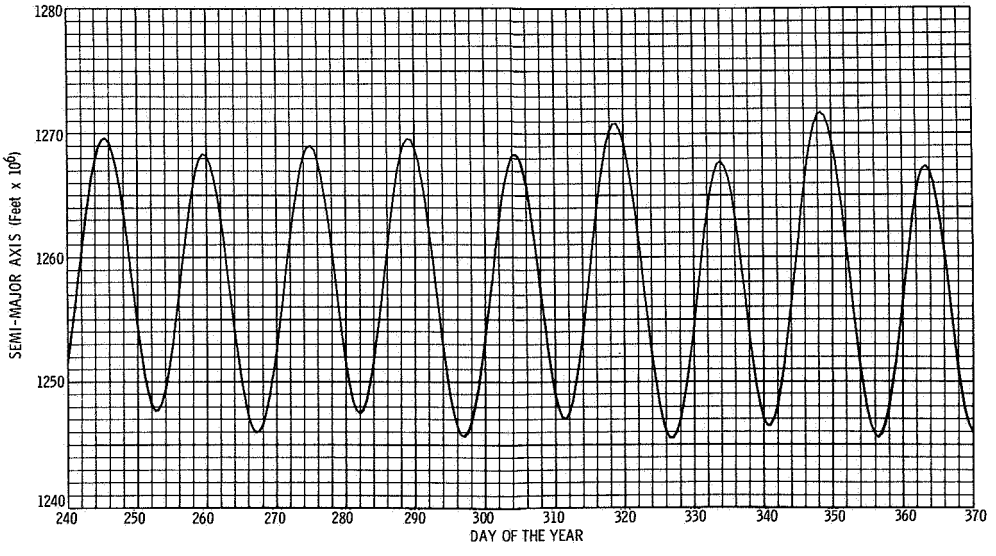
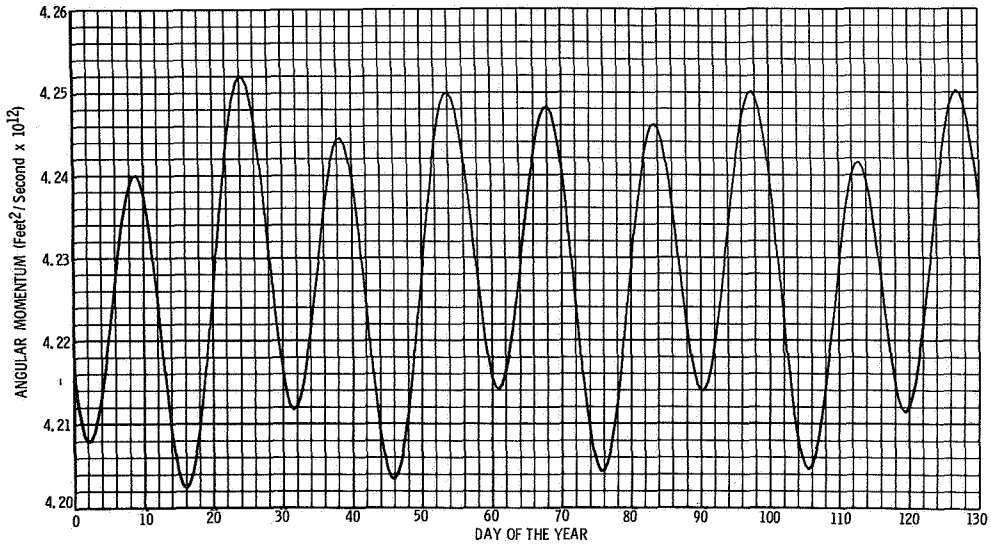
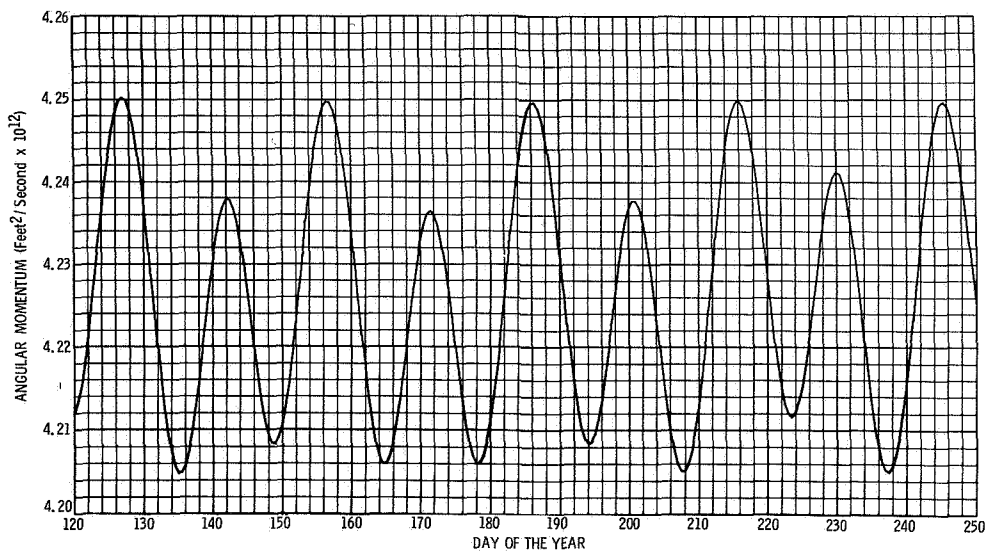


FIGURE B1982-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT





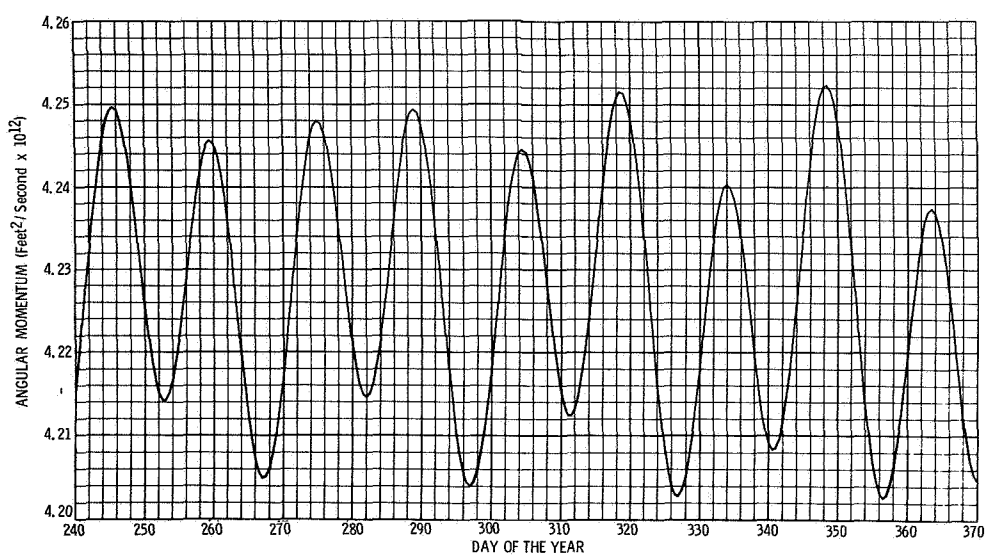
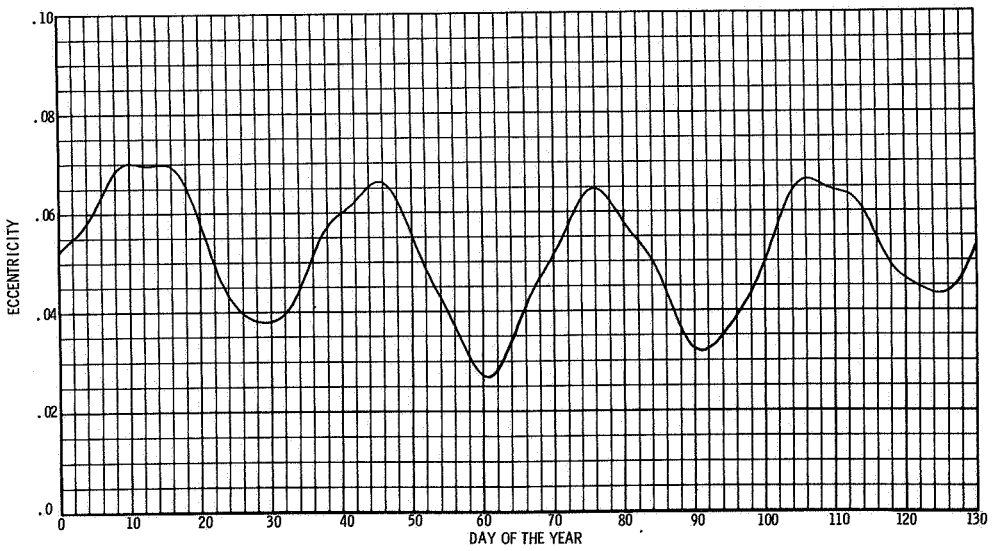
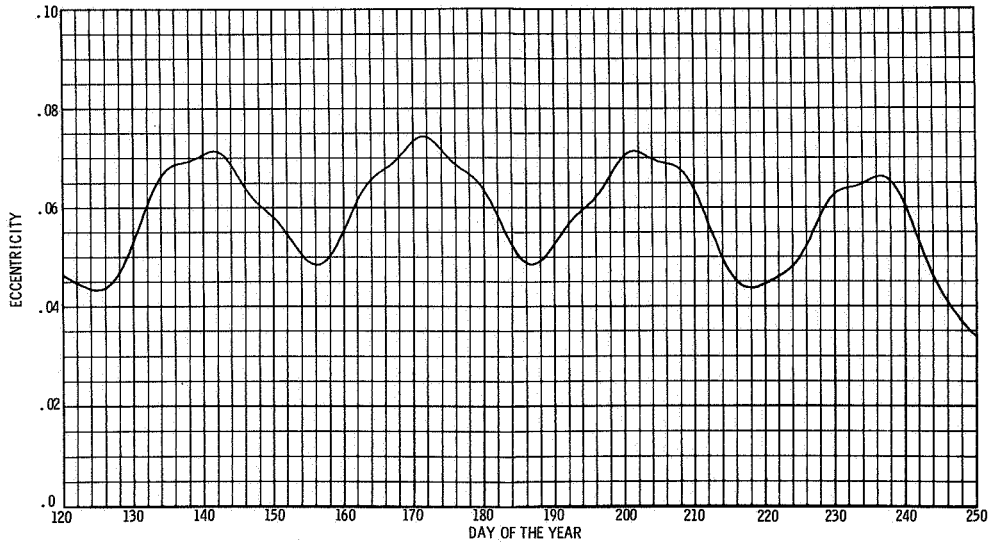


FIGURE B1982-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON





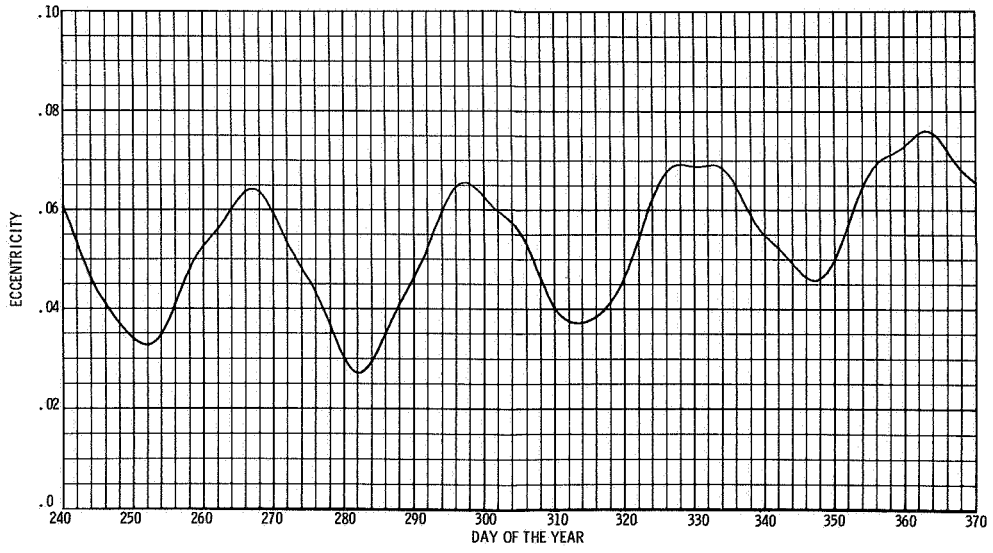
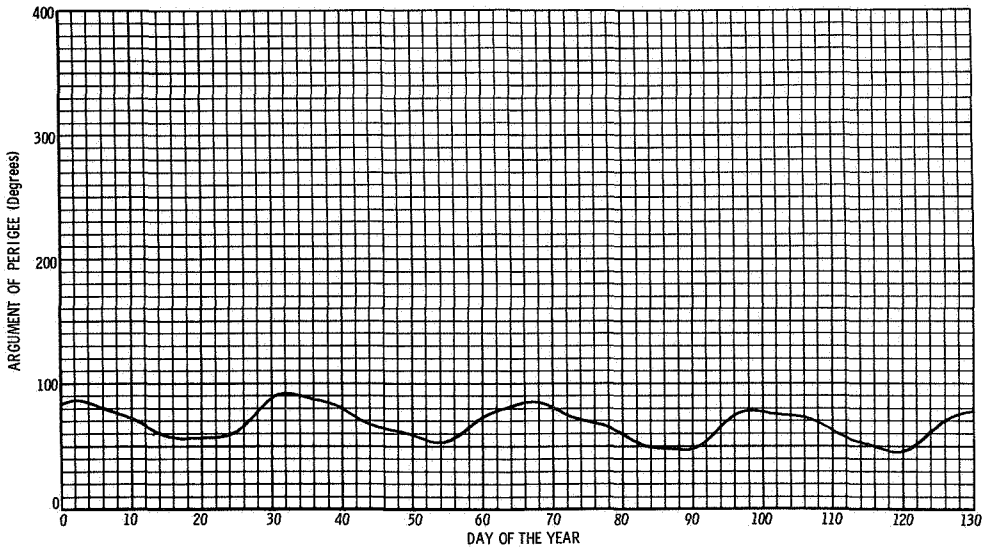
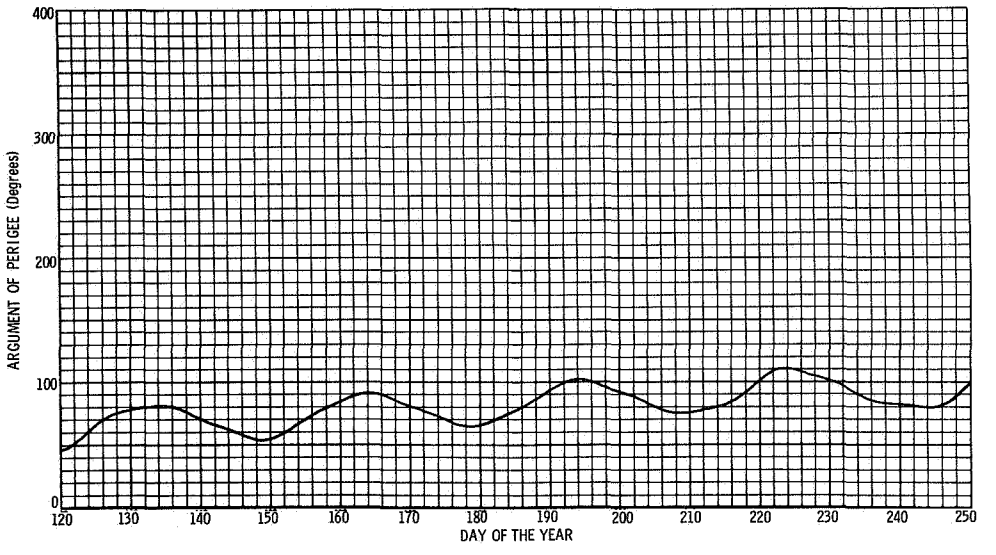


FIGURE B1982-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT





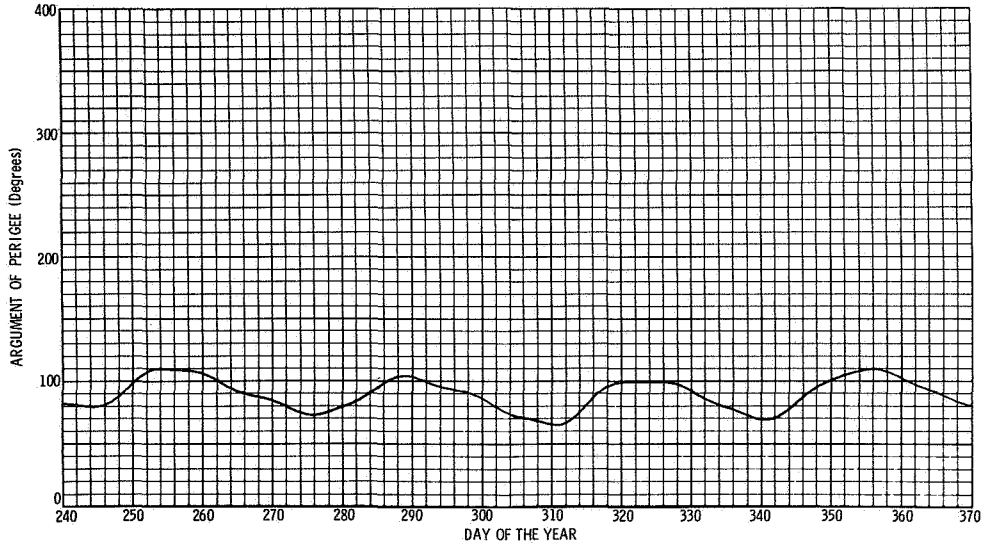
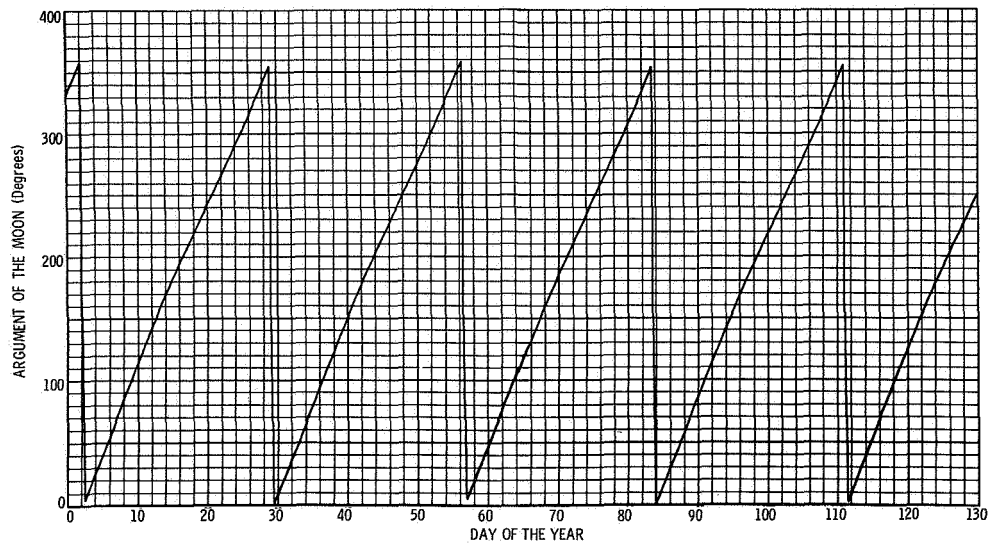
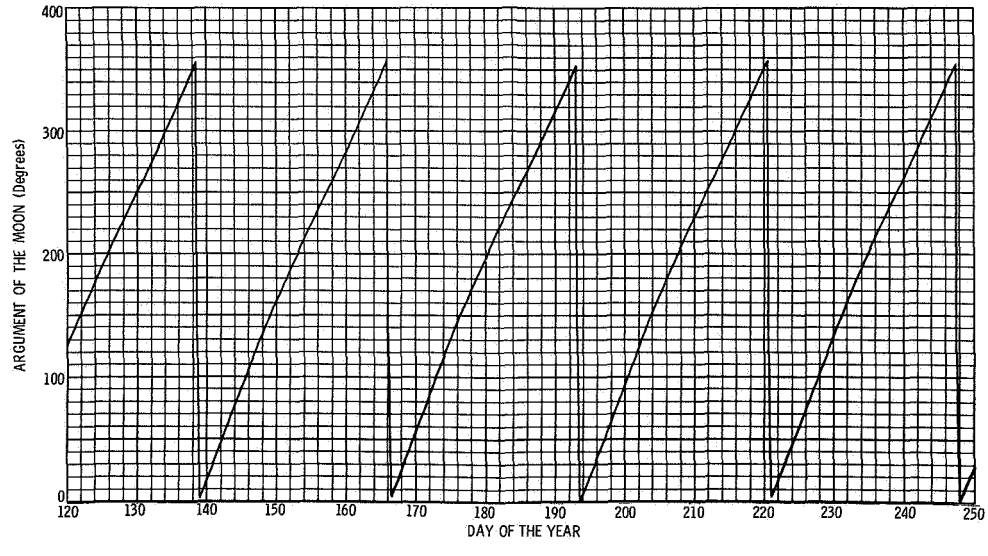


FIGURE B1982-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE





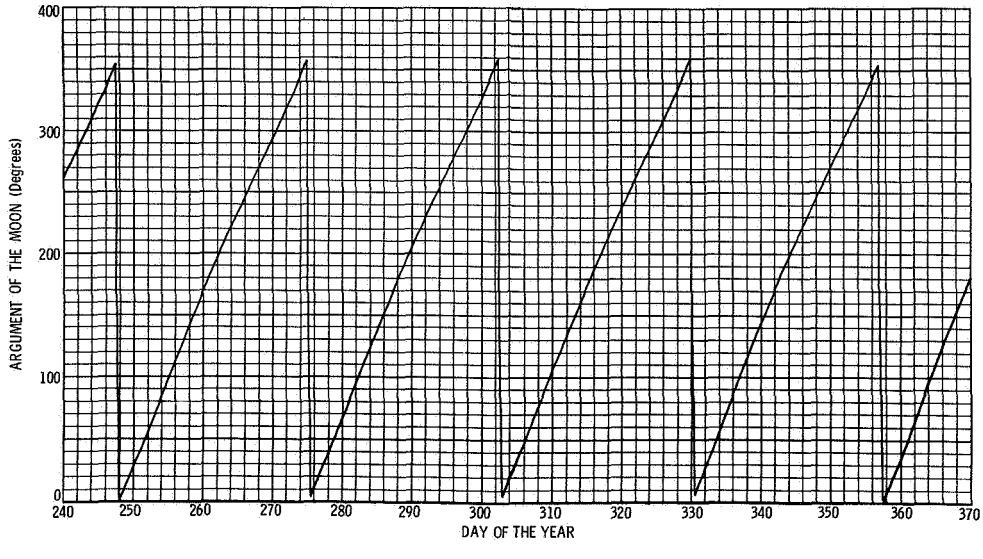


FIGURE B1982-10 ARGUMENT OF THE MOON'S POSITION

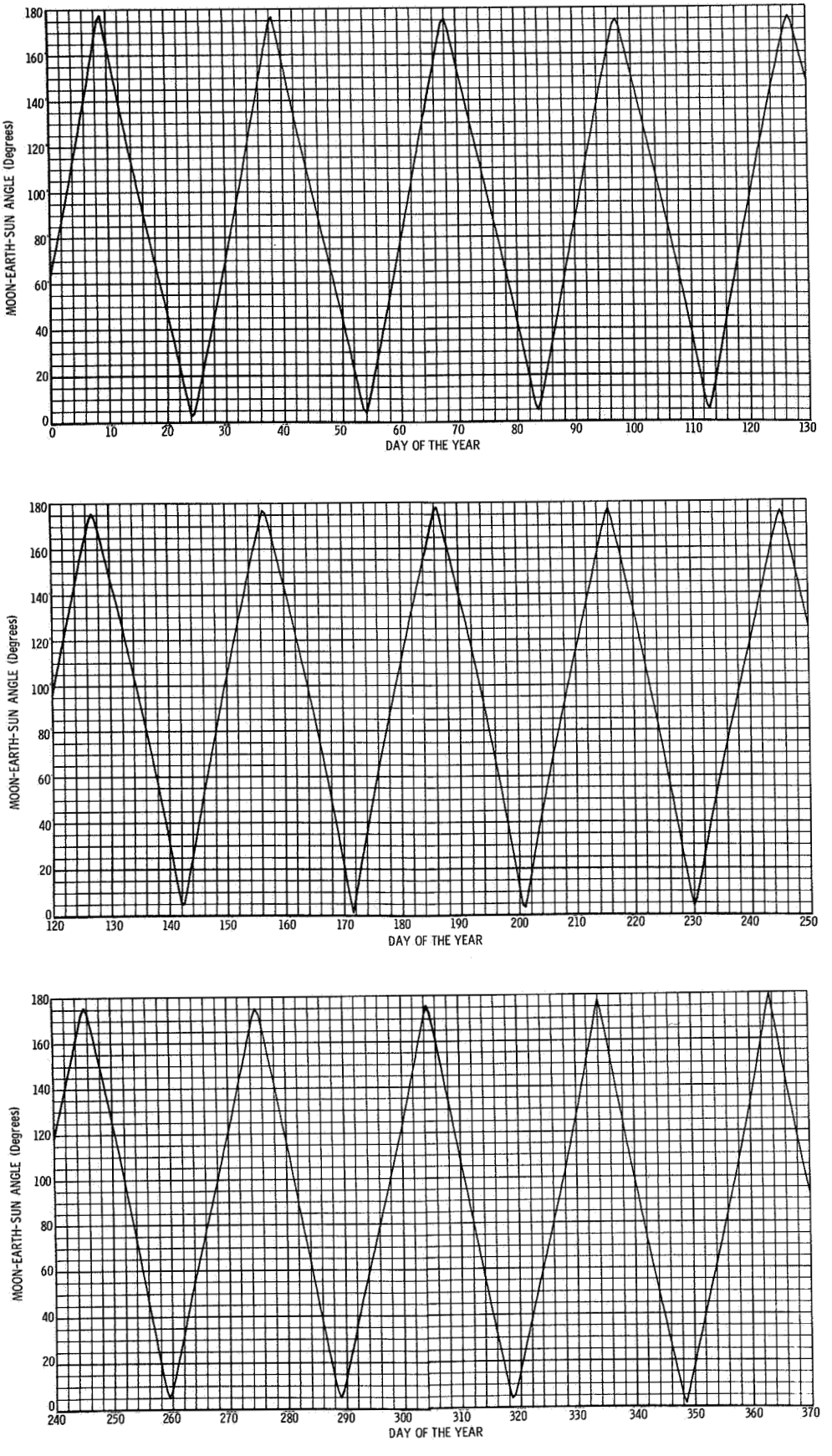
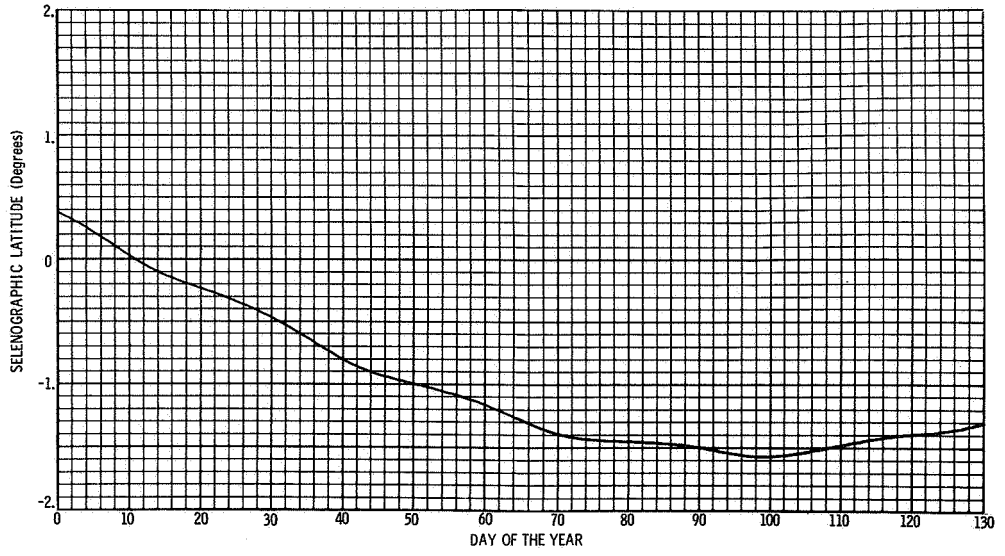
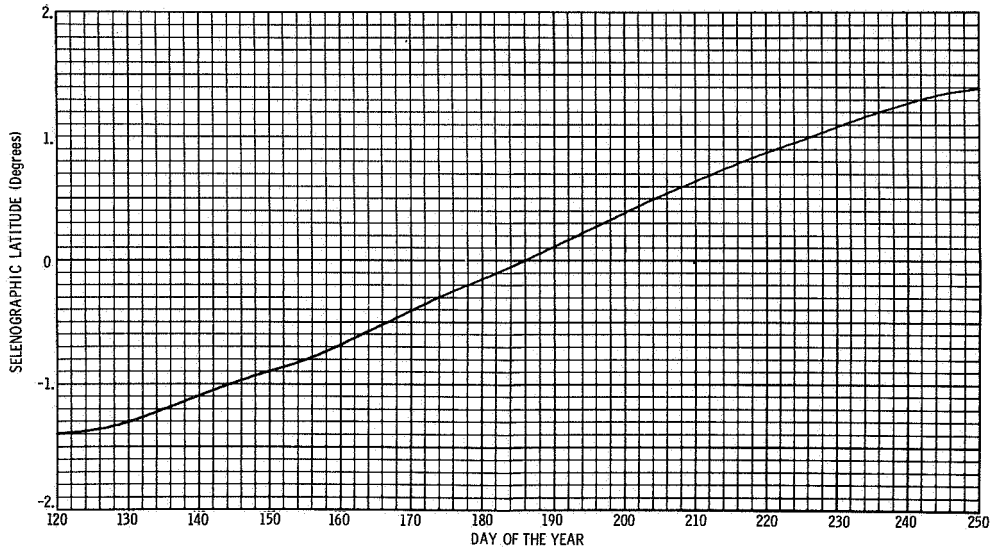


FIGURE B1982-11 MOON-EARTH-SUN ANGLE





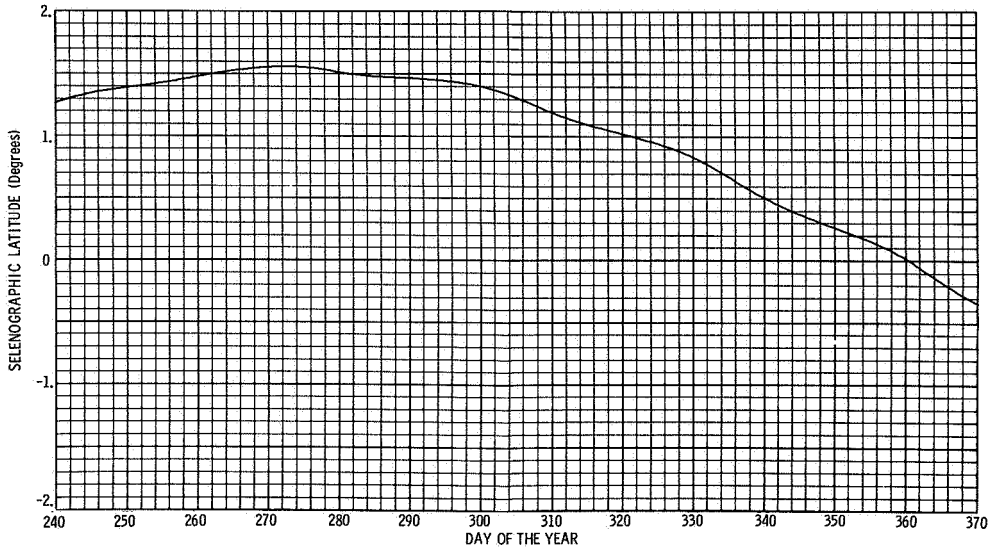
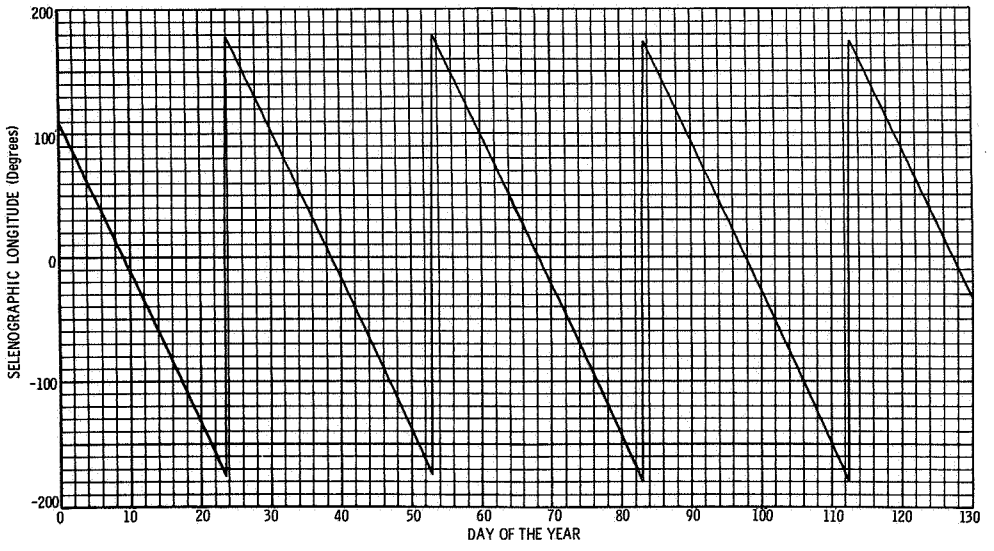
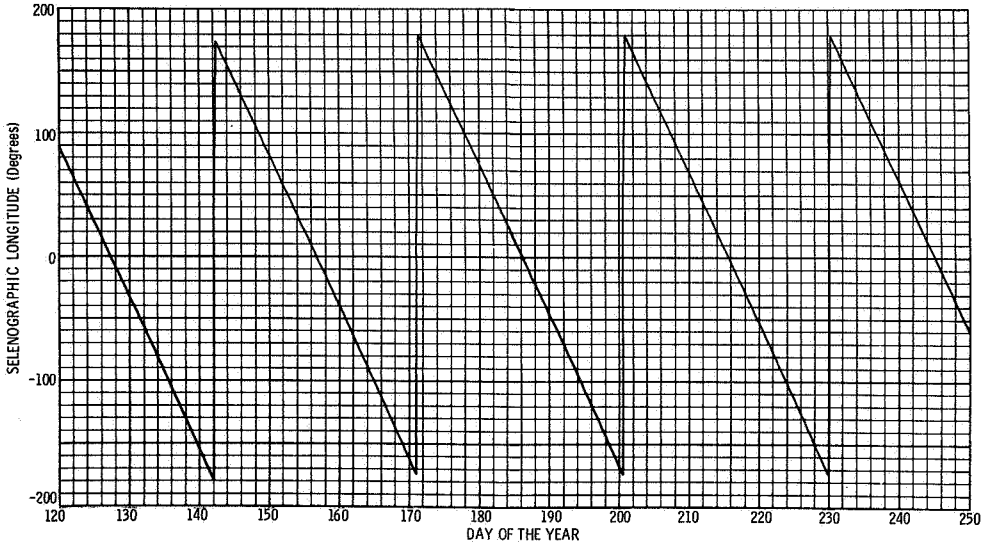


FIGURE B1982-12 SELENOGRAPHIC LATITUDE OF THE SUN





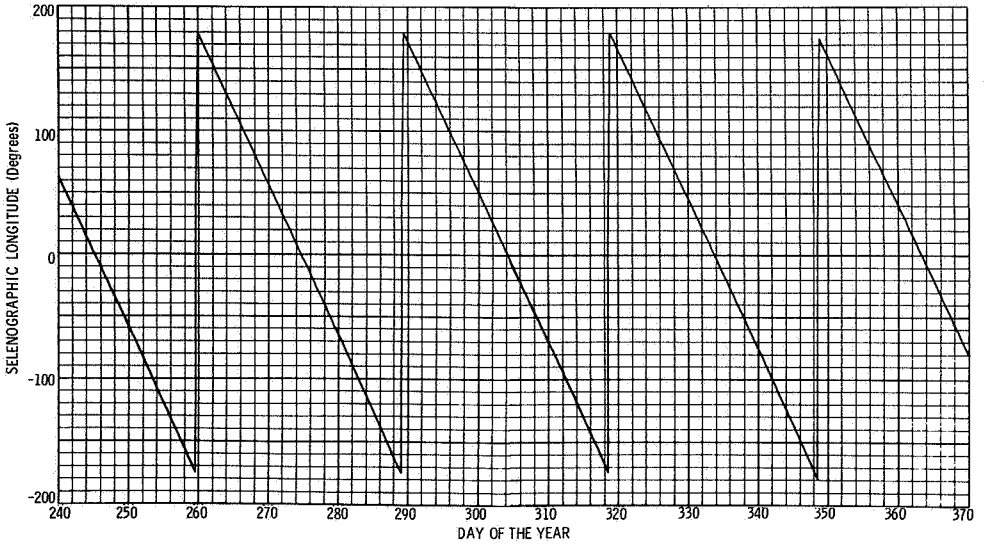
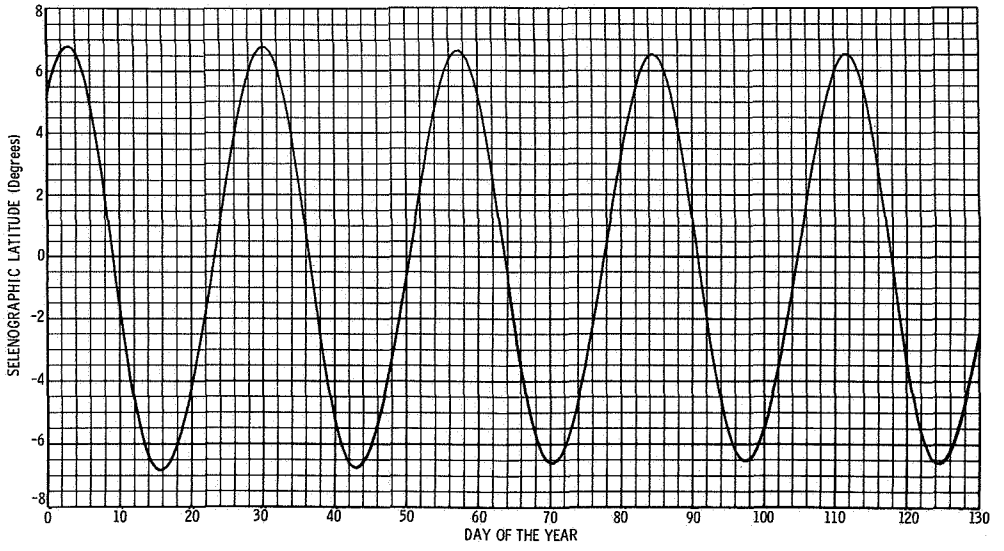
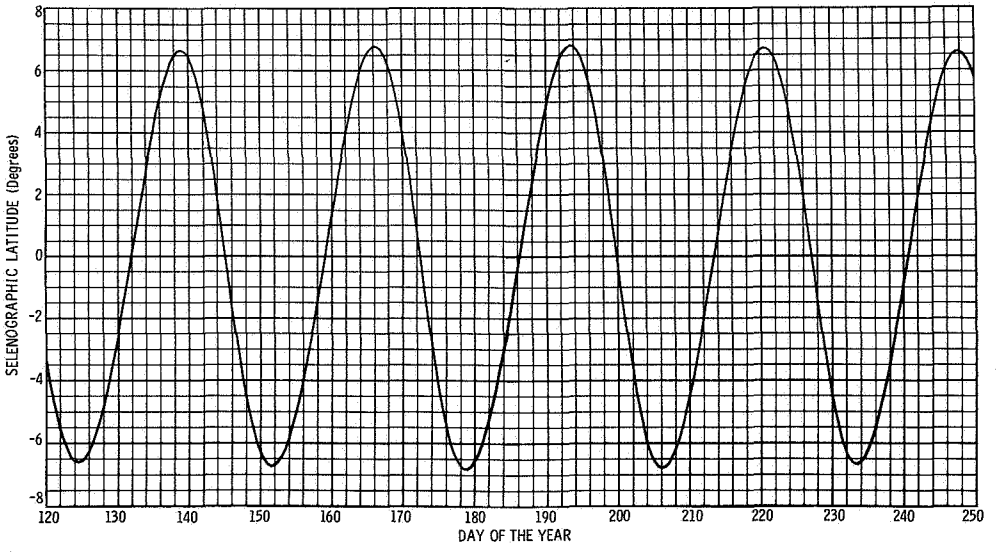


FIGURE B1982-13 SELENOGRAPHIC LONGITUDE OF THE SUN





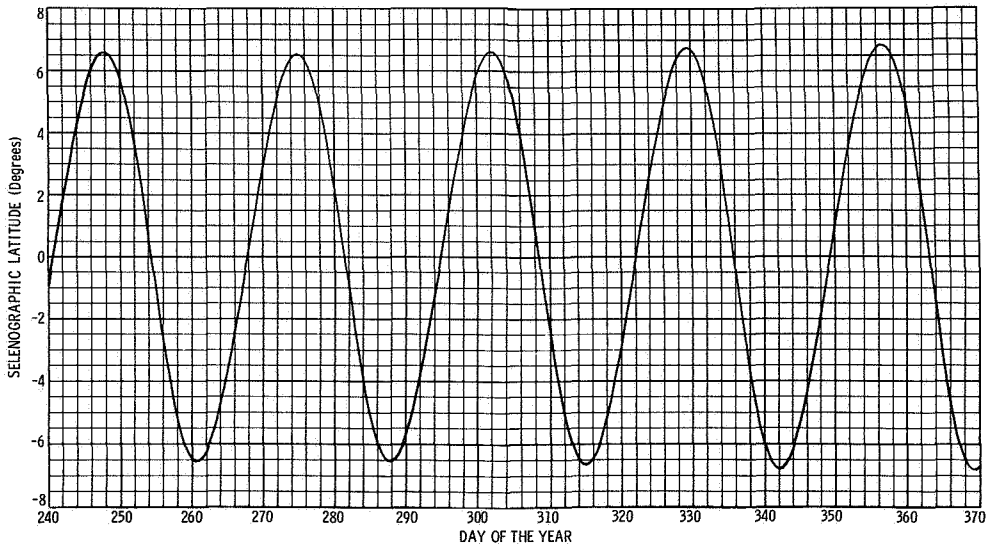


FIGURE B1982-14 SELENOGRAPHIC LATITUDE OF THE EARTH

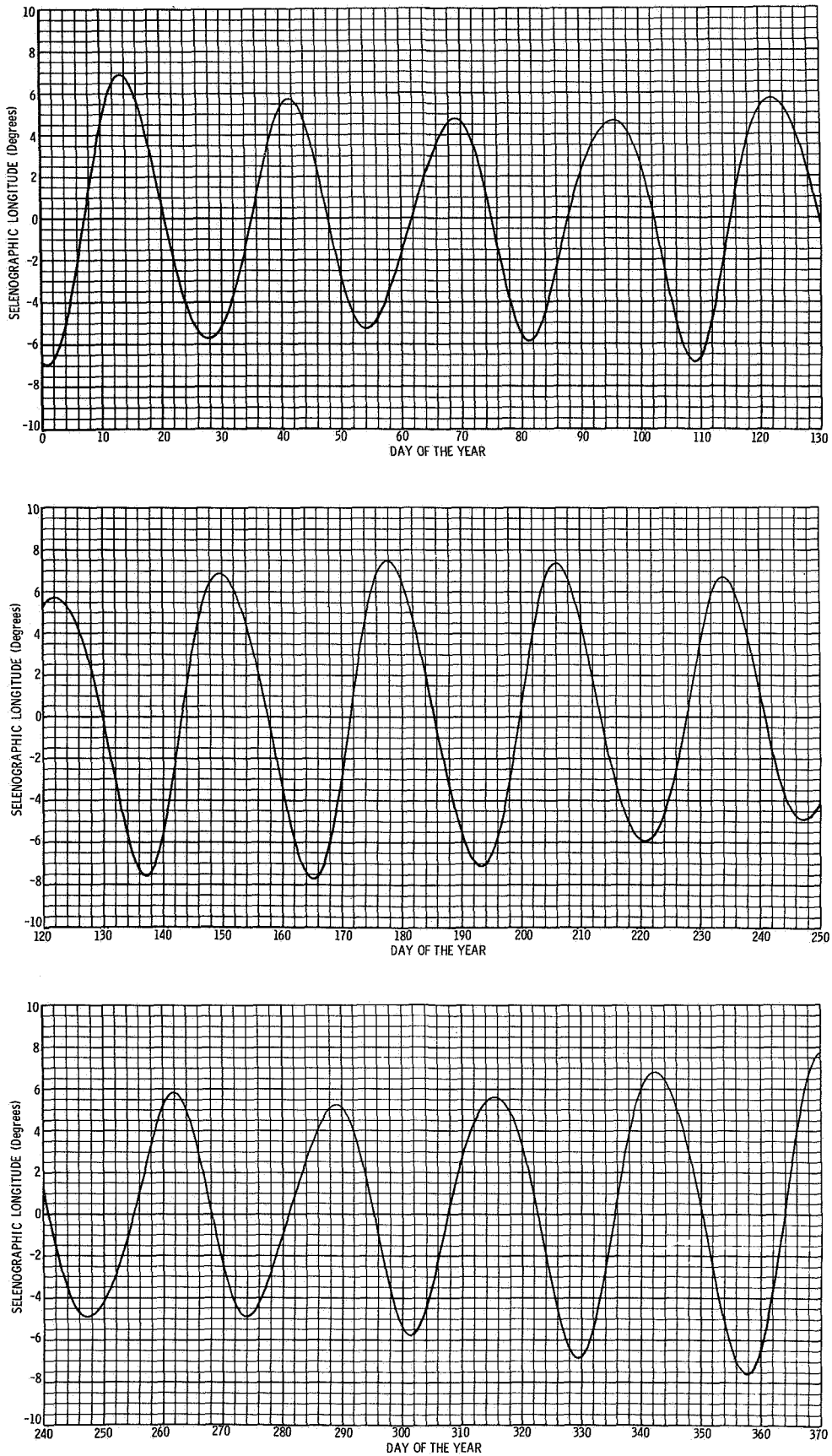


FIGURE B1982-15 SELENOGRAPHIC LONGITUDE OF THE EARTH

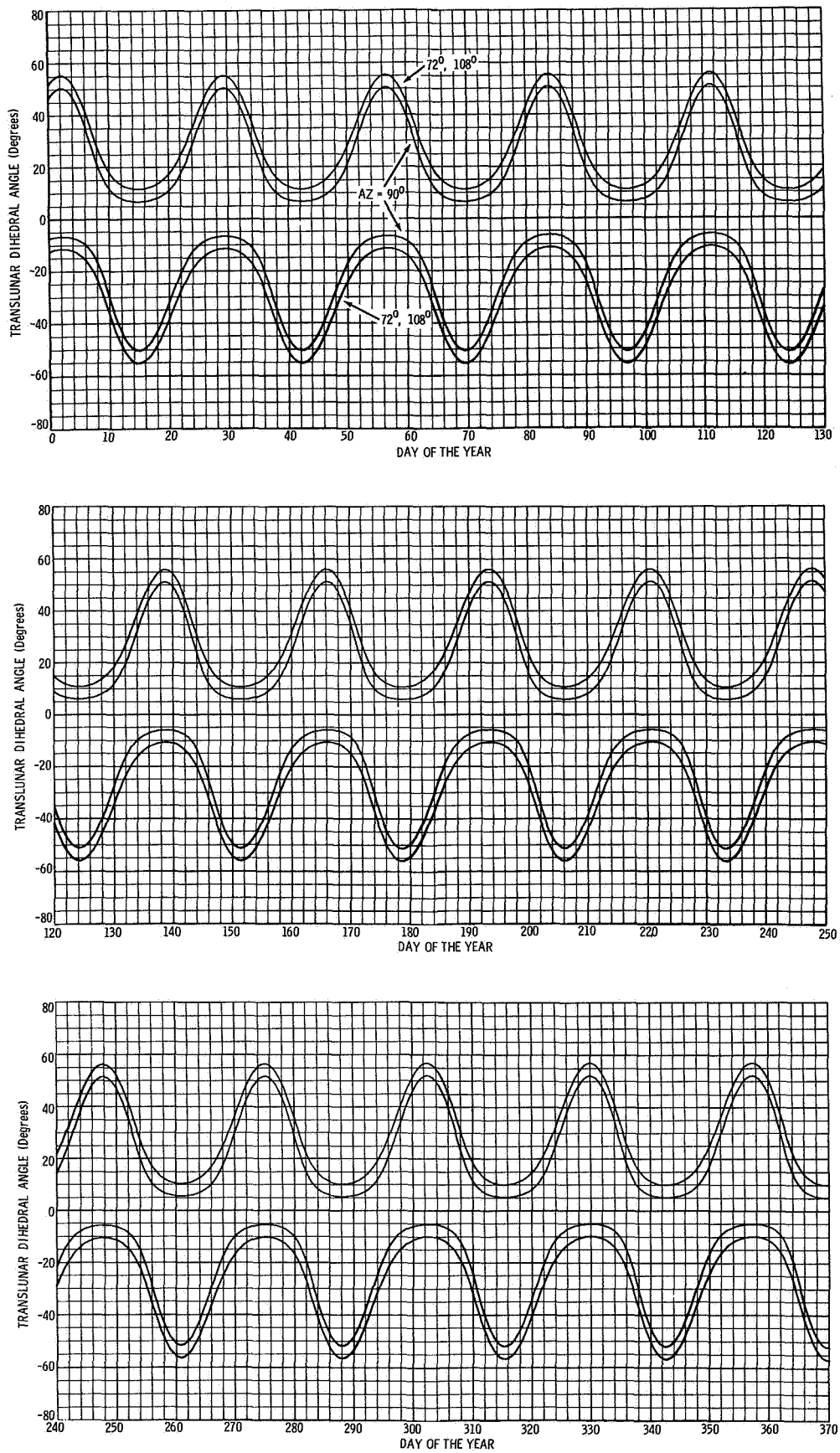
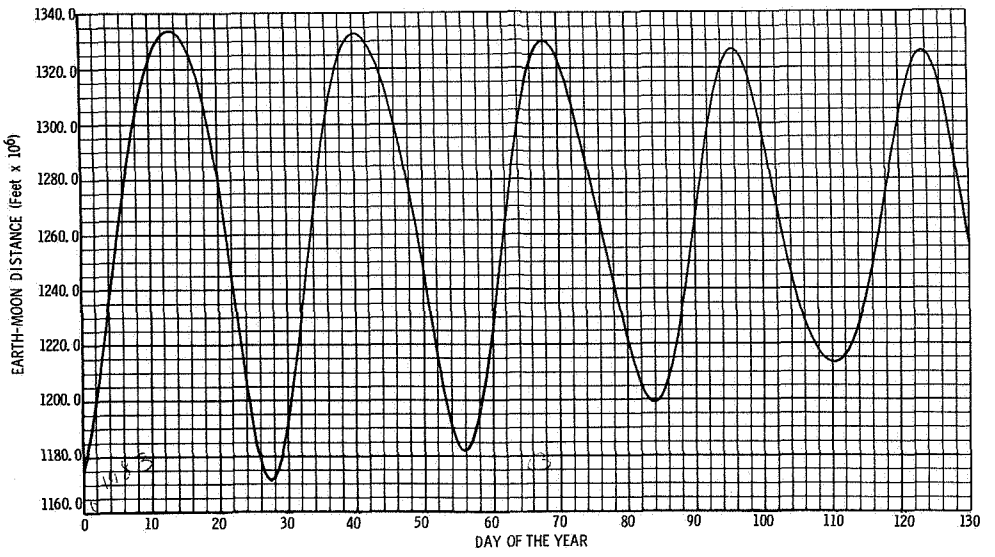
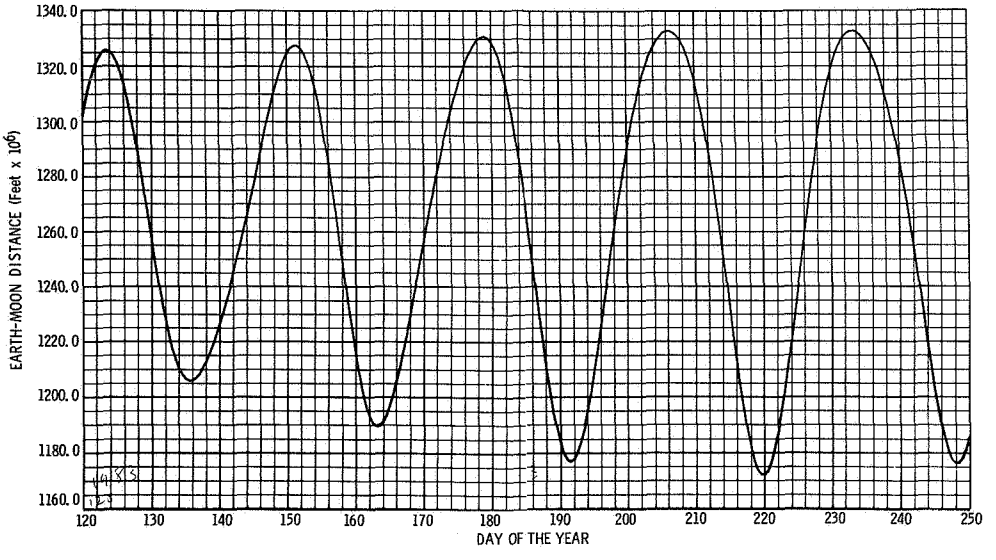


FIGURE B1982-16 TRANSLUNAR DIHEDRAL ANGLES

1983





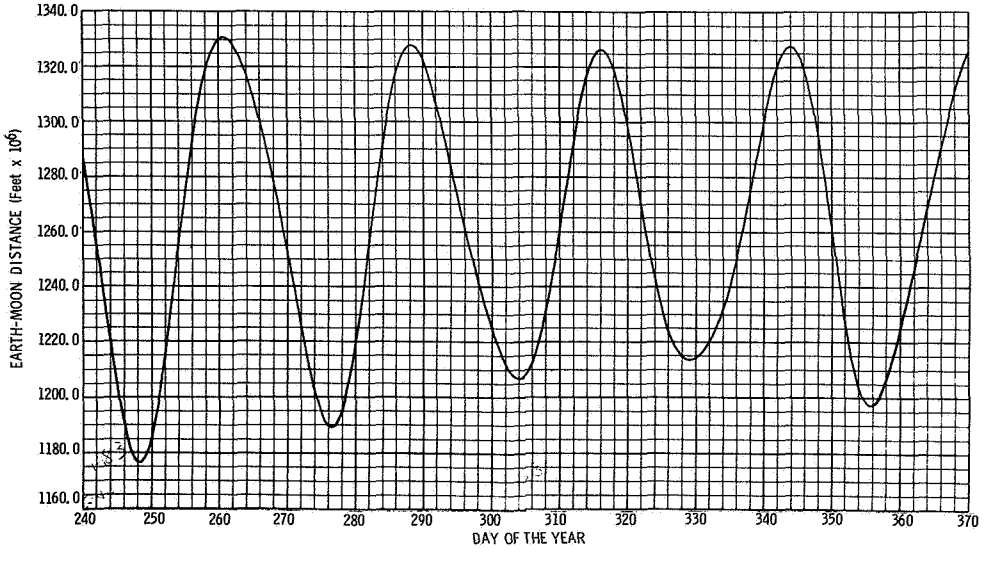
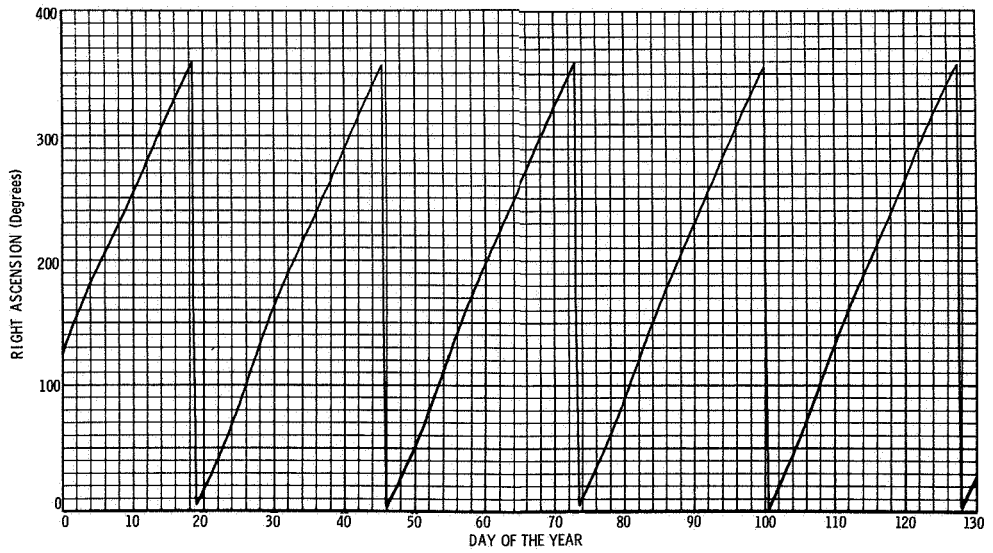
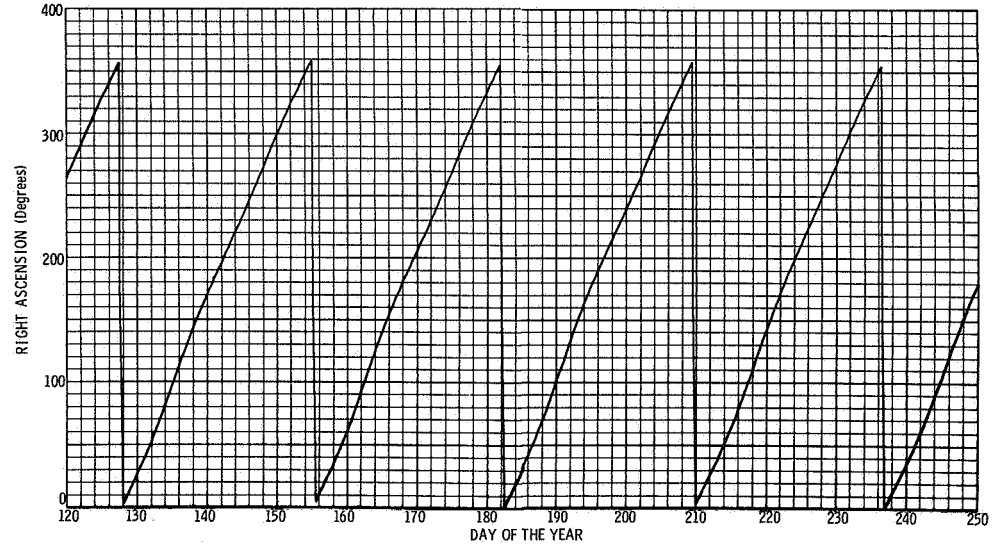


FIGURE B1983-1 EARTH-MOON DISTANCE





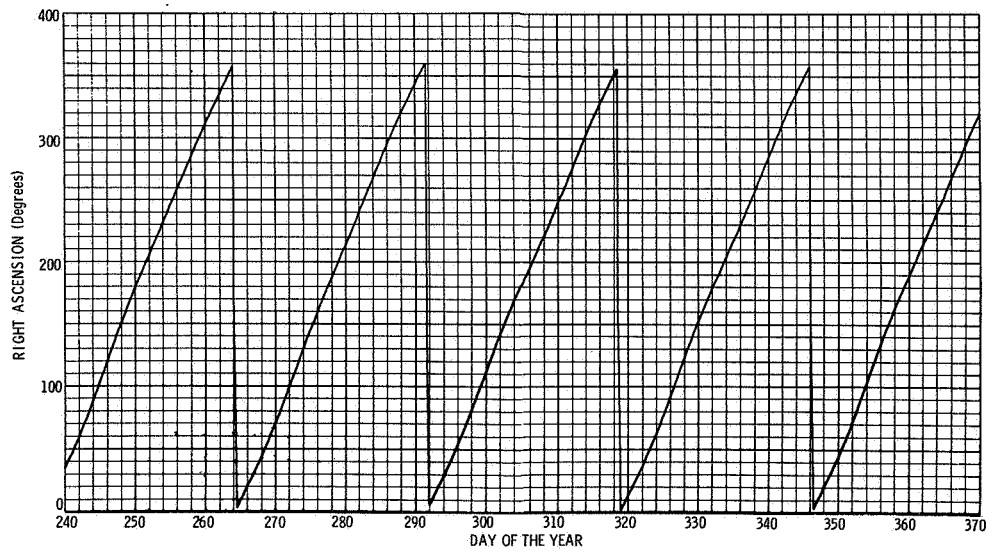


FIGURE B1983-2 RIGHT ASCENSION OF THE MOON

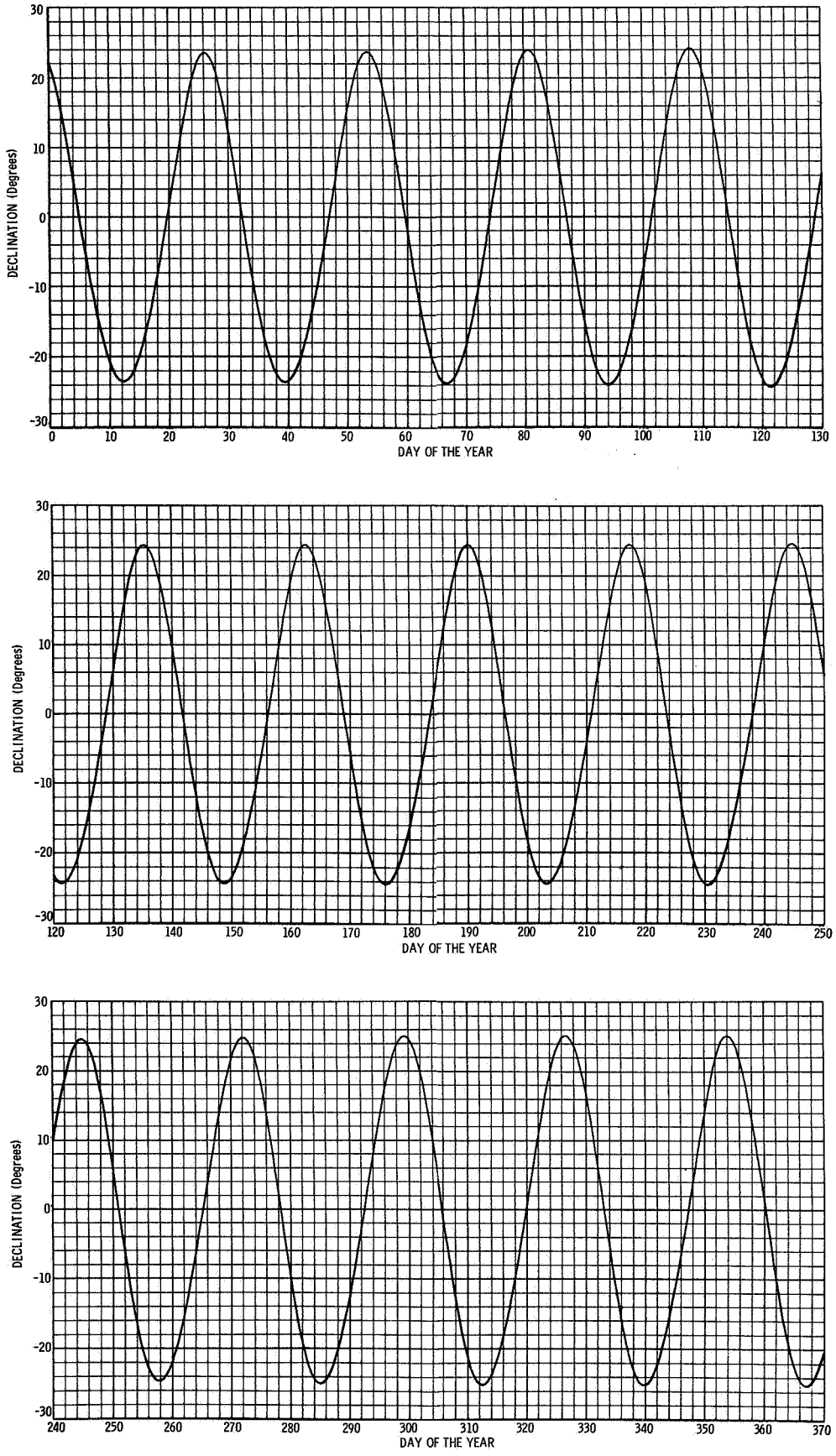
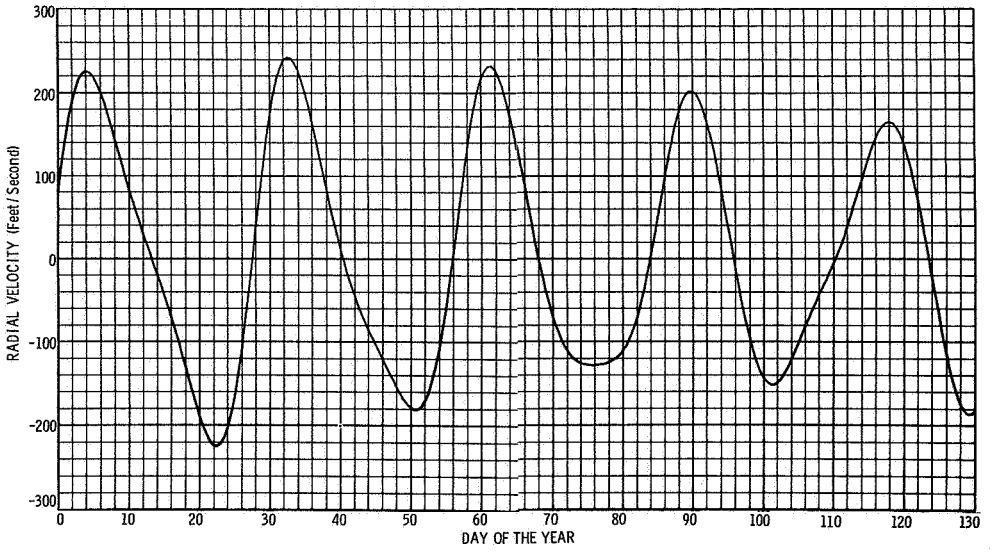
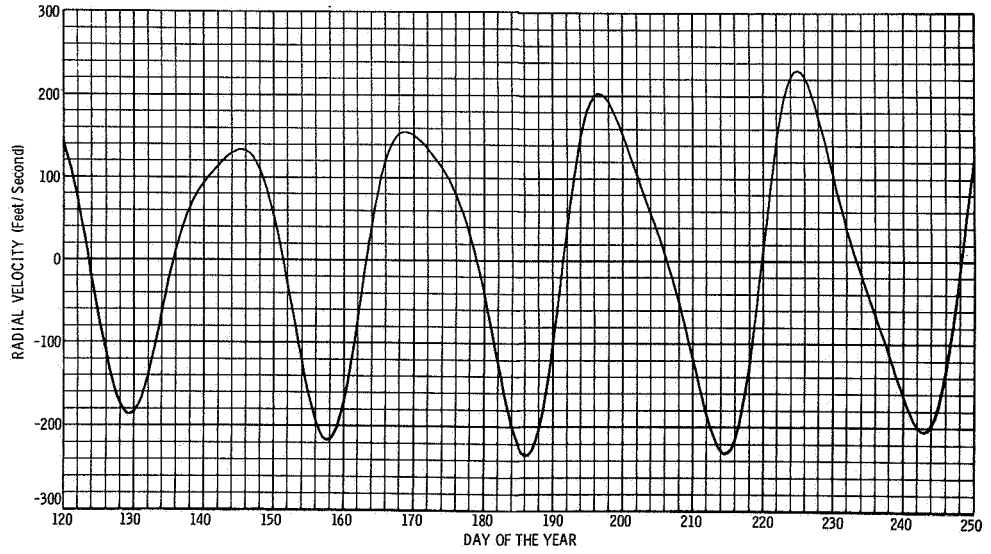


FIGURE B1983-3 DECLINATION OF THE MOON





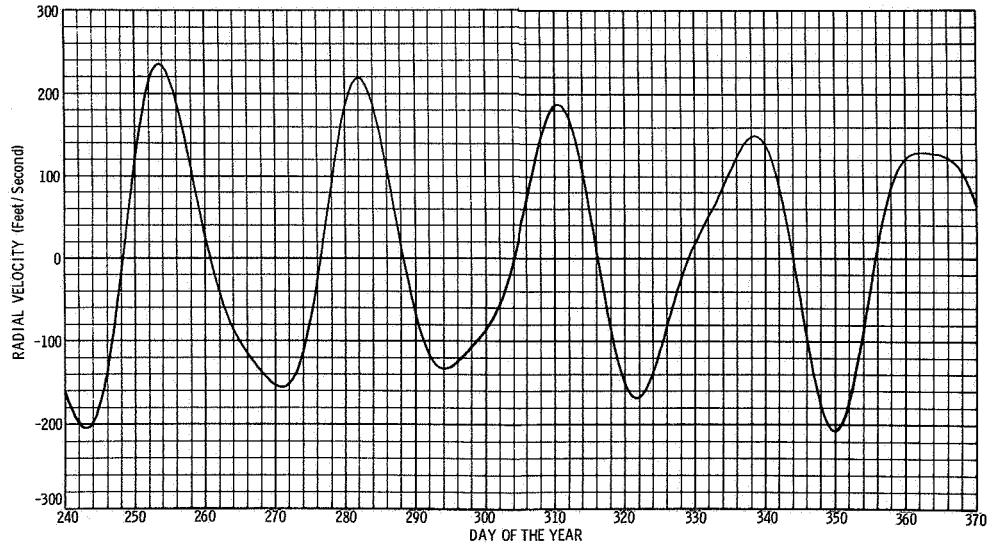
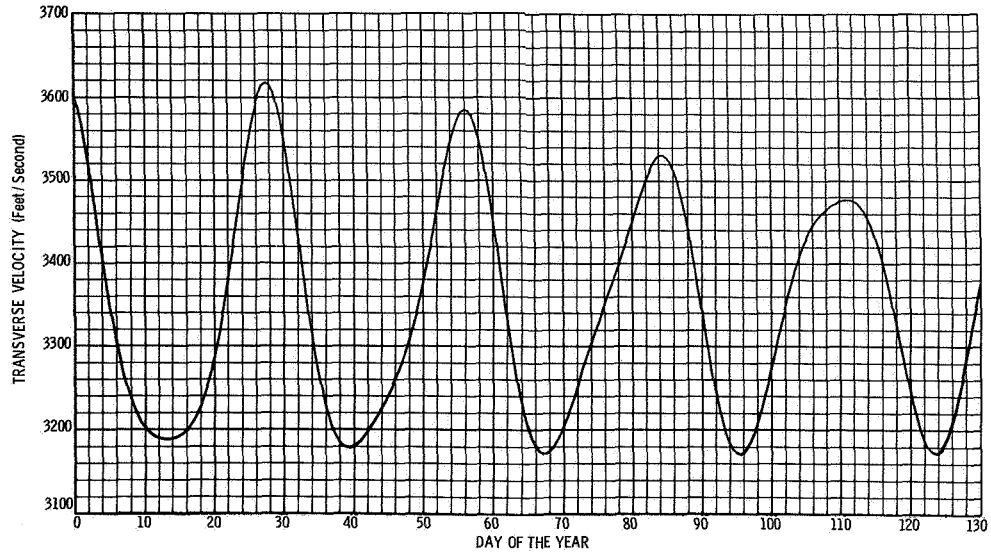
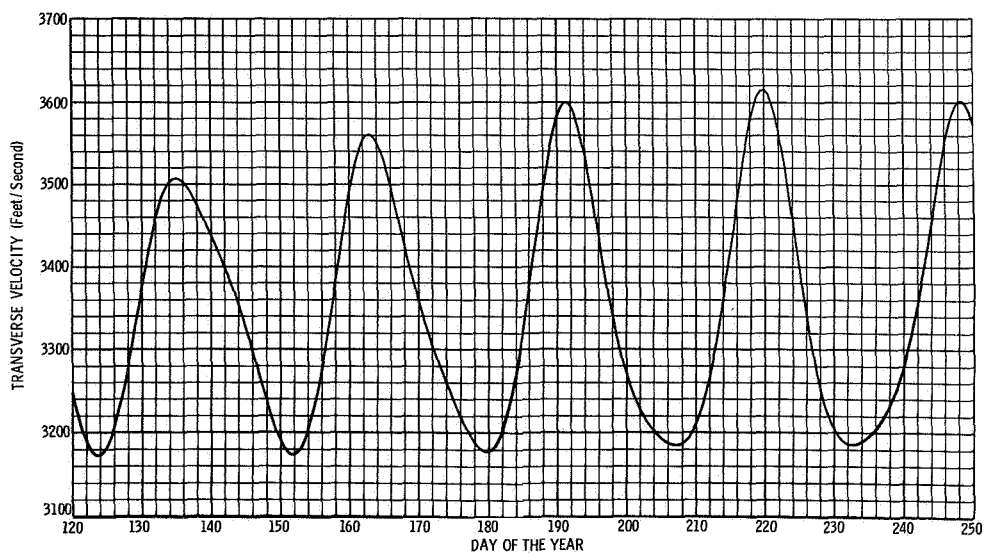


FIGURE B1983-4 RADIAL VELOCITY OF THE MOON





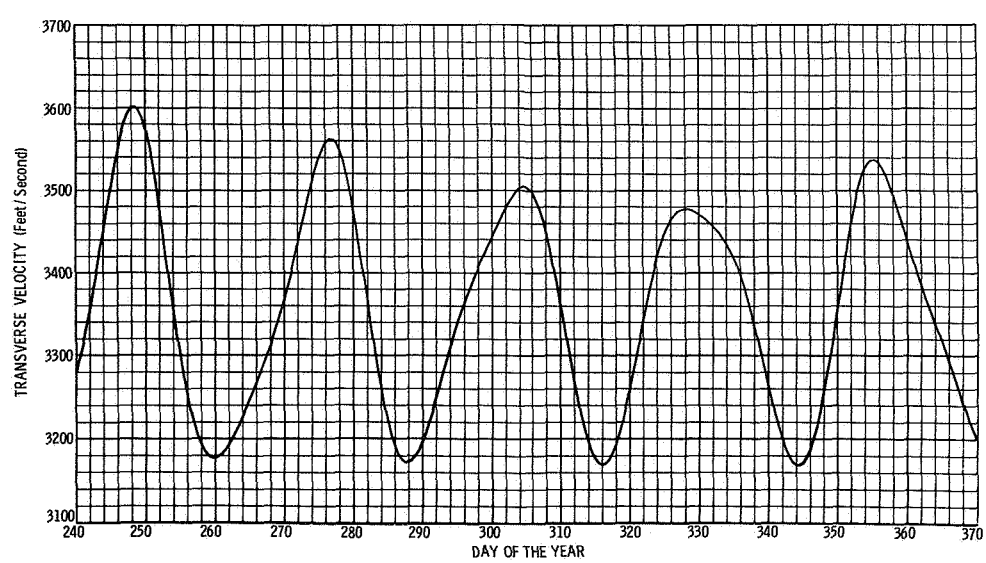
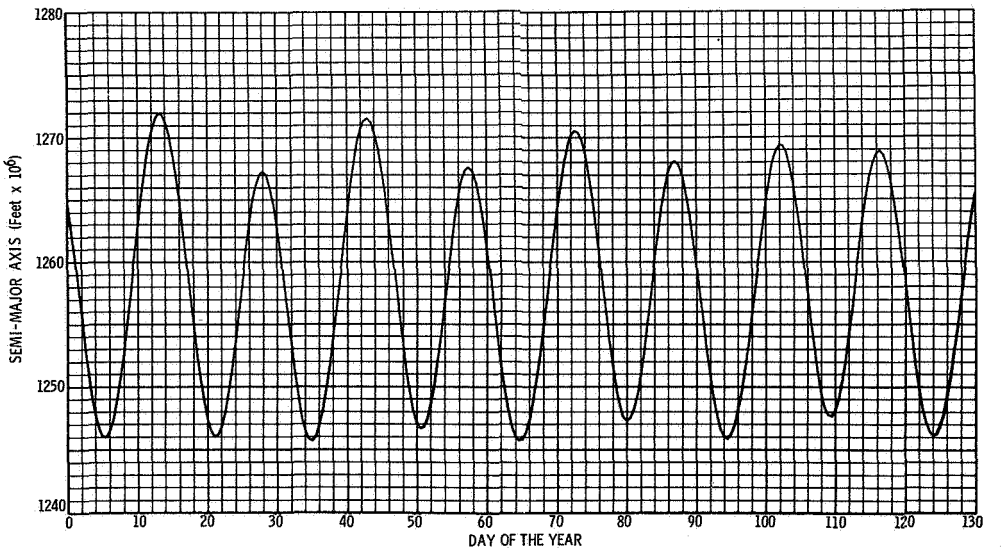
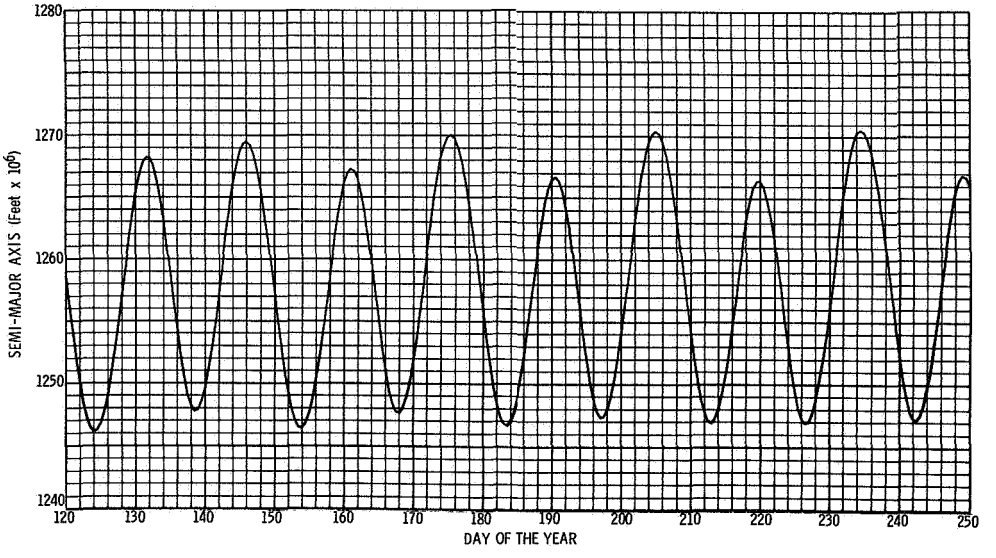


FIGURE B1983-5 TRANSVERSE VELOCITY OF THE MOON





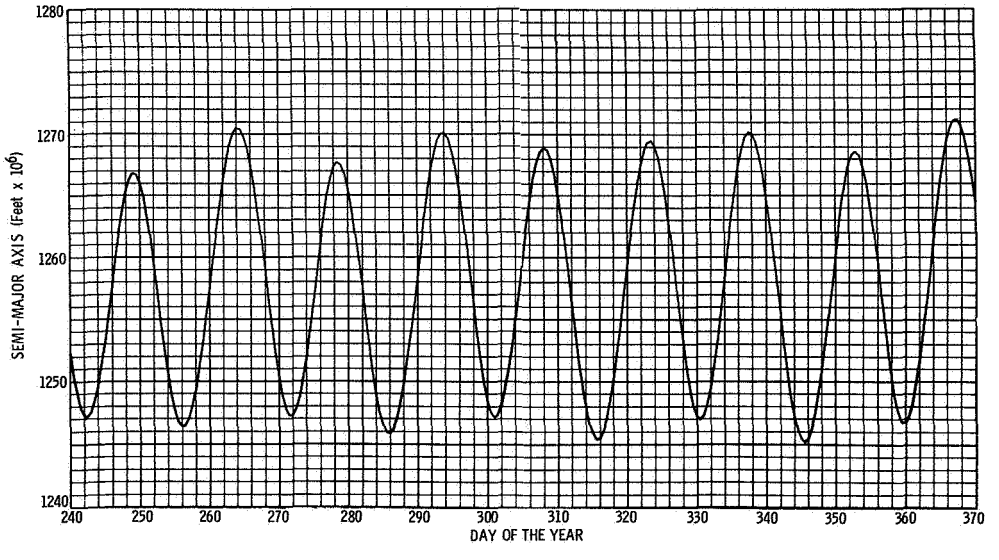


FIGURE B1983-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT

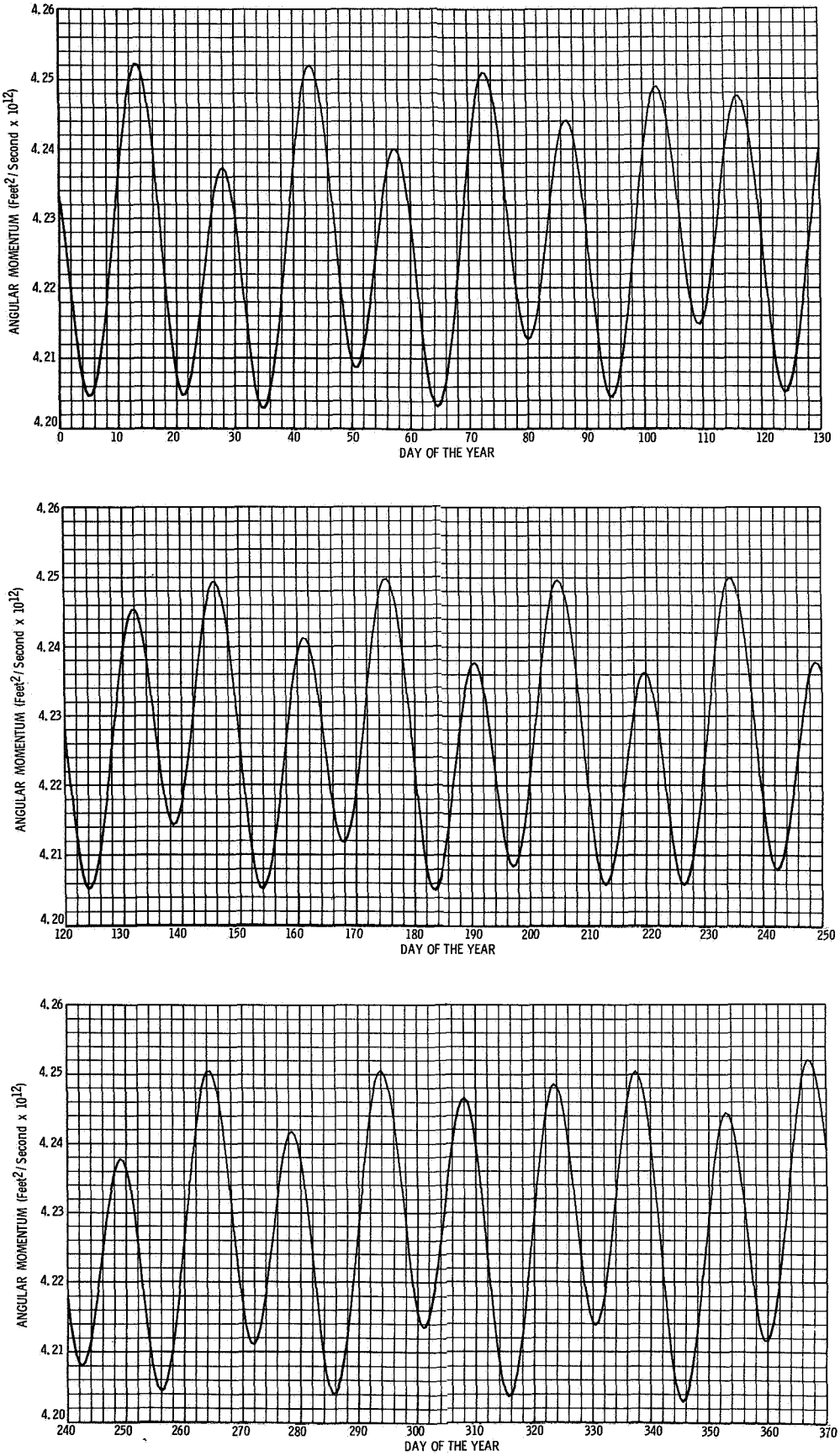


FIGURE B1983-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON

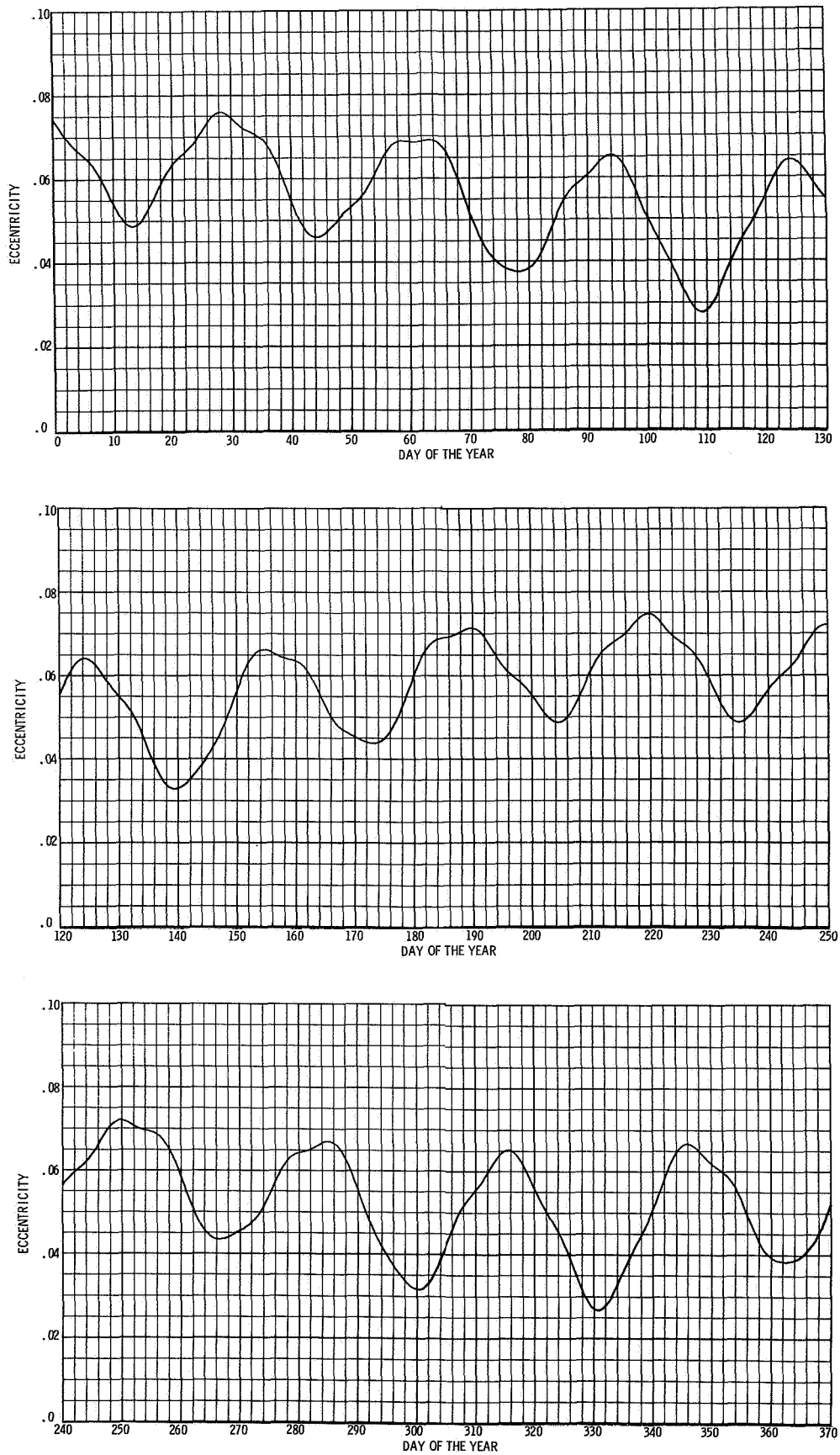
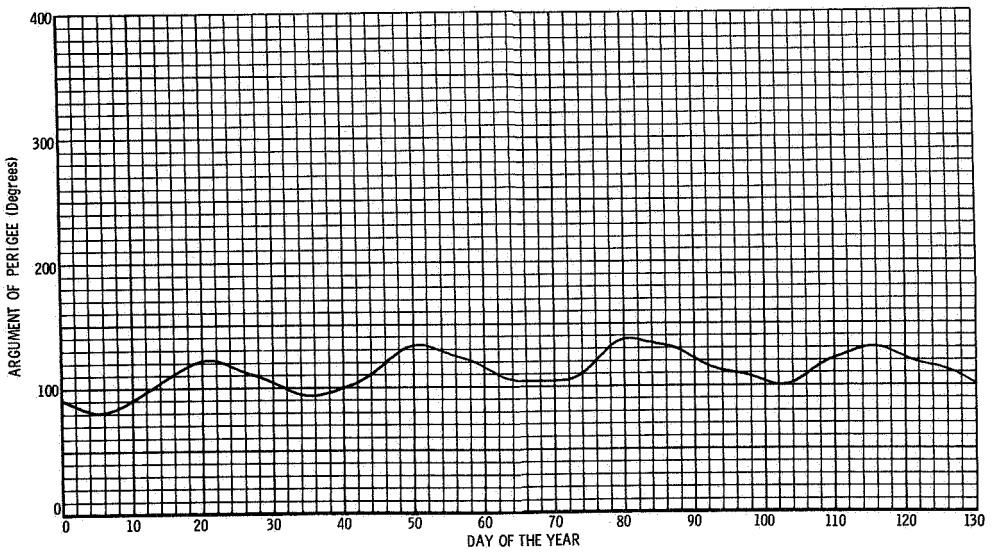
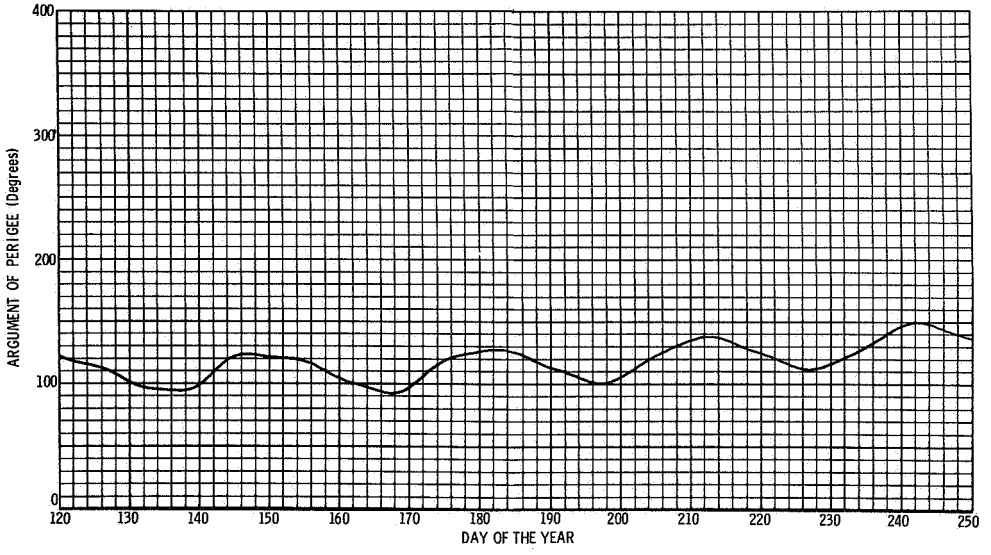


FIGURE B1983-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT





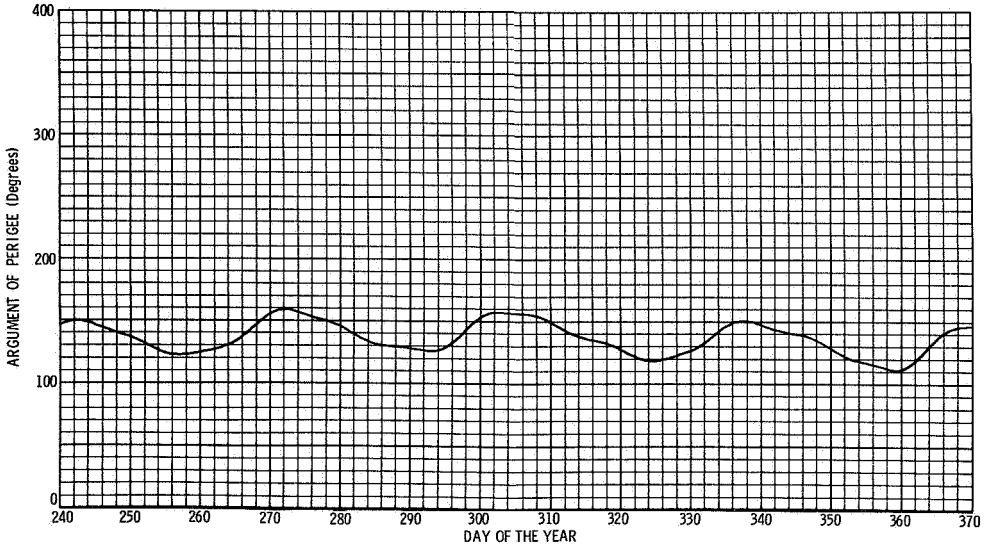
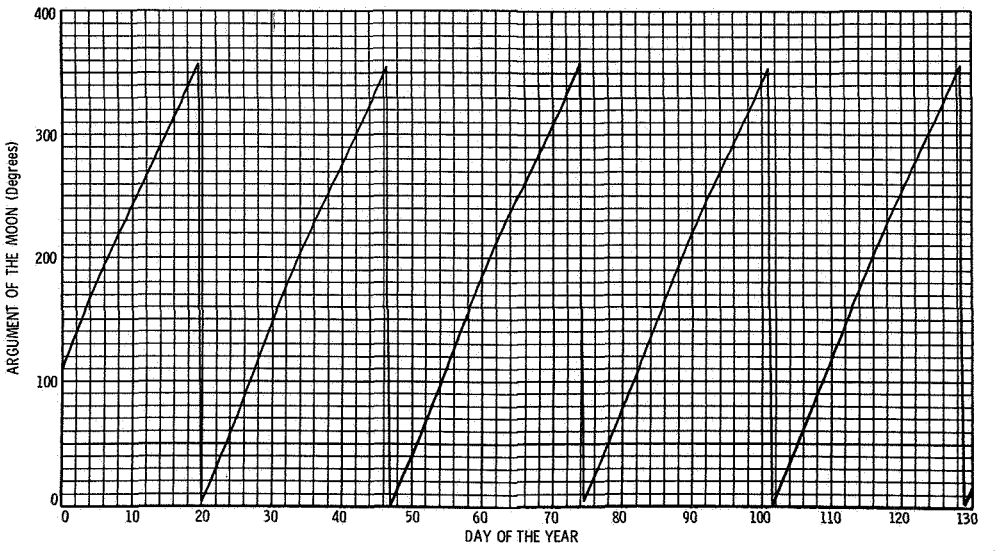
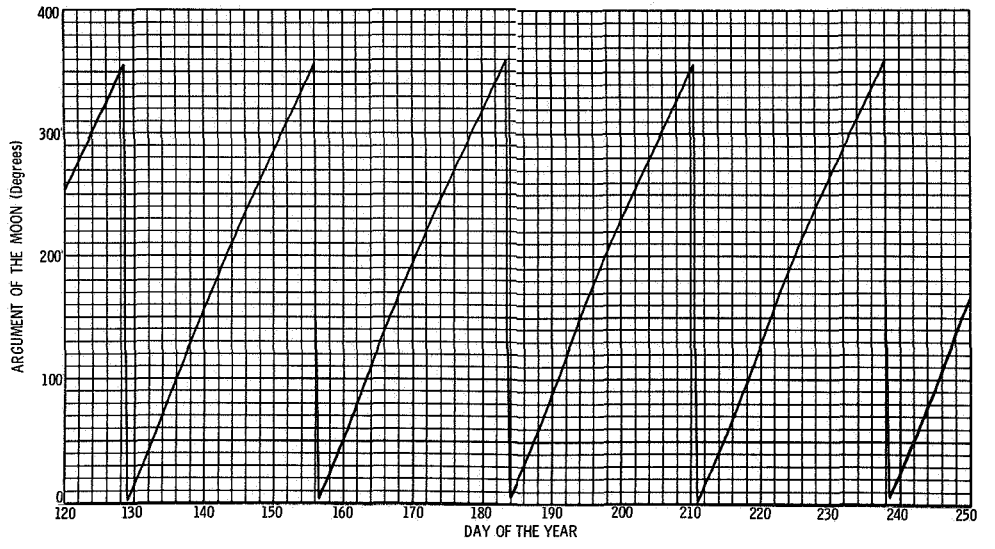


FIGURE B1983-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE





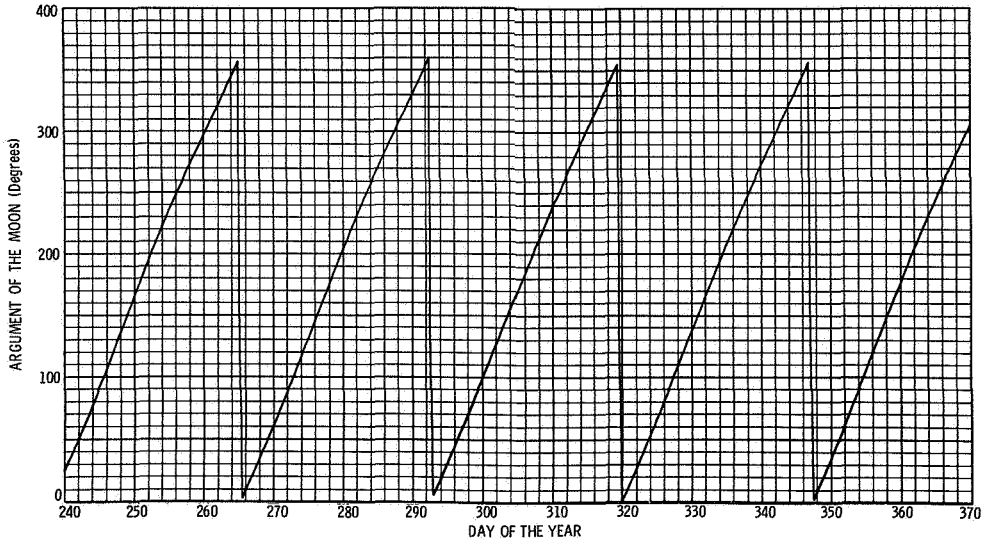


FIGURE B1983-10 ARGUMENT OF THE MOON'S POSITION

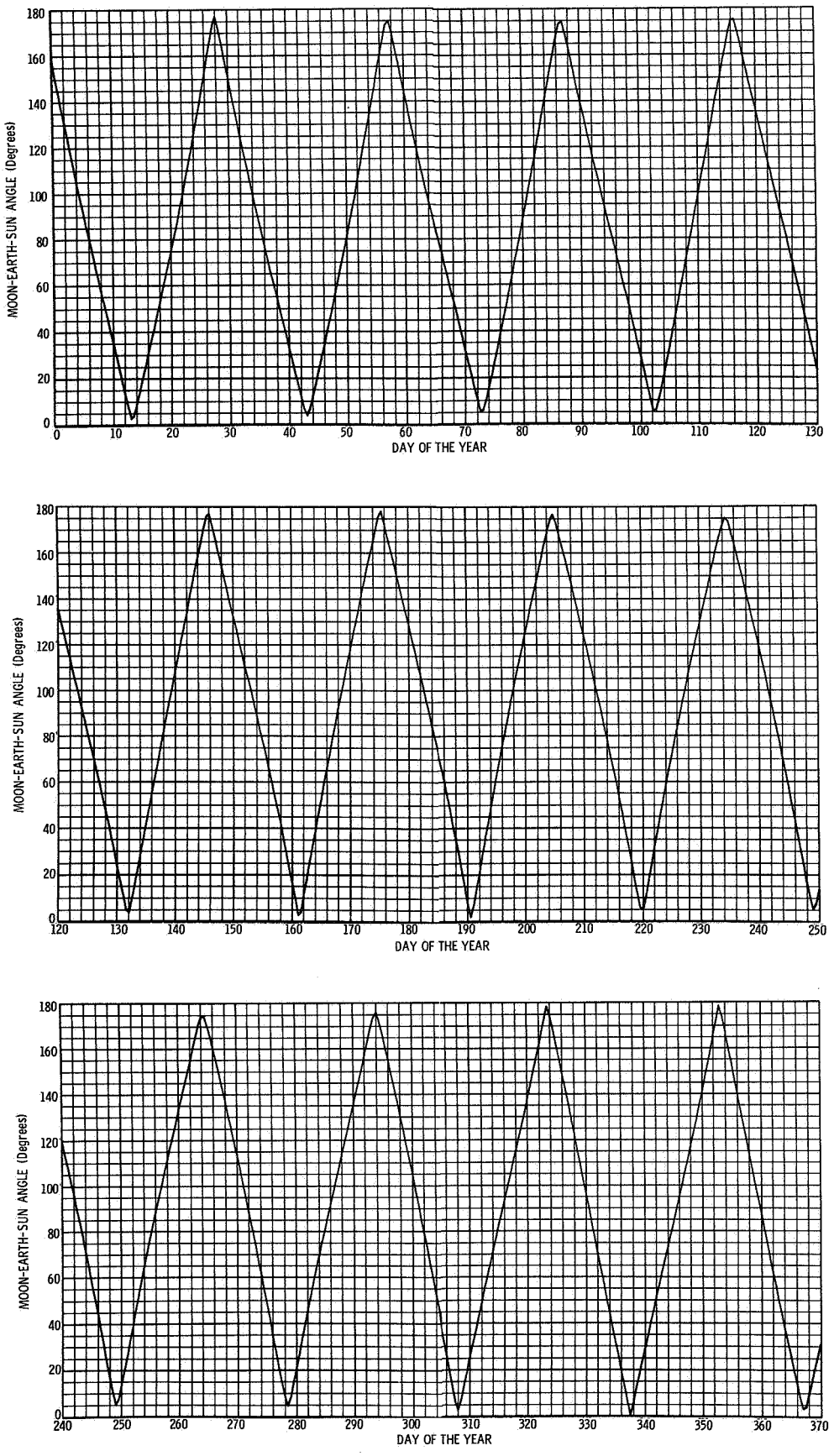


FIGURE B1983-11 MOON-EARTH-SUN ANGLE

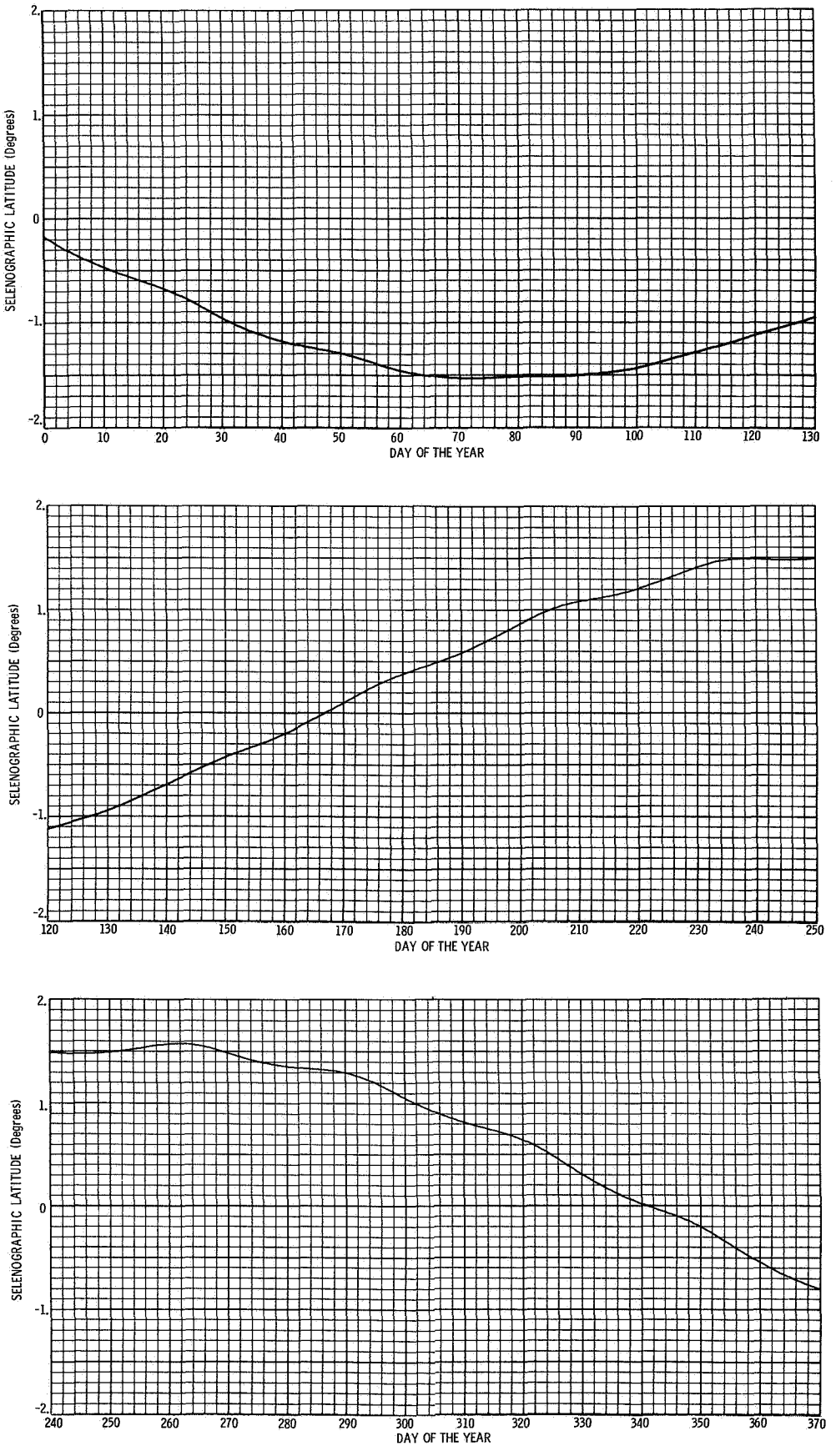
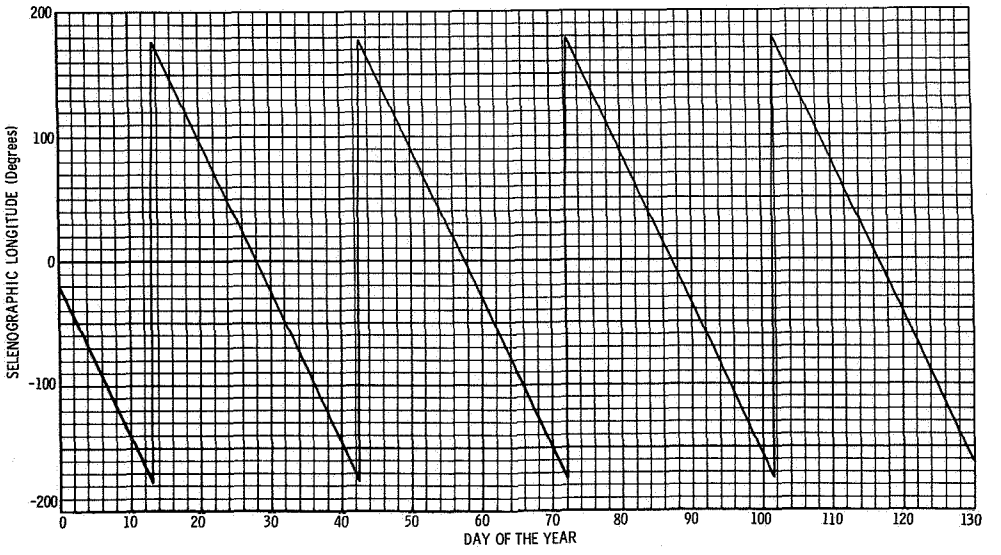
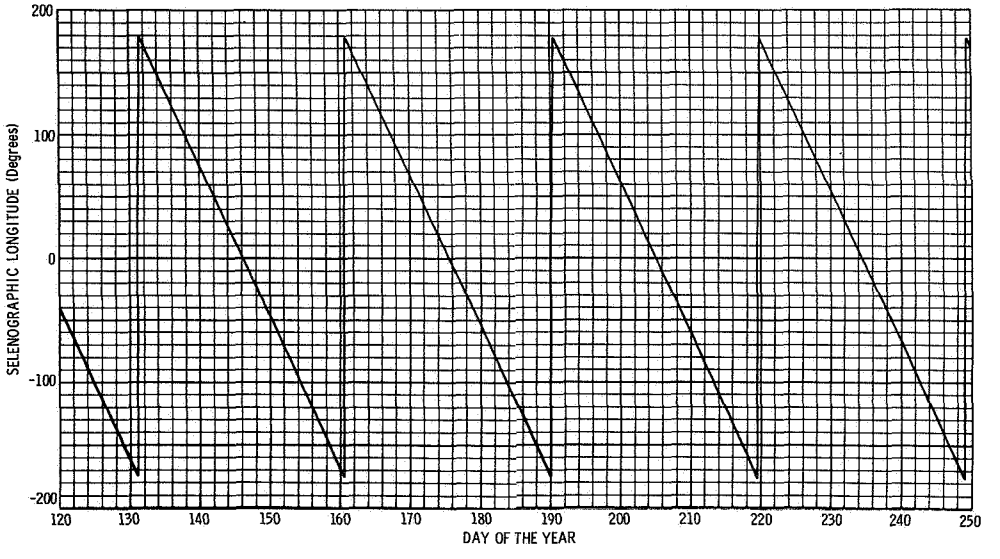


FIGURE B1983-12 SELENOGRAPHIC LATITUDE OF THE SUN





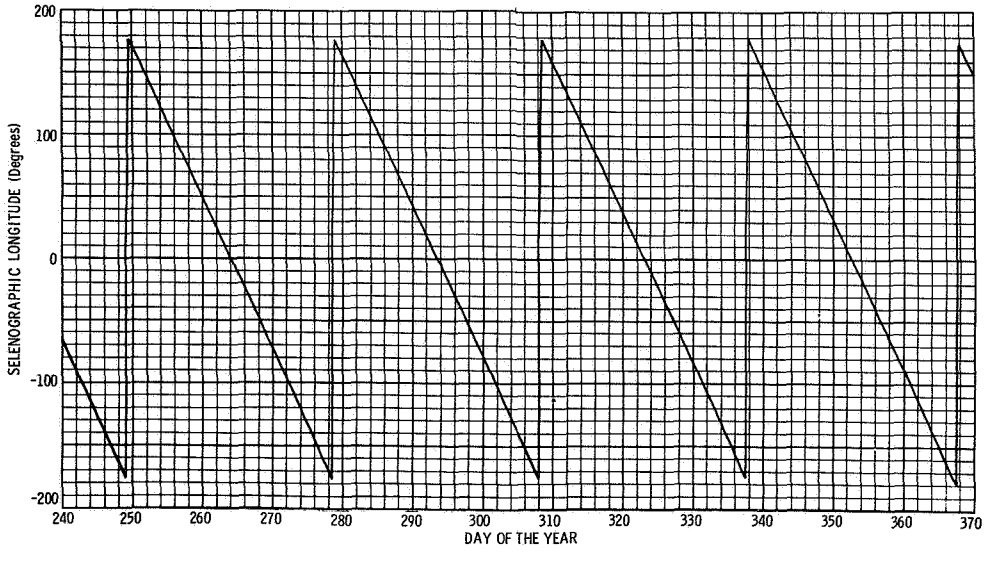


FIGURE B1983-13 SELENOGRAPHIC LONGITUDE OF THE SUN

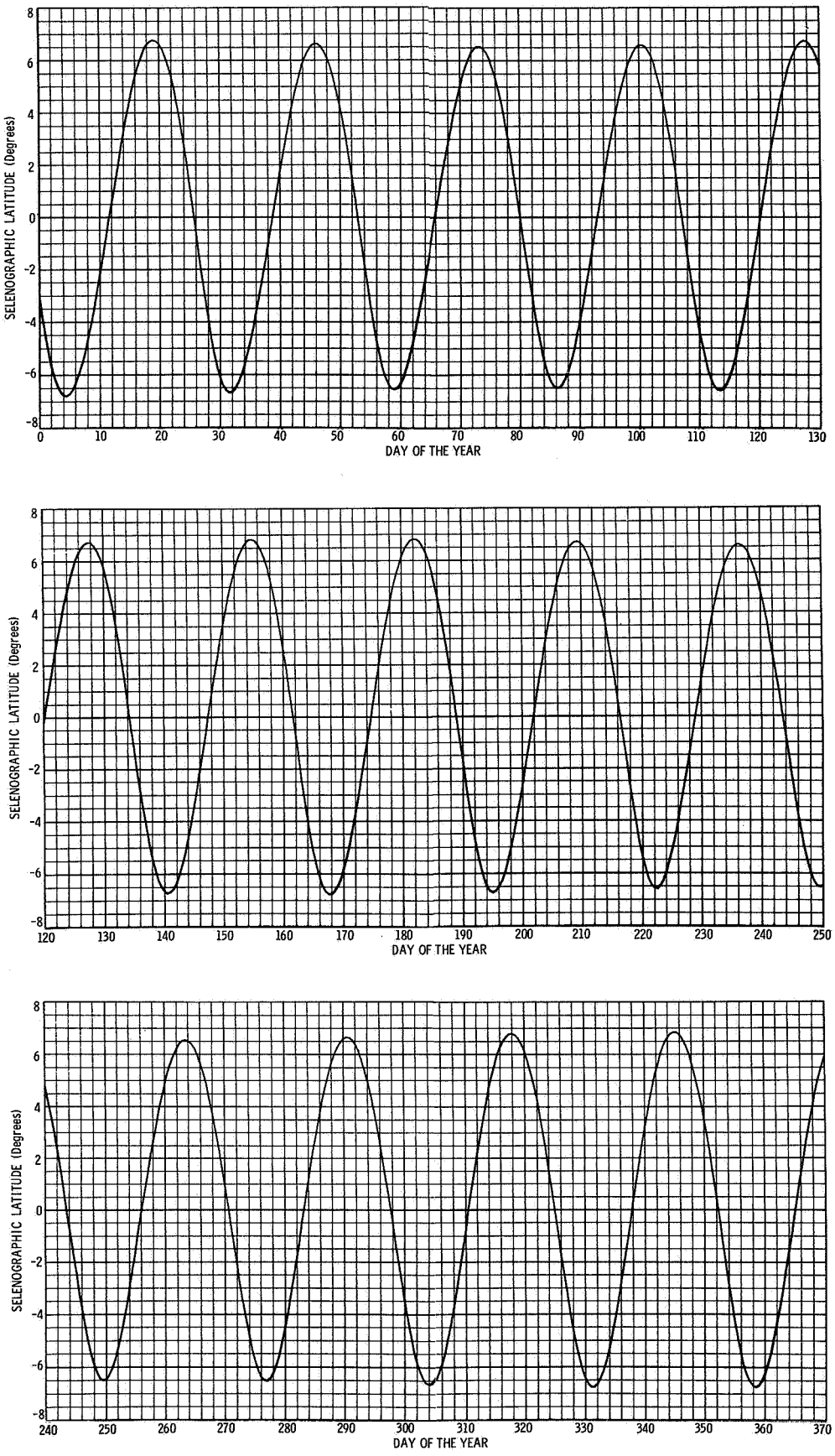
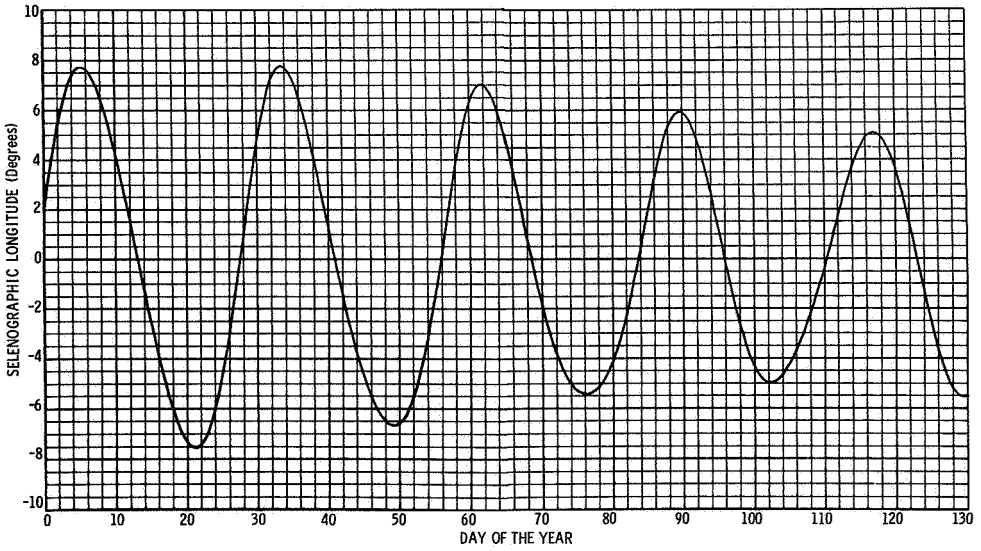
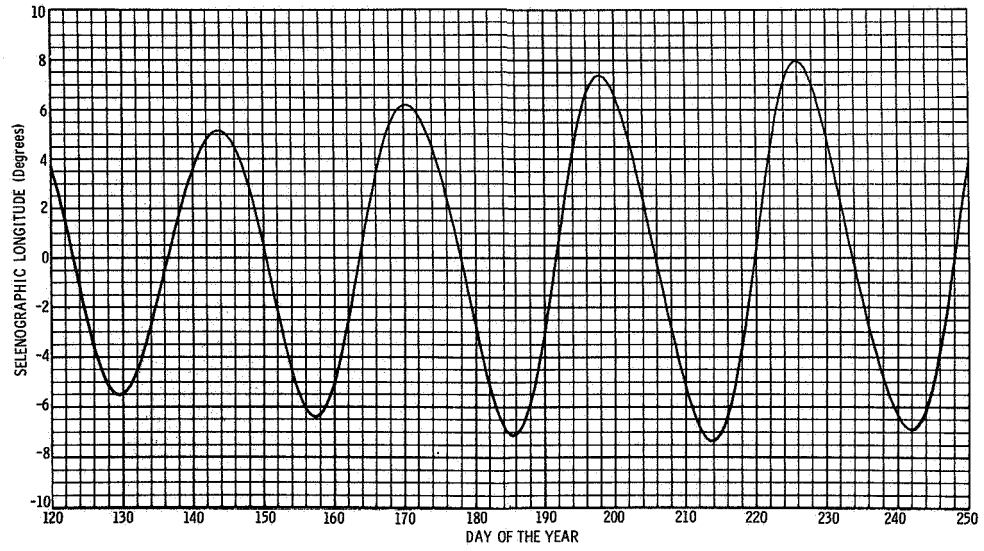


FIGURE B1983-14 SELENOGRAPHIC LATITUDE OF THE EARTH





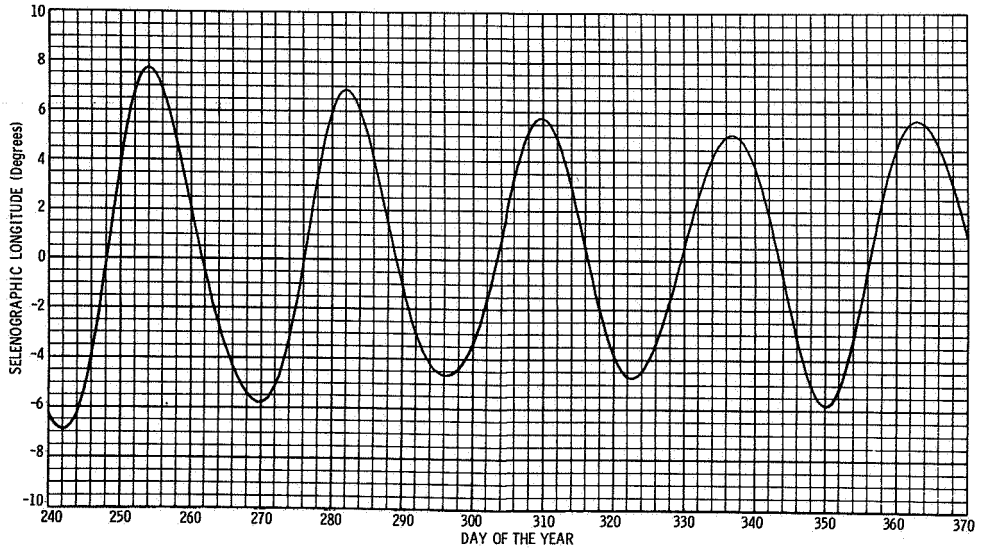
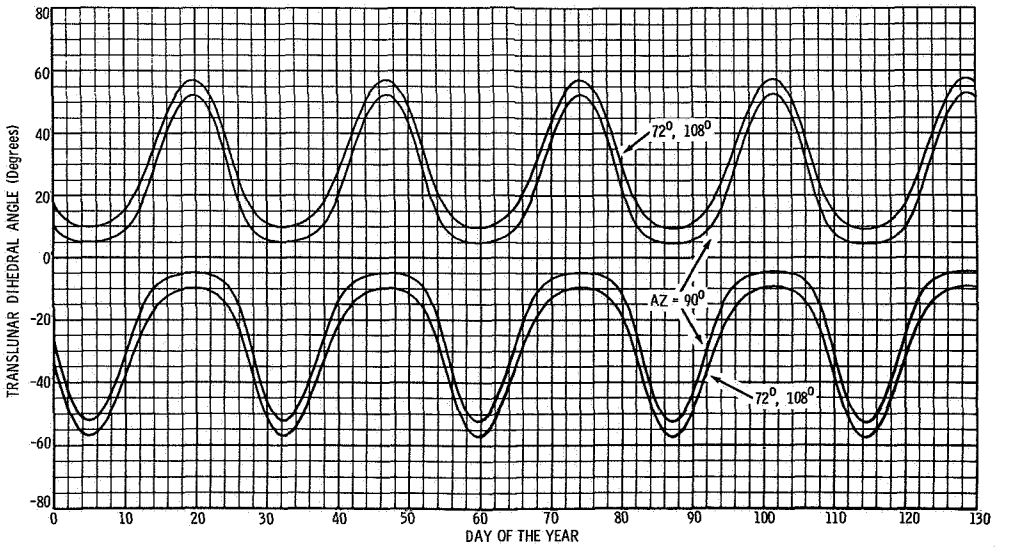
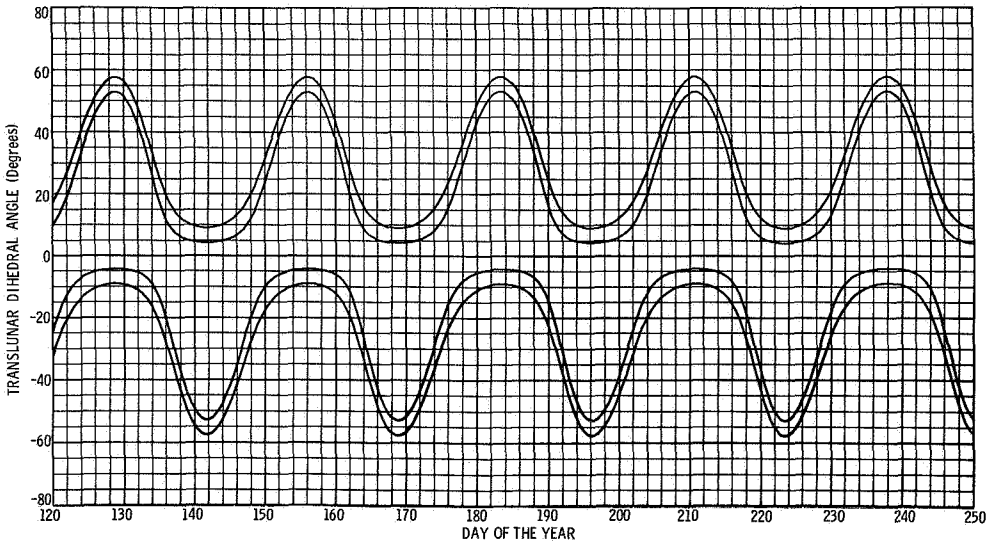


FIGURE B1983-15 SELENOGRAPHIC LONGITUDE OF THE EARTH





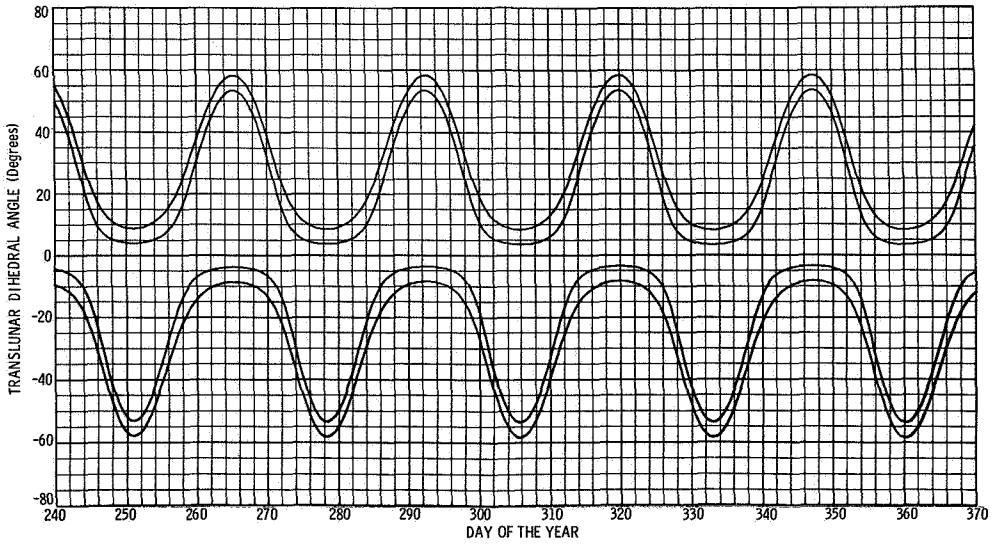
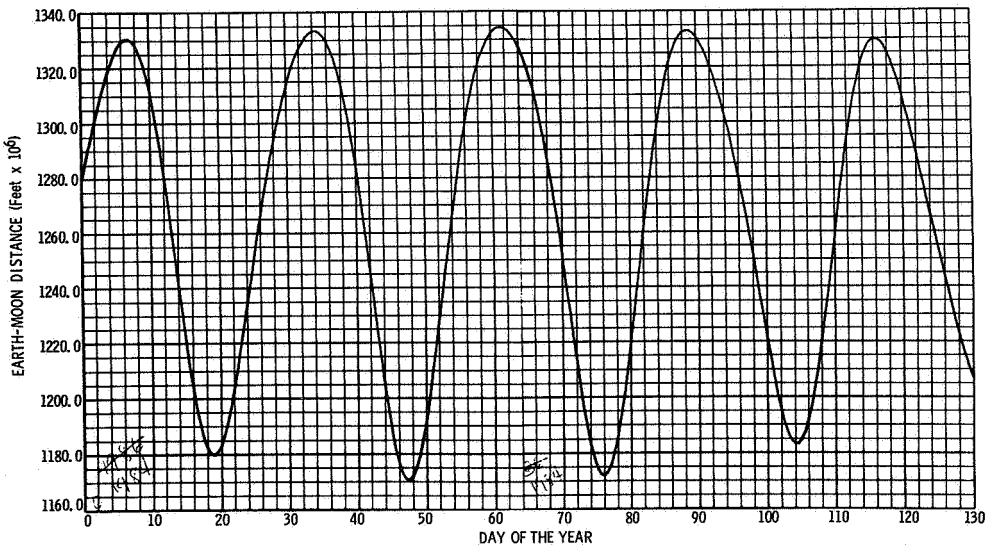
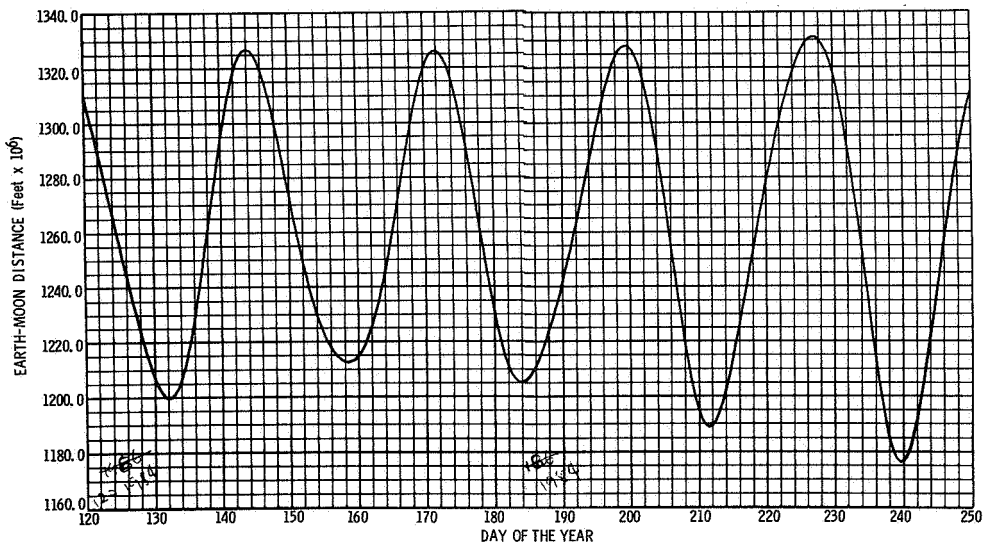


FIGURE B1983-16 TRANSLUNAR DIHEDRAL ANGLES

1984





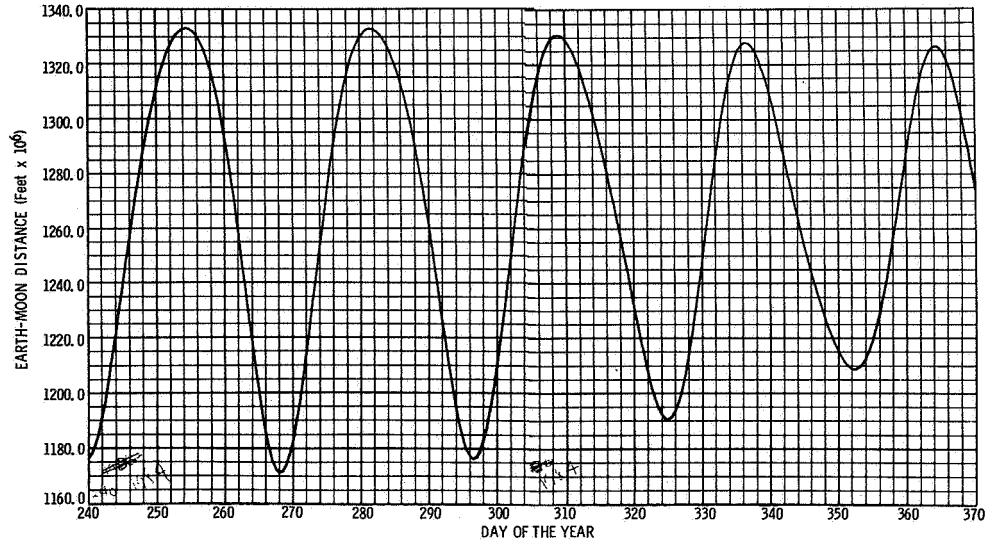
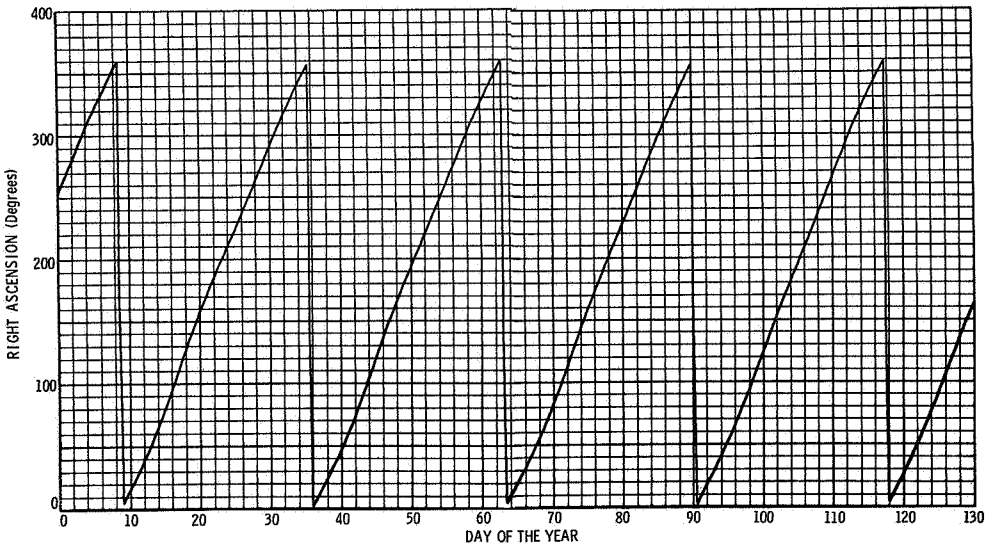
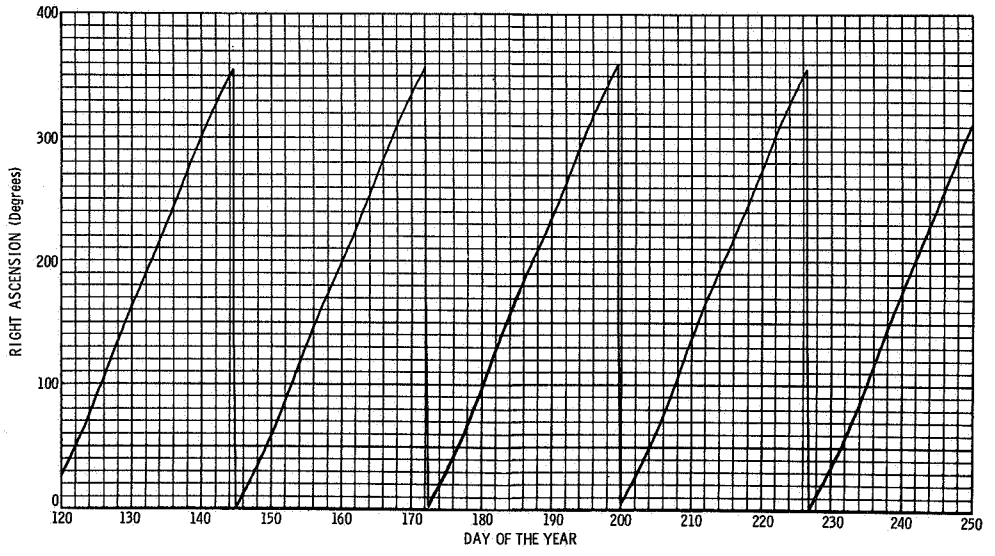


FIGURE B1984-1 EARTH-MOON DISTANCE





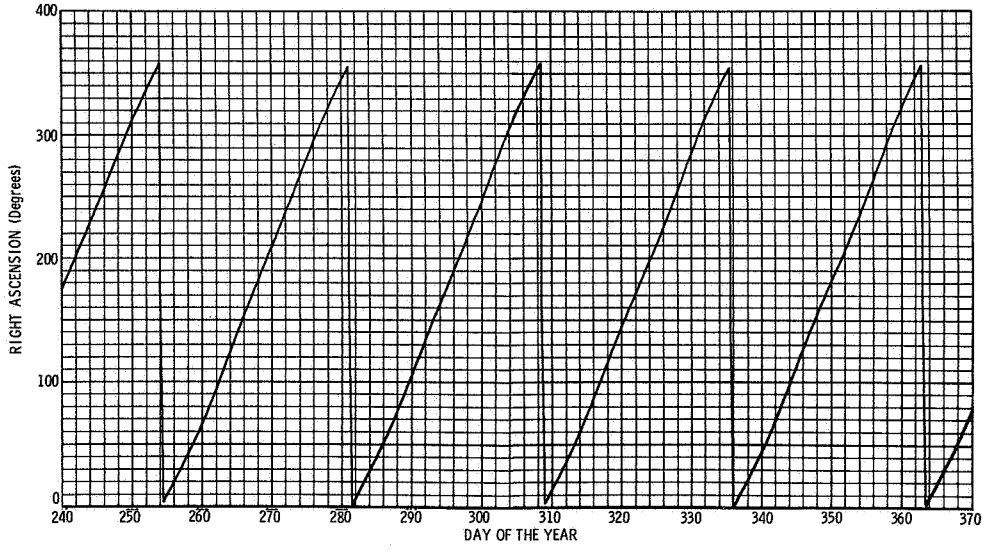
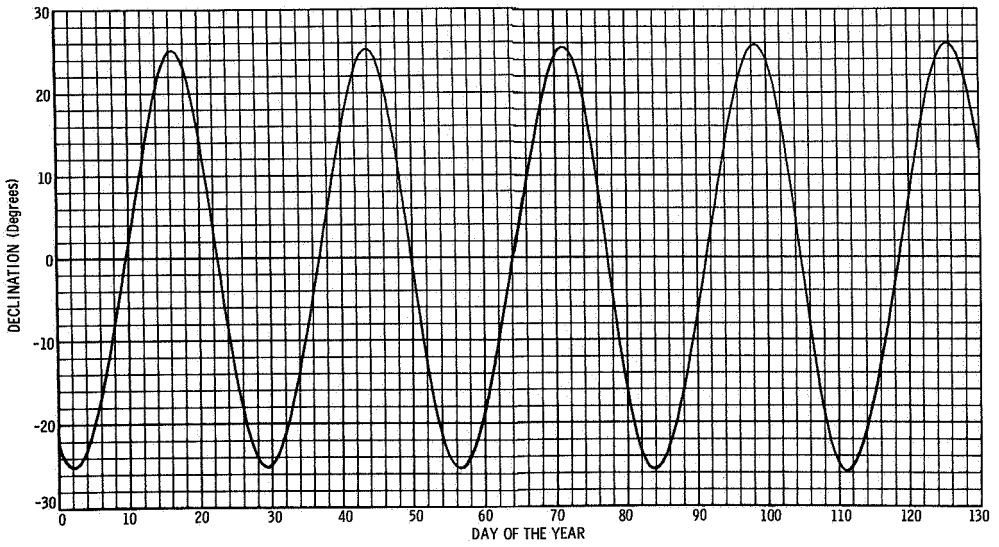
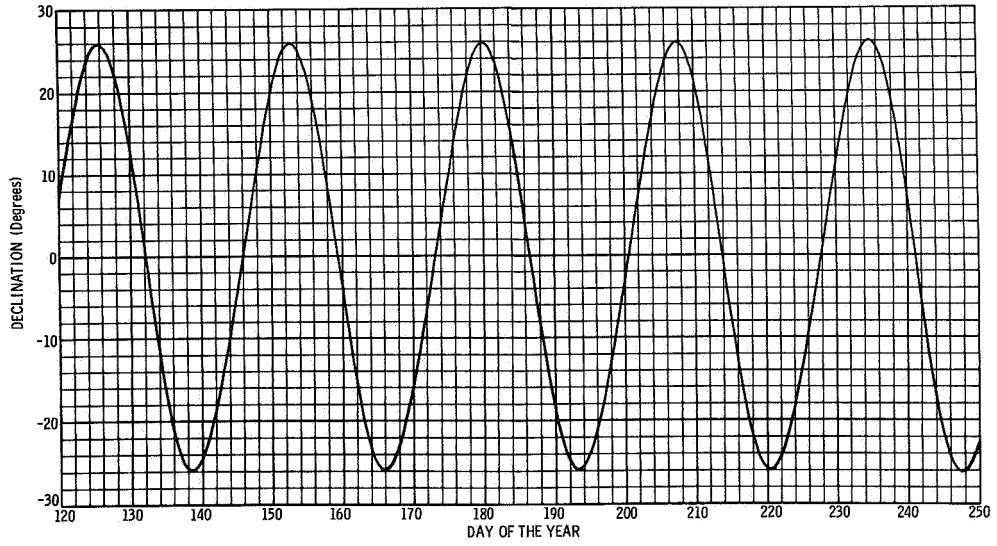


FIGURE B1984-2 RIGHT ASCENSION OF THE MOON





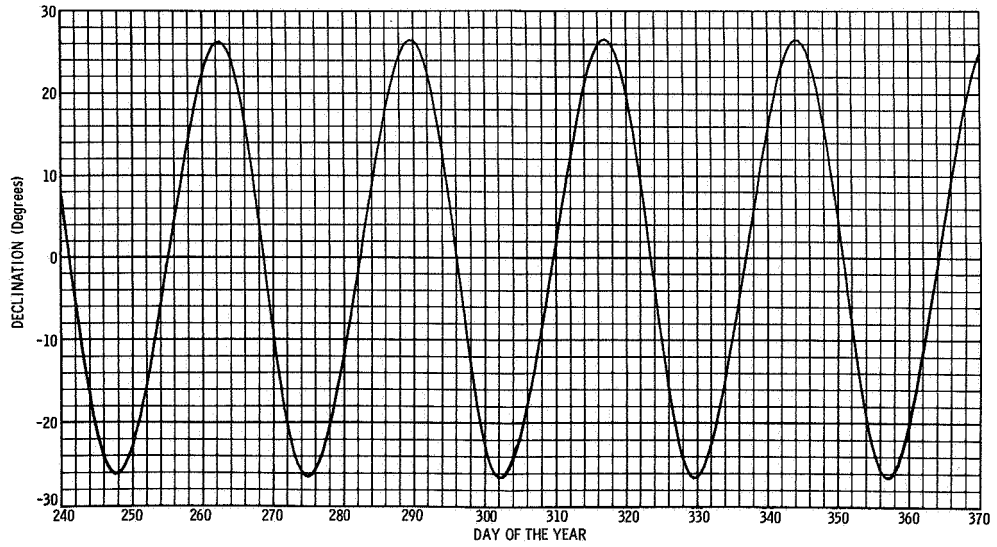
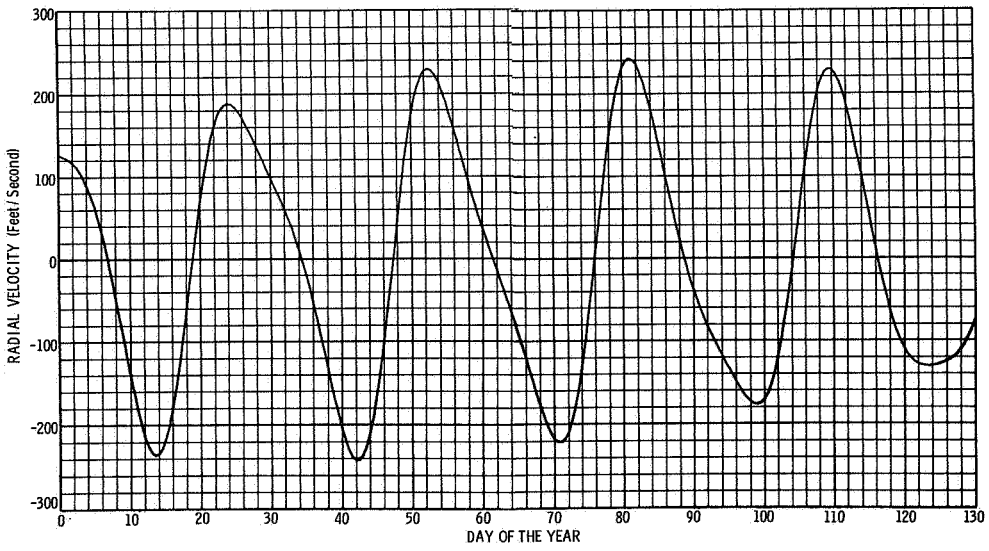
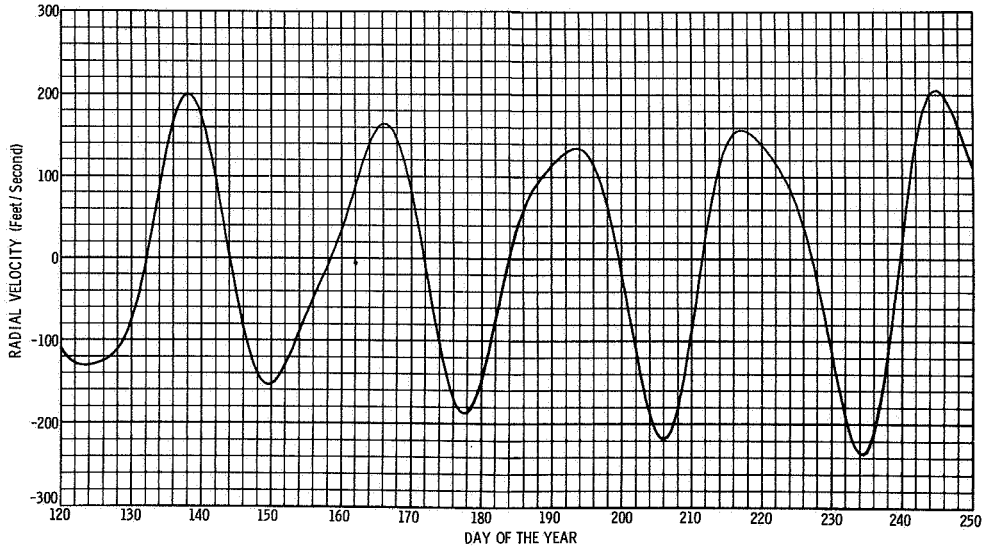


FIGURE B1984-3 DECLINATION OF THE MOON





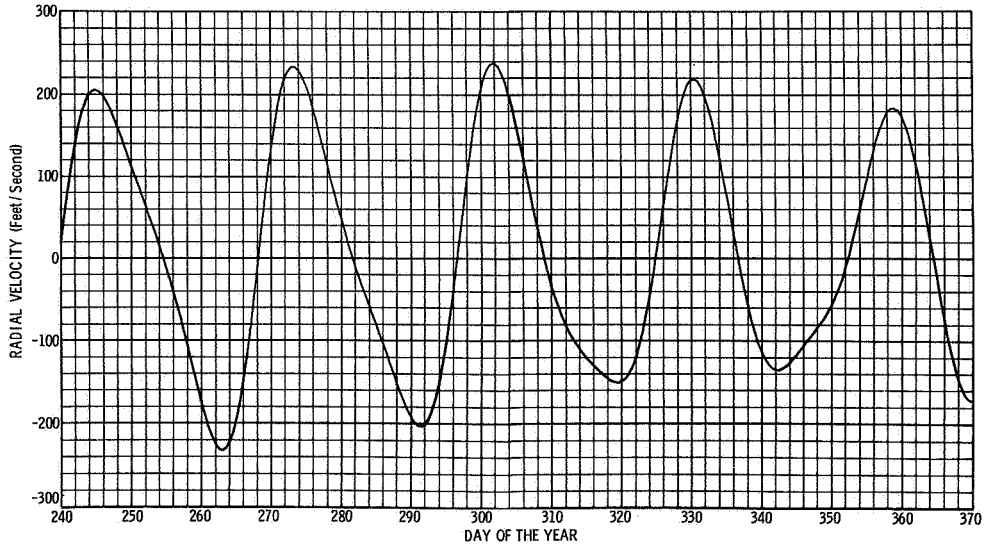
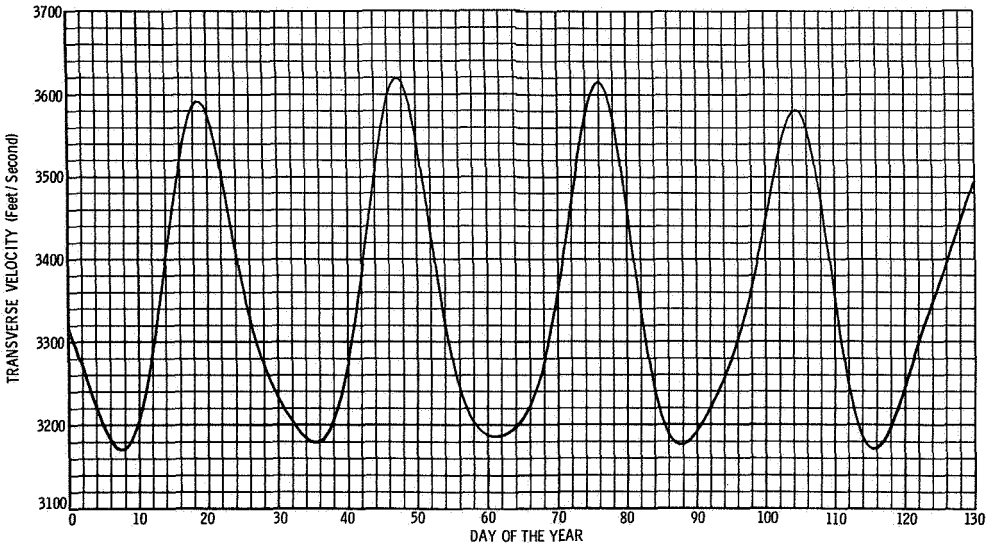
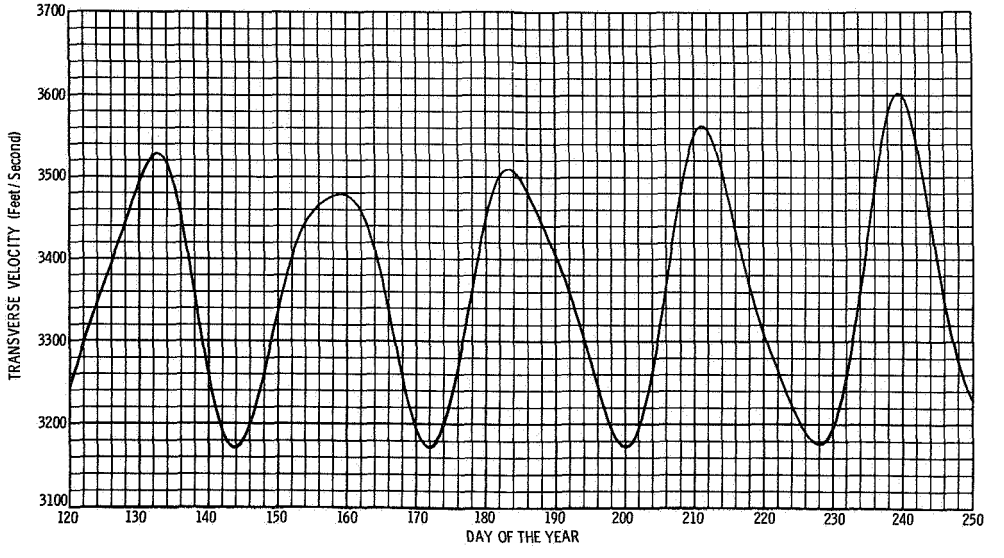


FIGURE B1984-4 RADIAL VELOCITY OF THE MOON





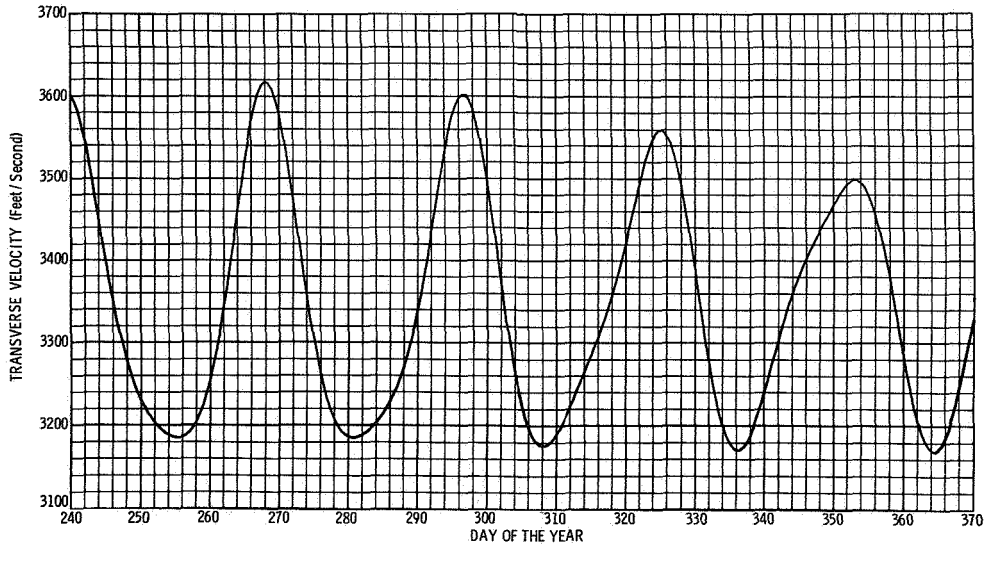
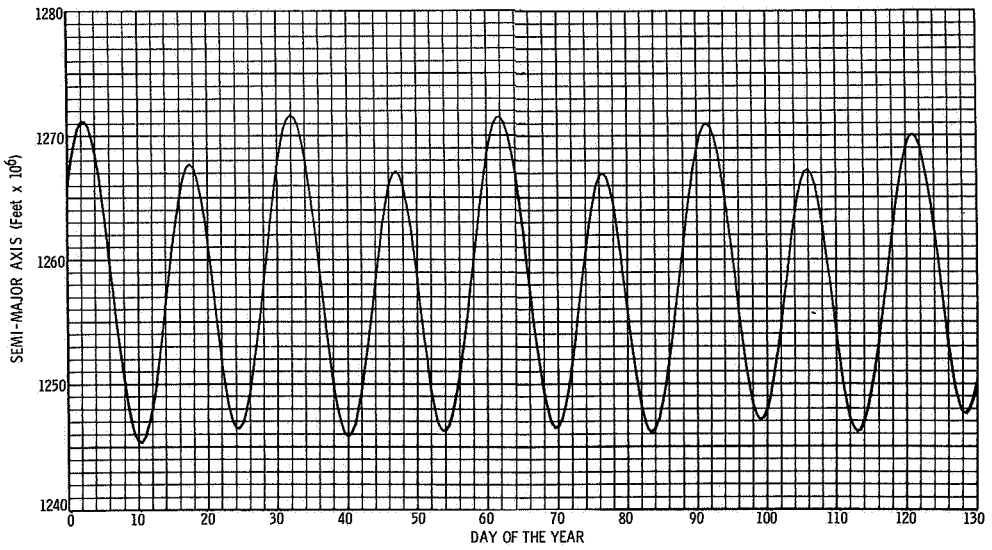
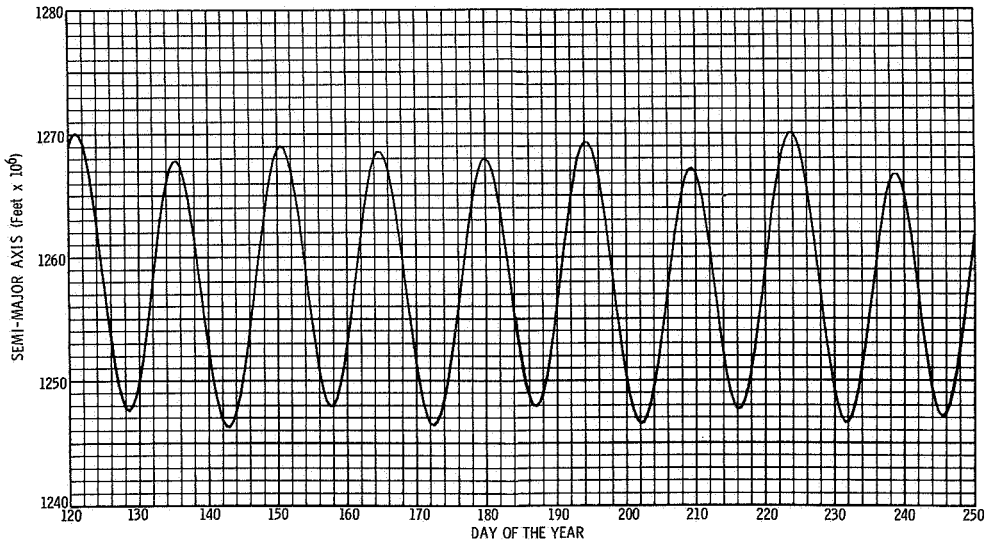


FIGURE B1984-5 TRANSVERSE VELOCITY OF THE MOON





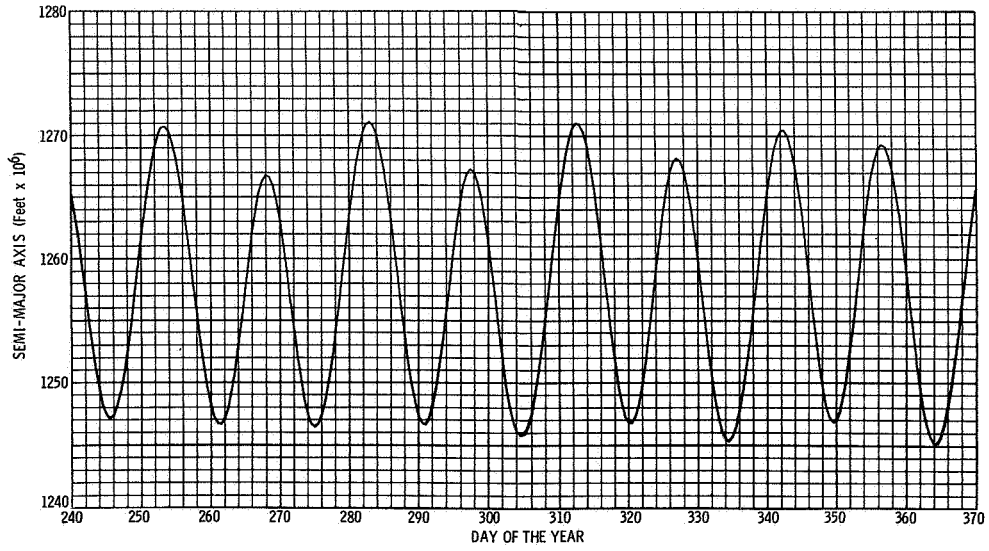


FIGURE B1984-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT

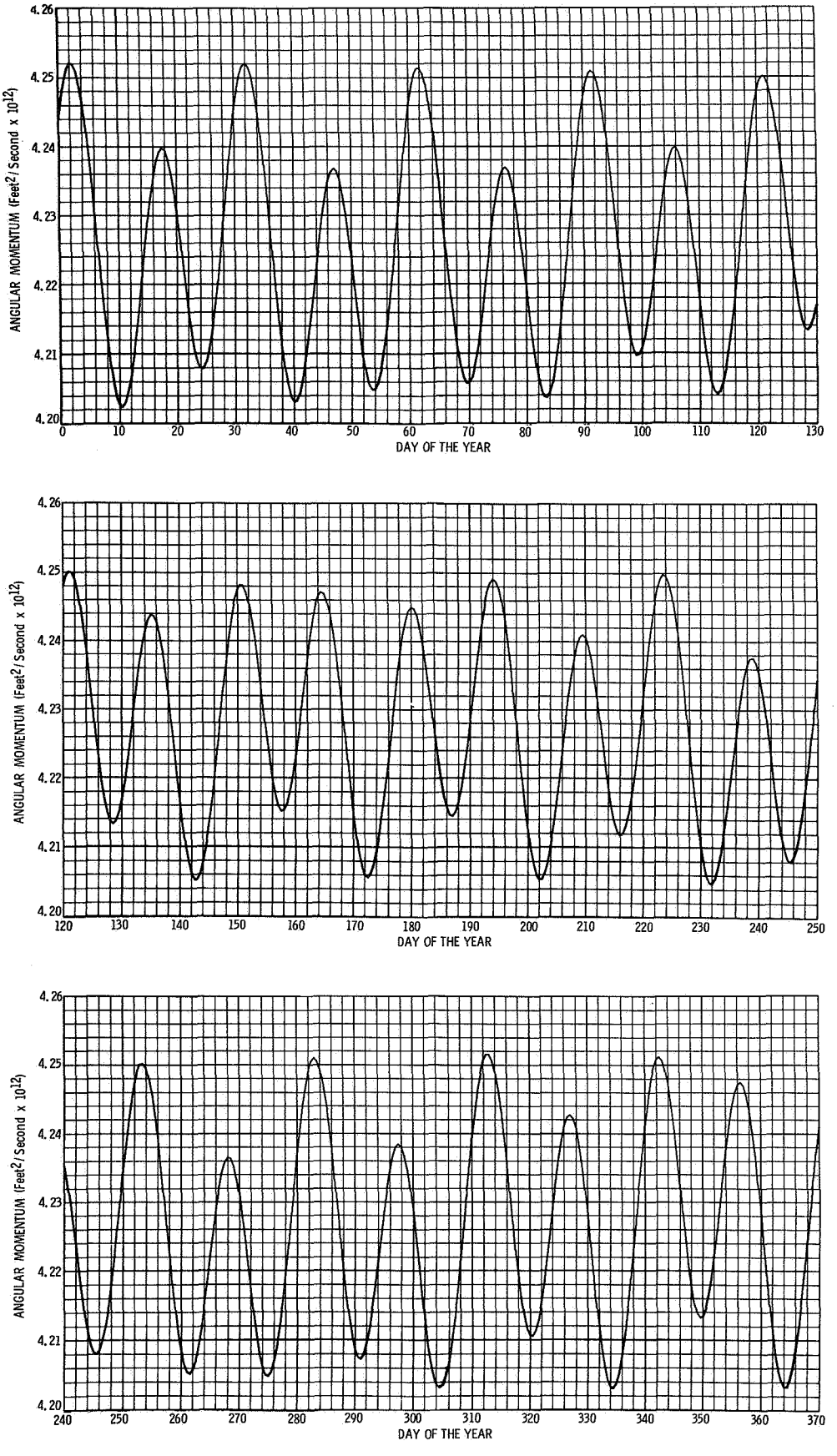


FIGURE B1984-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON

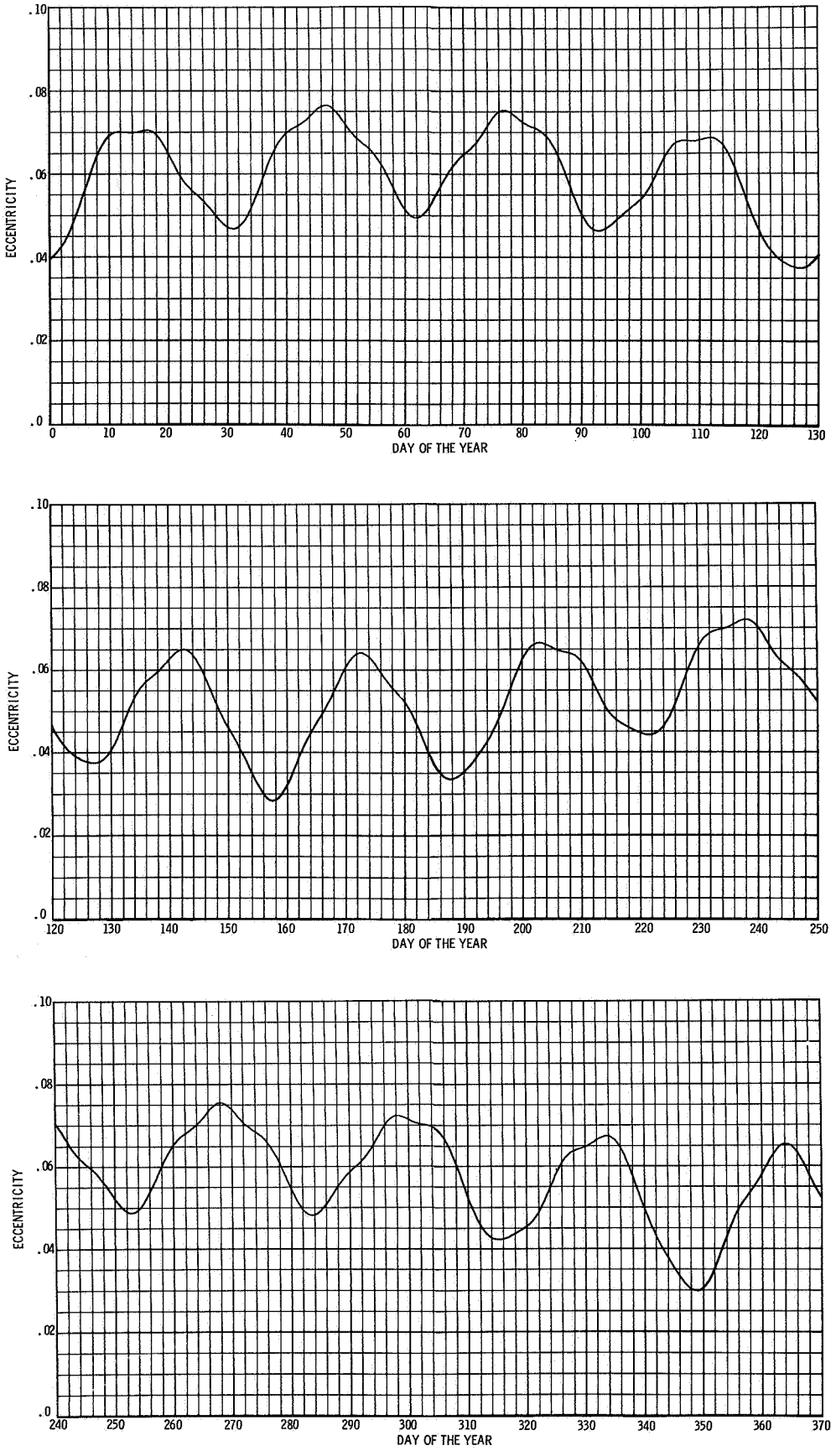


FIGURE B1984-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT

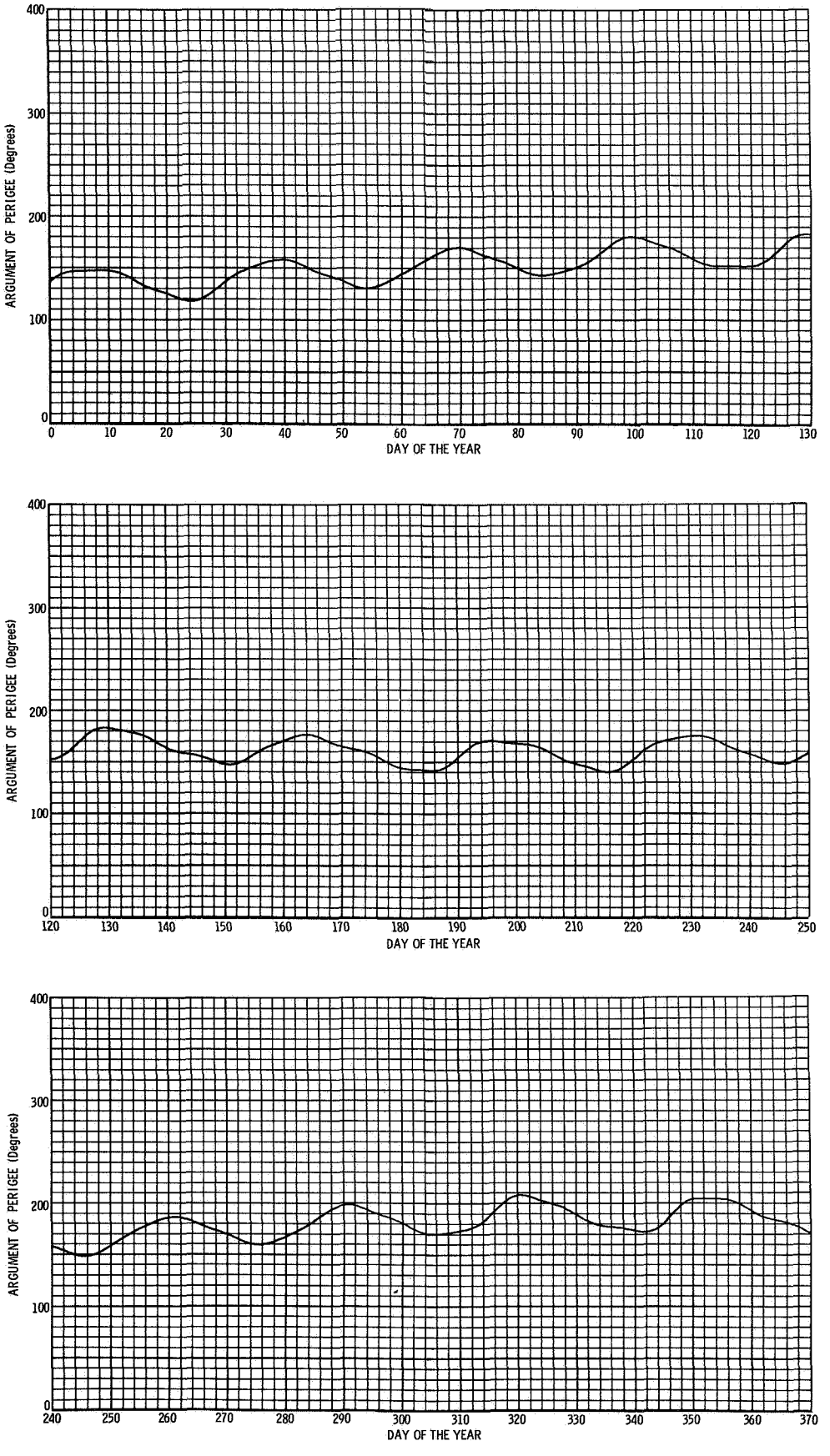
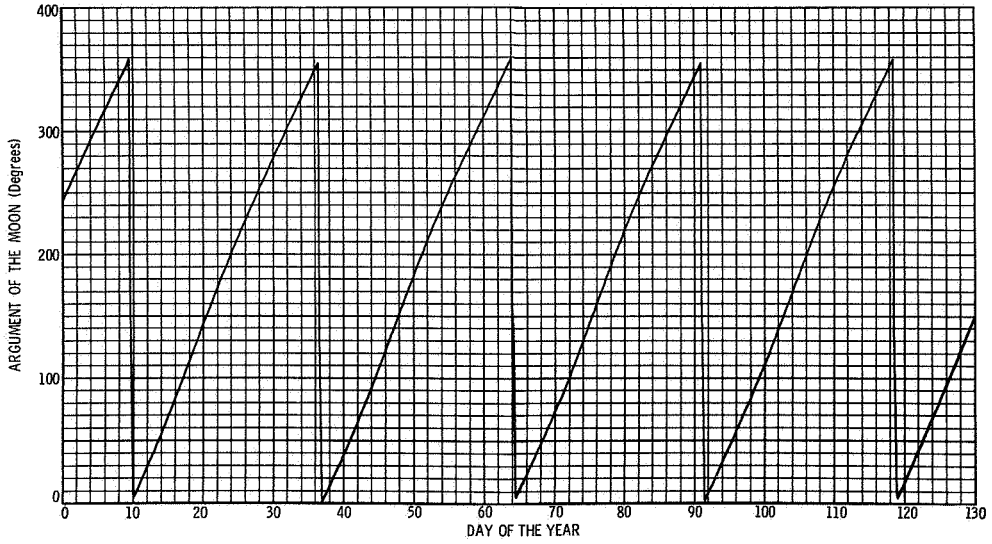
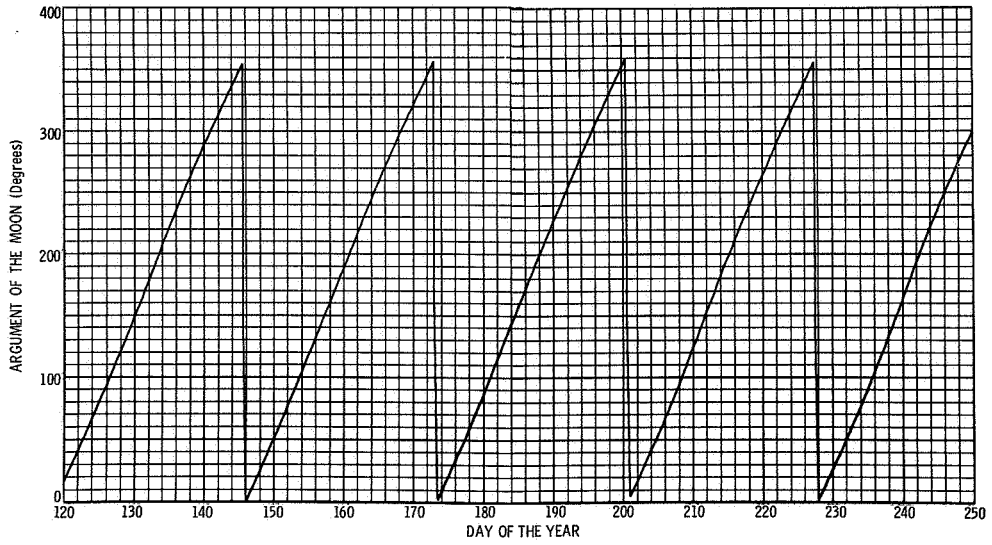


FIGURE B1984-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE





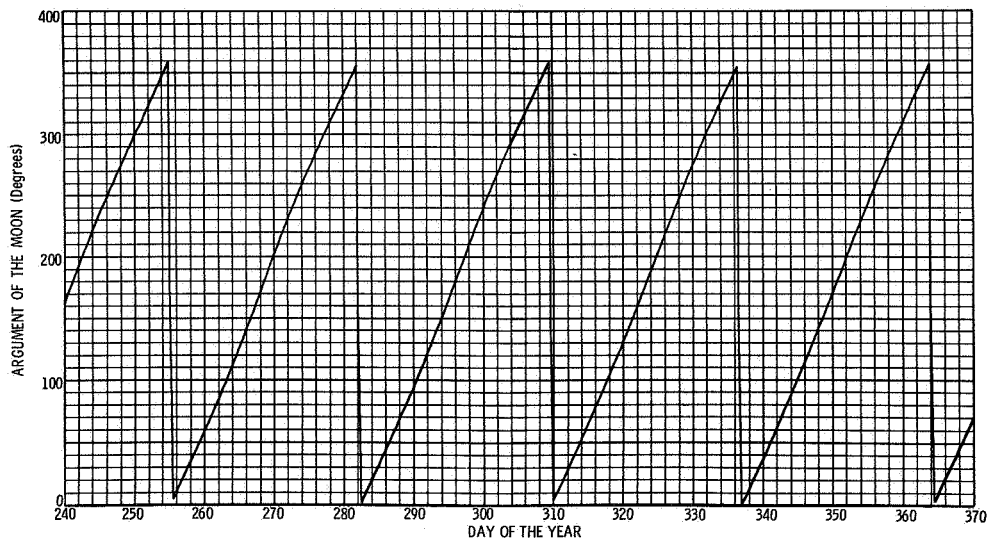
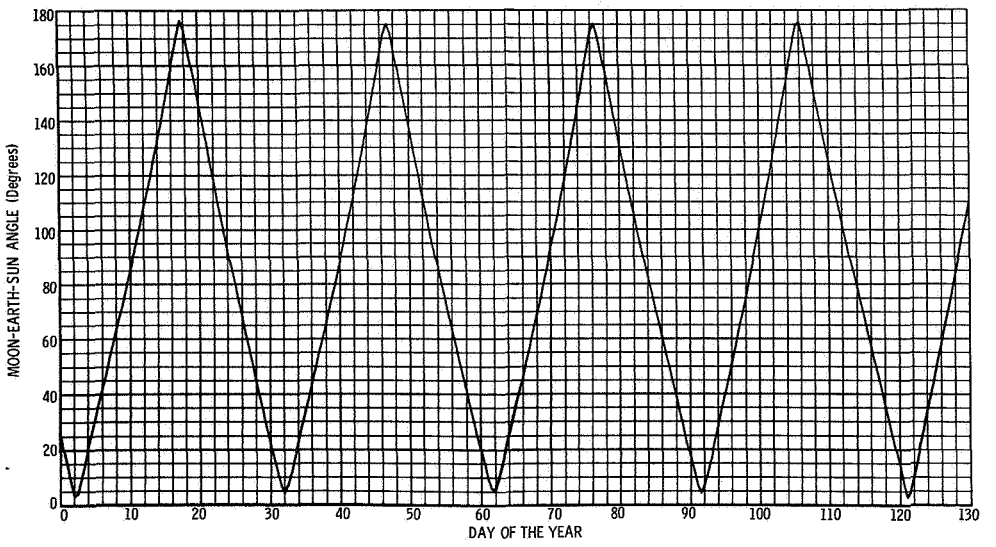
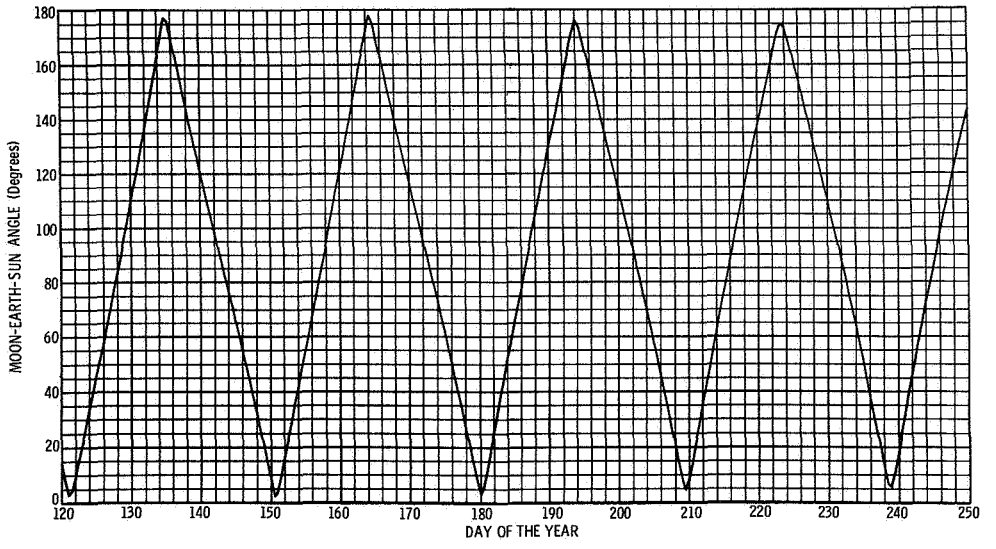


FIGURE B1984-10 ARGUMENT OF THE MOON'S POSITION





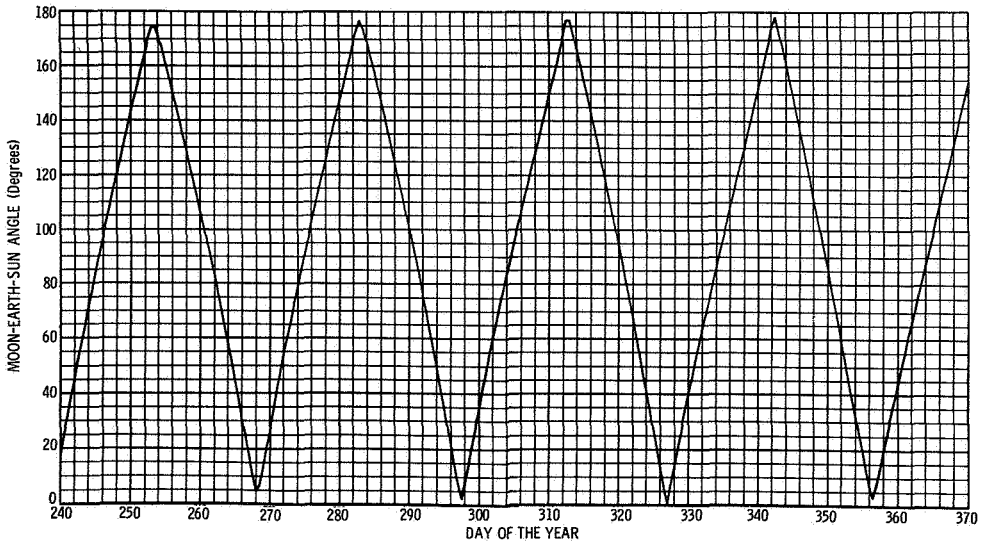
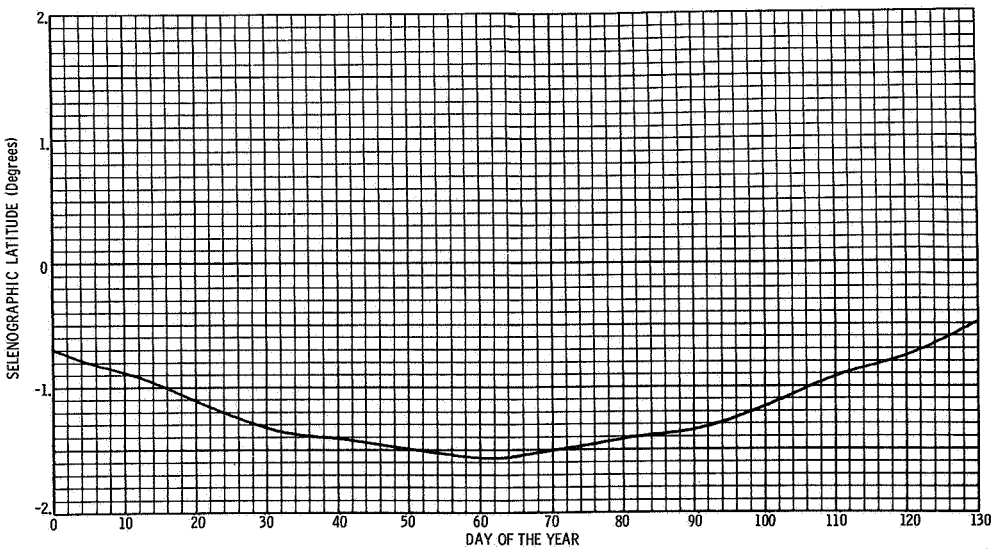
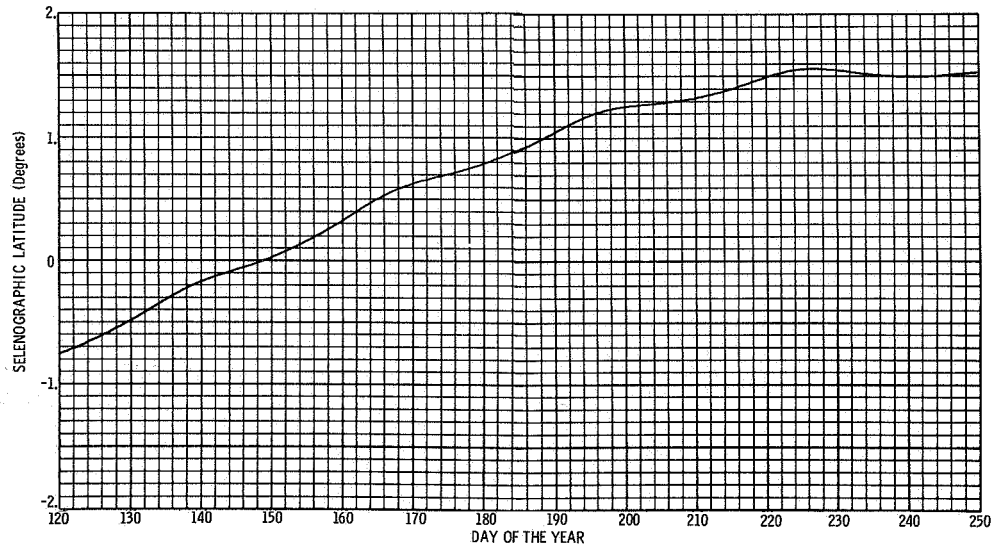


FIGURE B1984-11 MOON-EARTH-SUN ANGLE





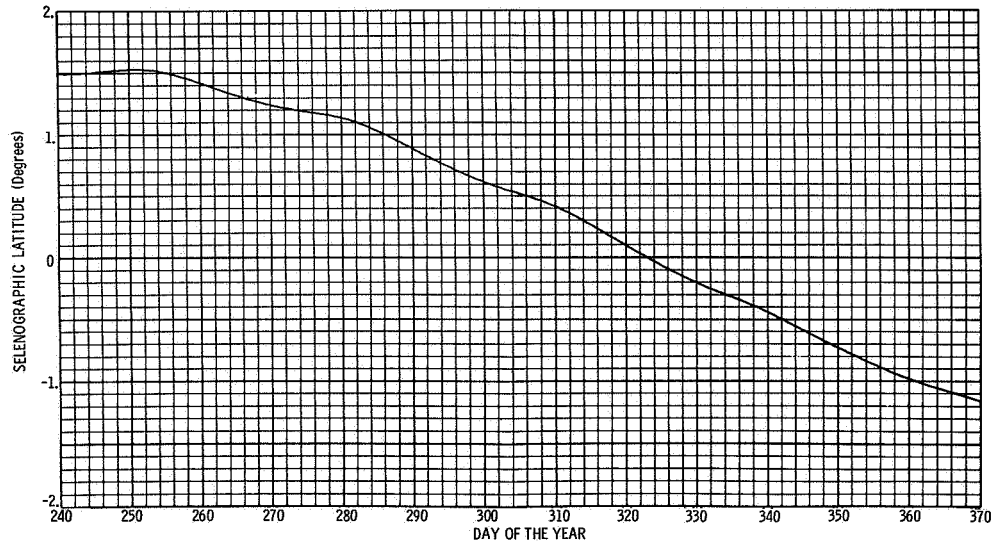
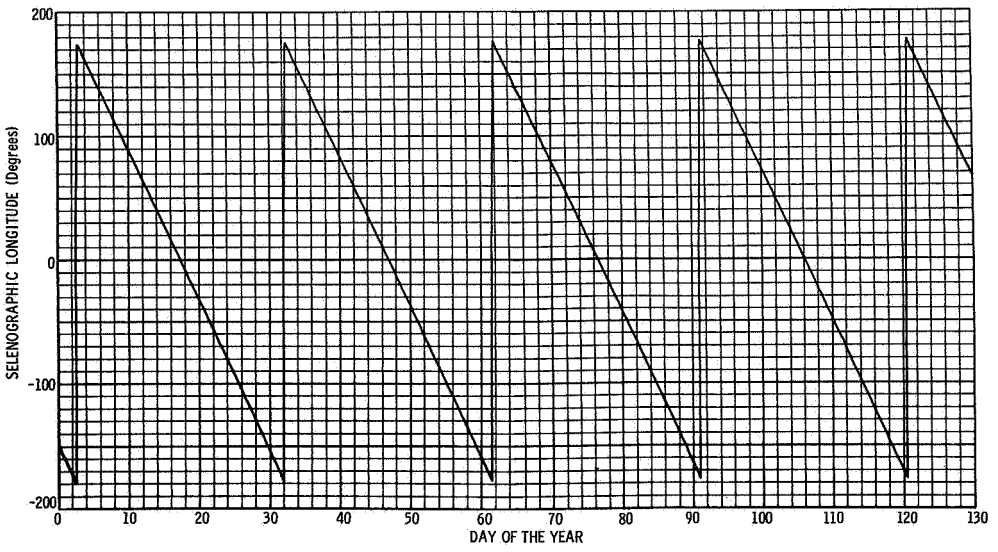
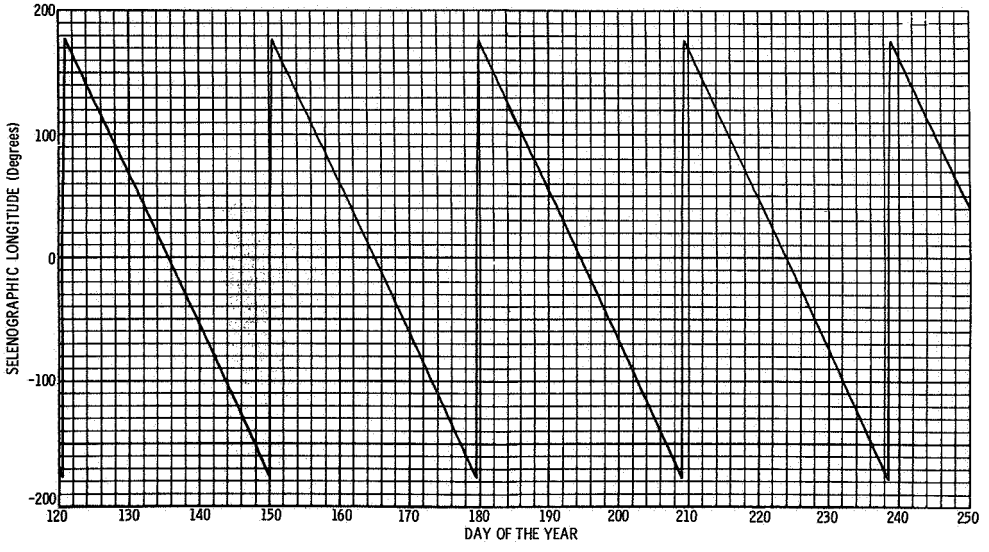


FIGURE B1984-12 SELENOGRAPHIC LATITUDE OF THE SUN





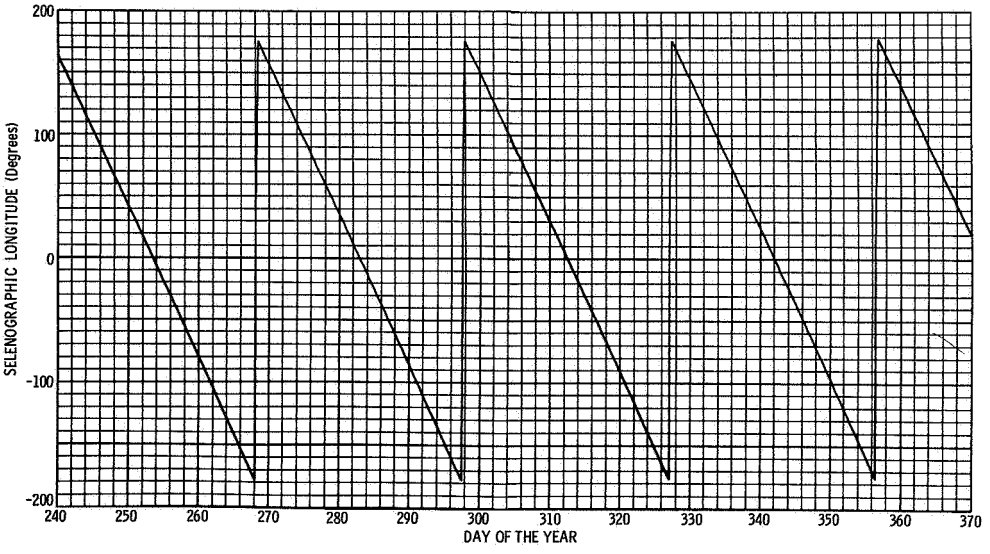
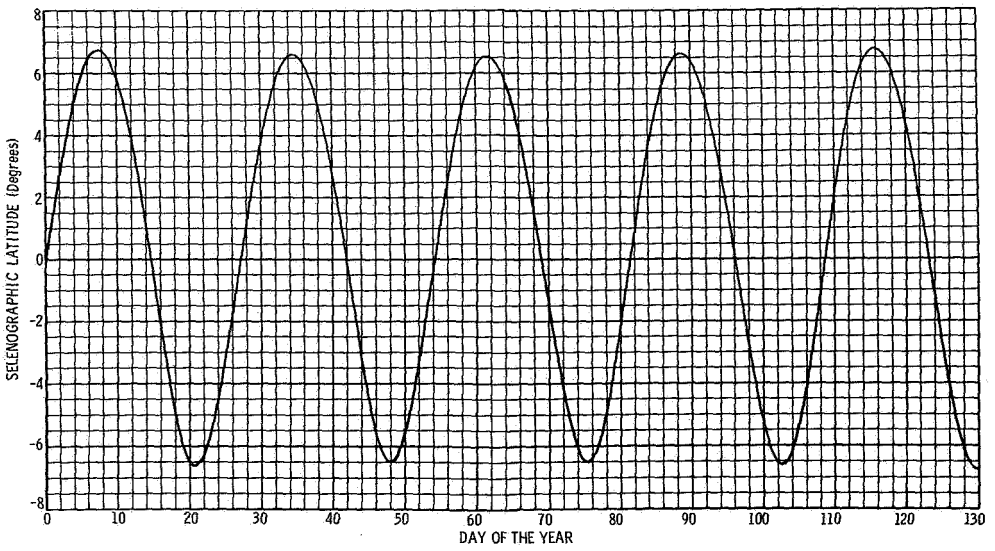
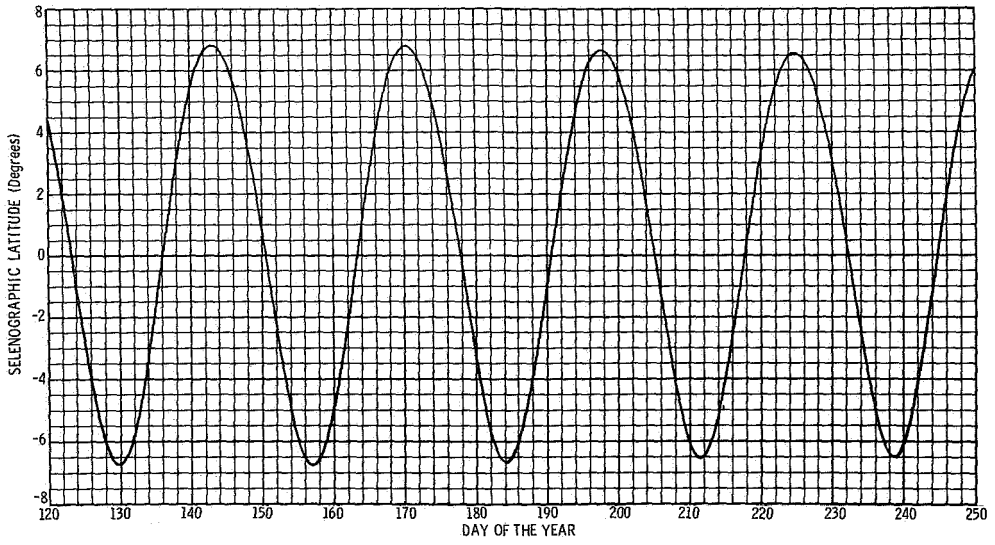


FIGURE B1984-13 SELENOGRAPHIC LONGITUDE OF THE SUN





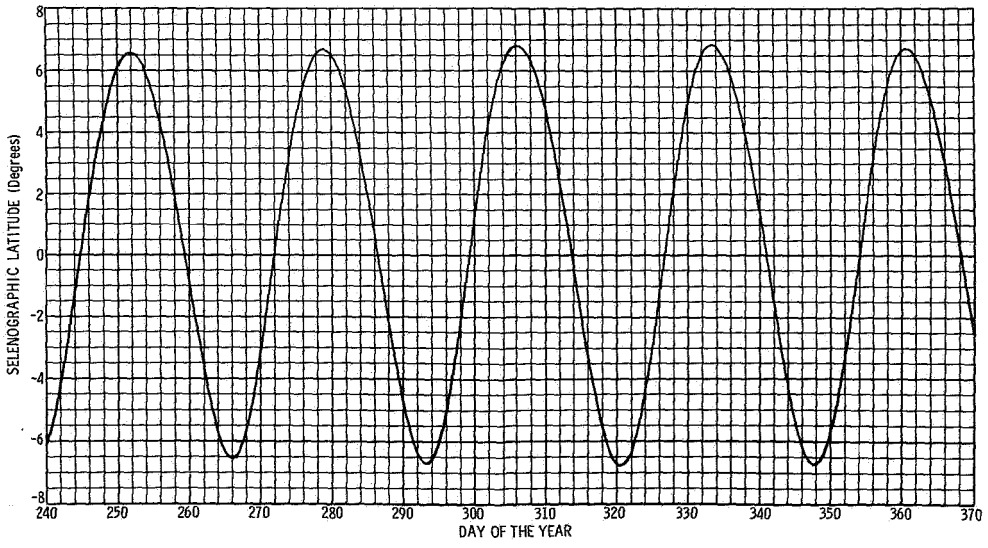


FIGURE B1984-14 SELENOGRAPHIC LATITUDE OF THE EARTH

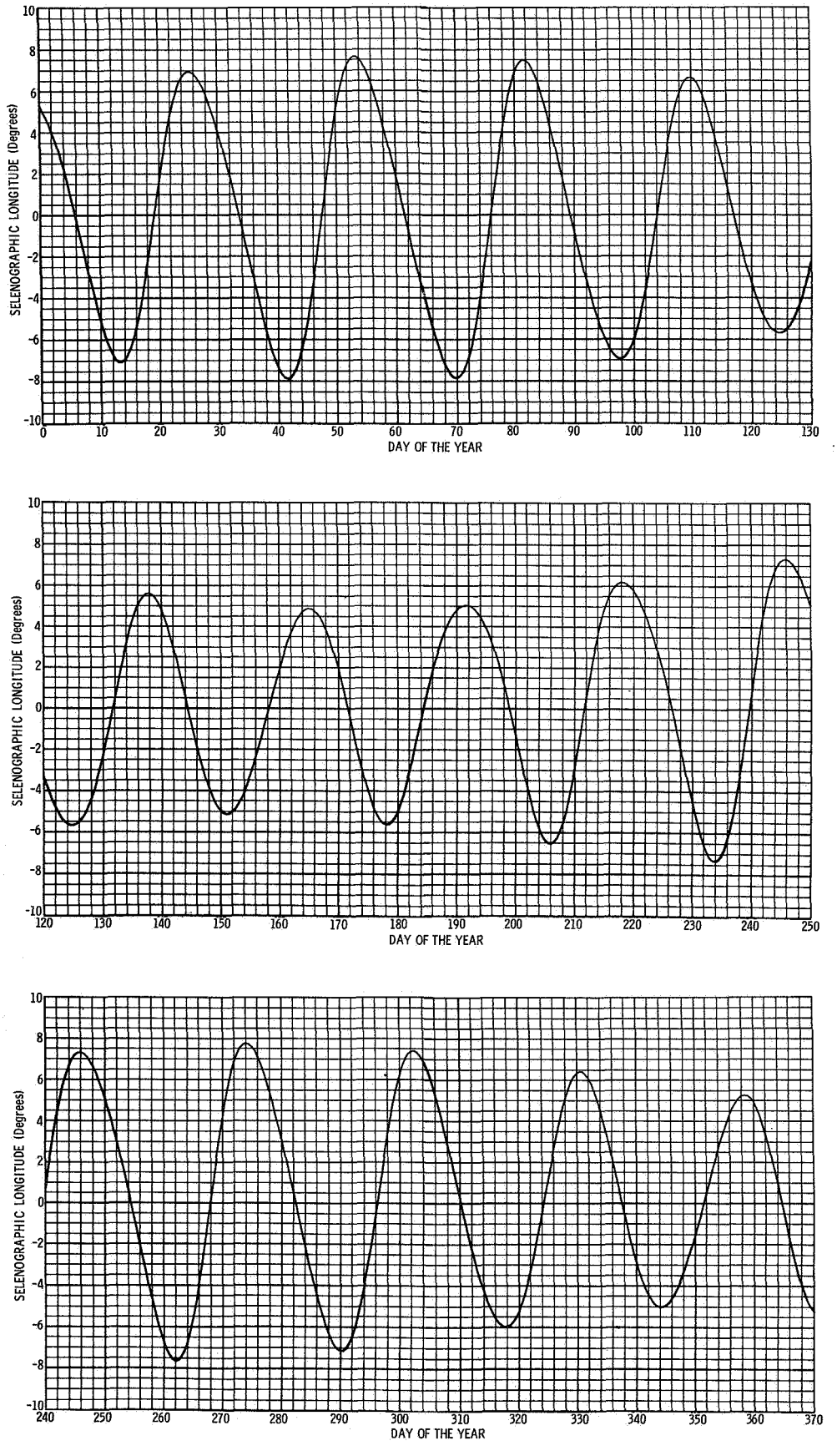


FIGURE B1984-15 SELENOGRAPHIC LONGITUDE OF THE EARTH

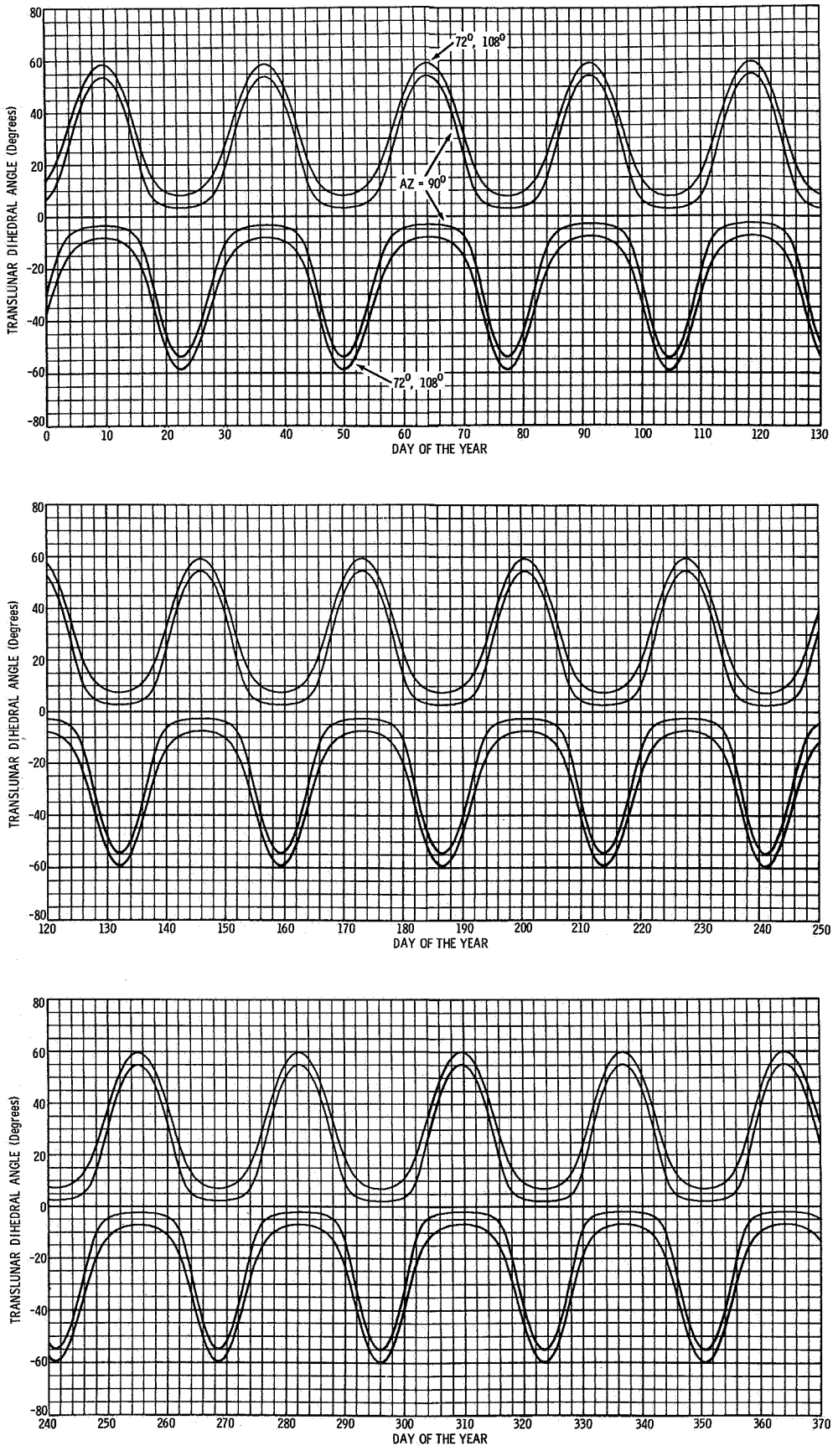


FIGURE B1984-16 TRANSLUNAR DIHEDRAL ANGLES

1985

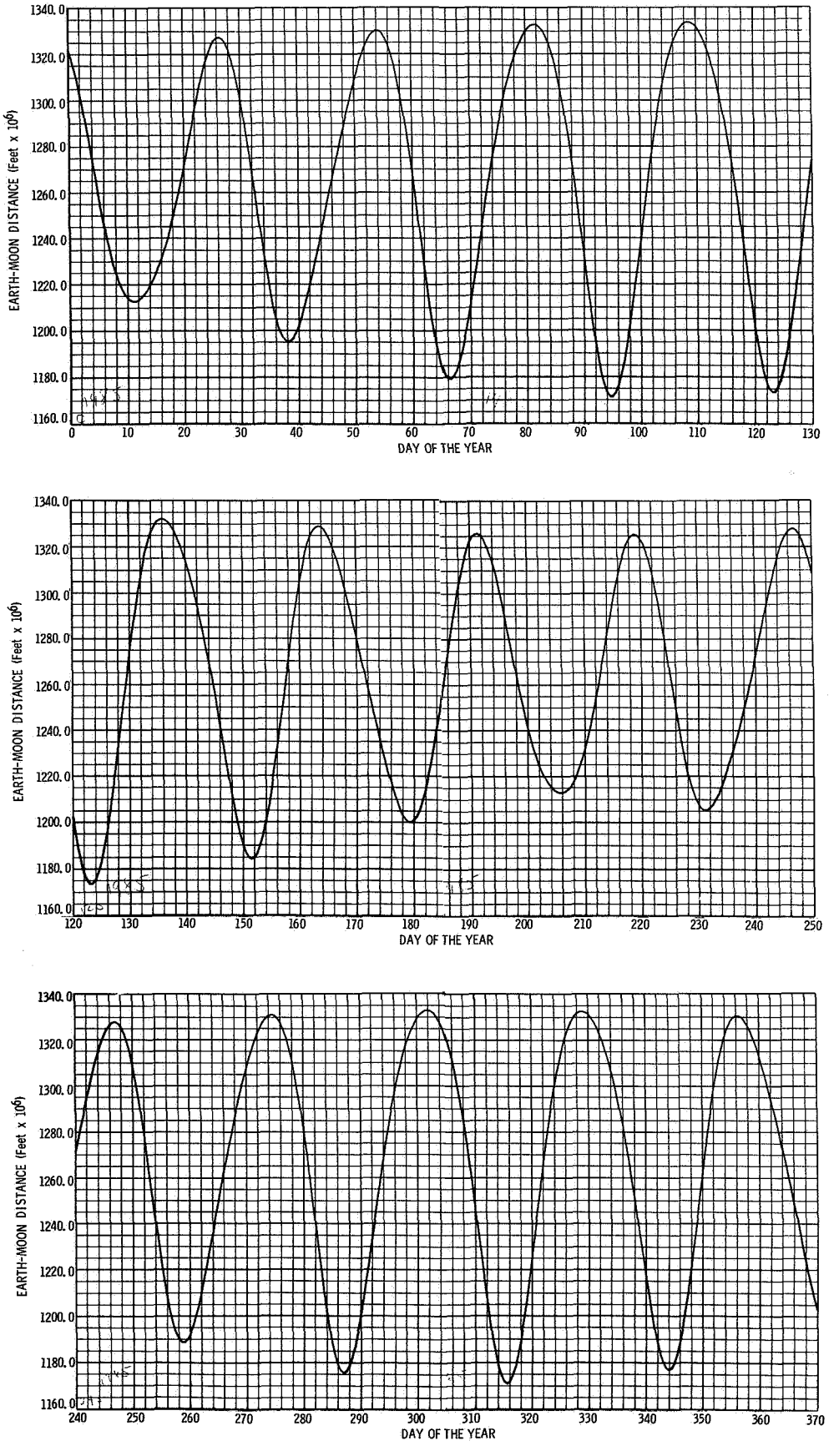
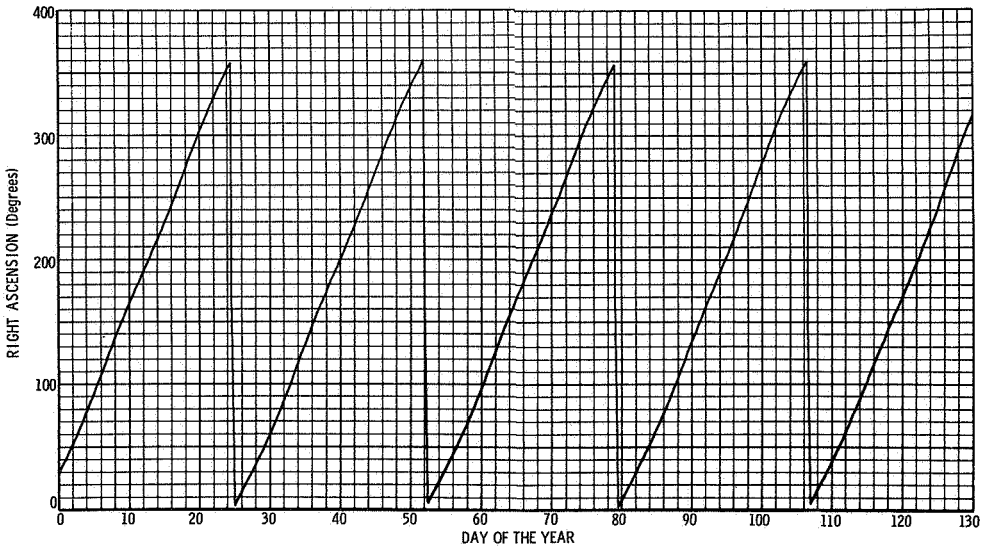
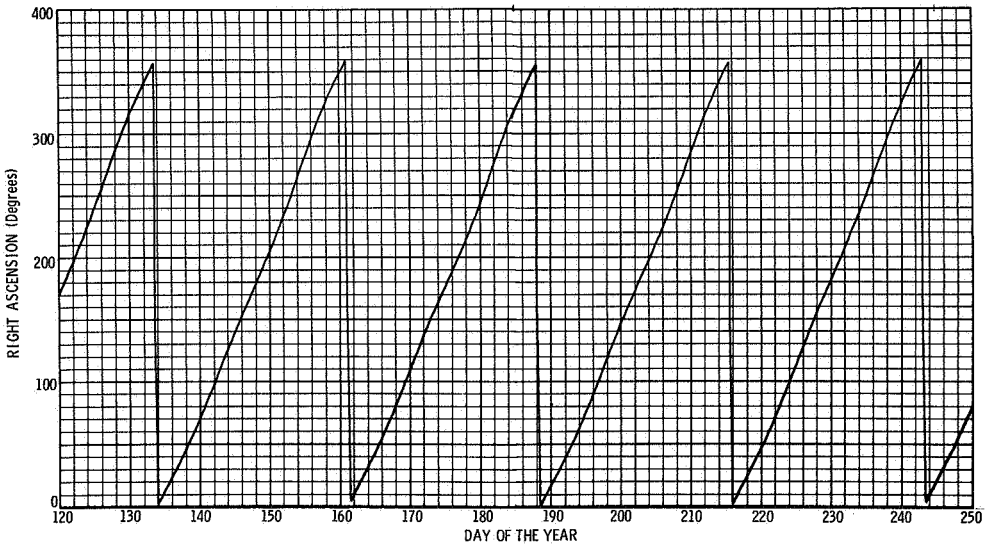


FIGURE B1985-1 EARTH-MOON DISTANCE





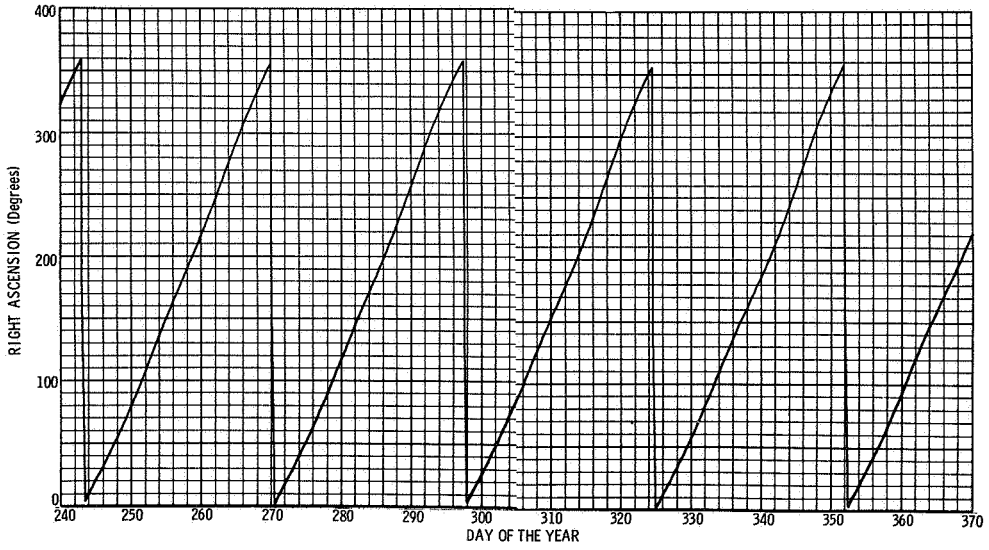


FIGURE B1985-2 RIGHT ASCENSION OF THE MOON

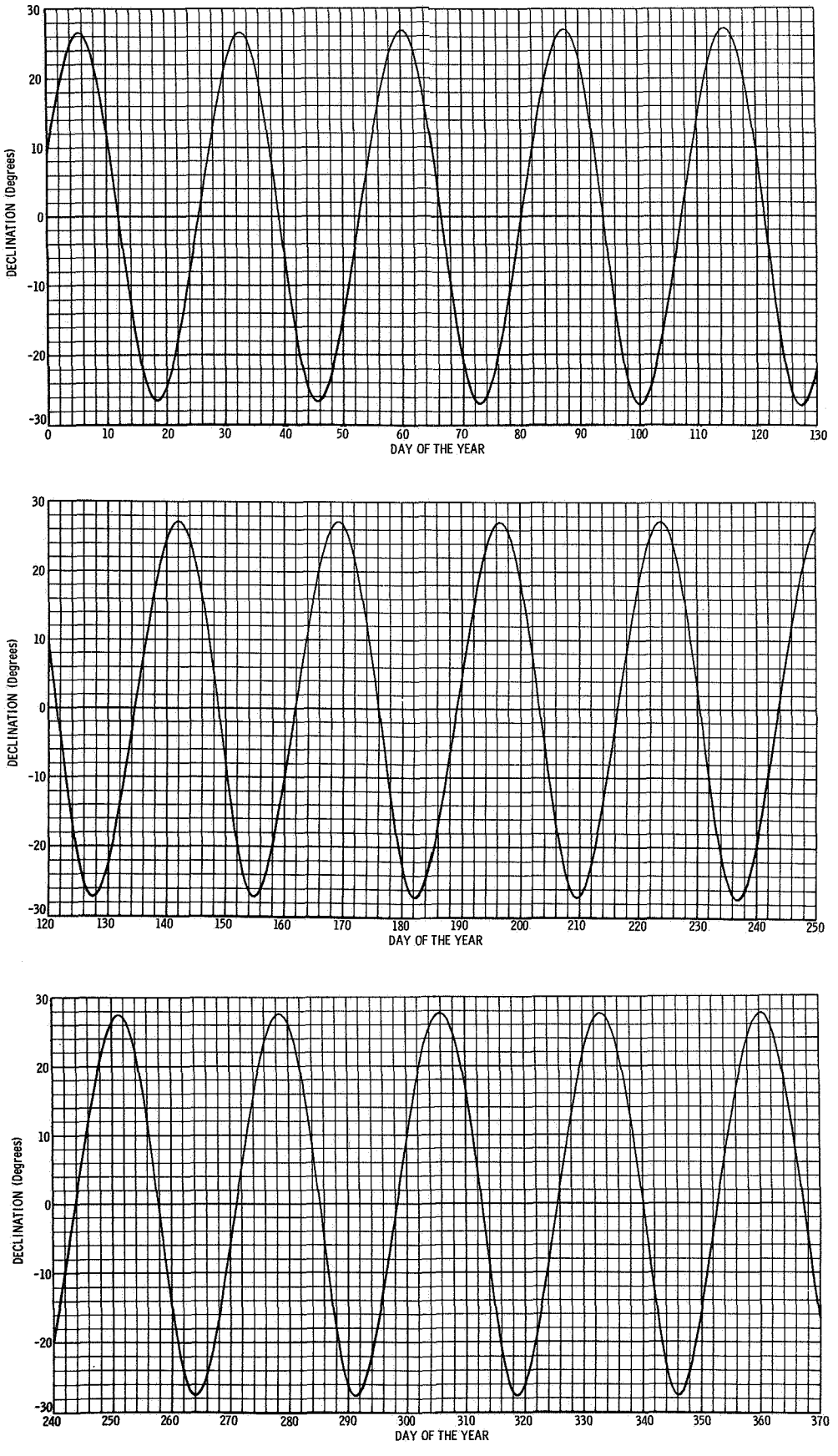


FIGURE B1985-3 DECLINATION OF THE MOON

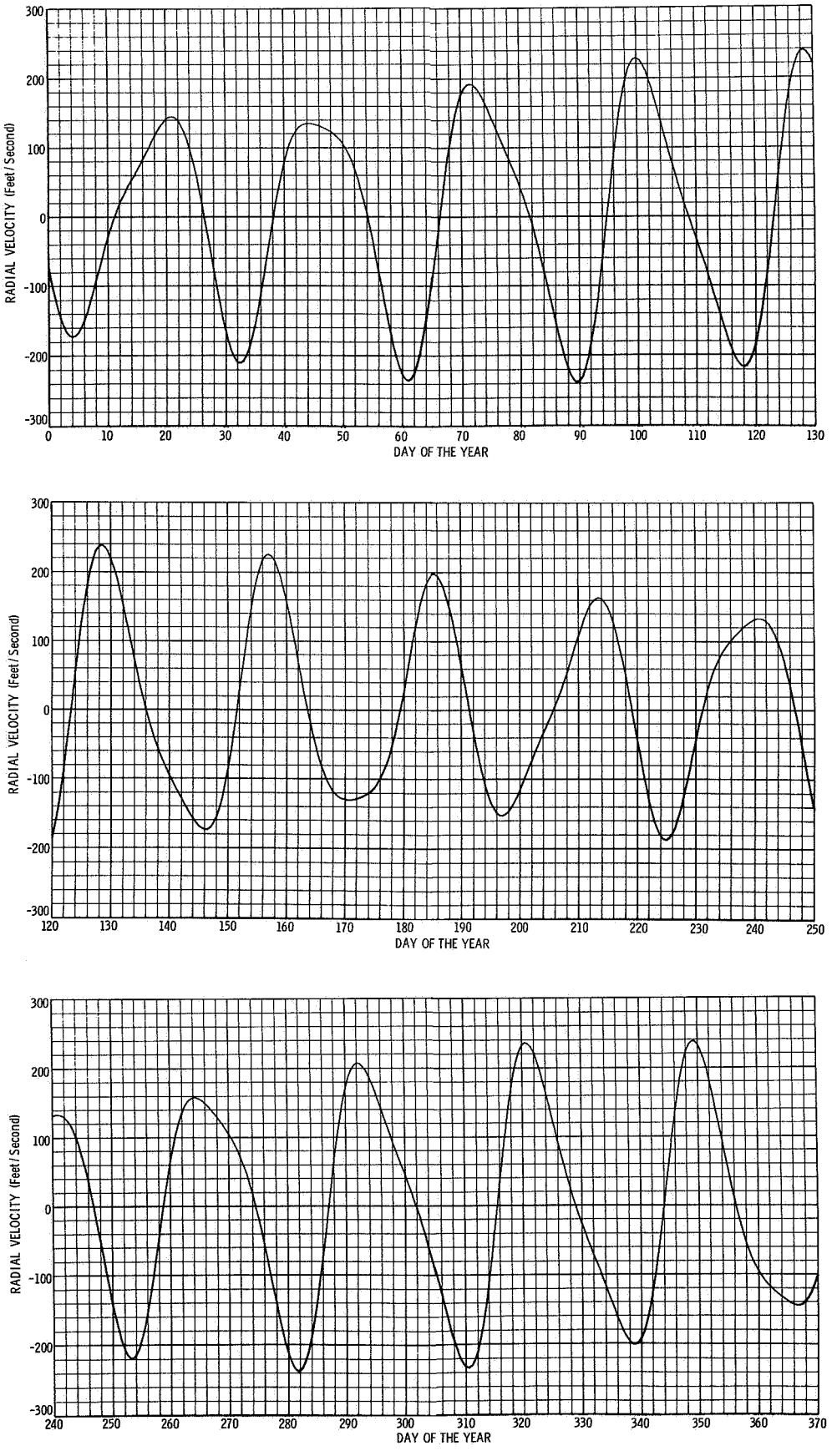


FIGURE B1985-4 RADIAL VELOCITY OF THE MOON

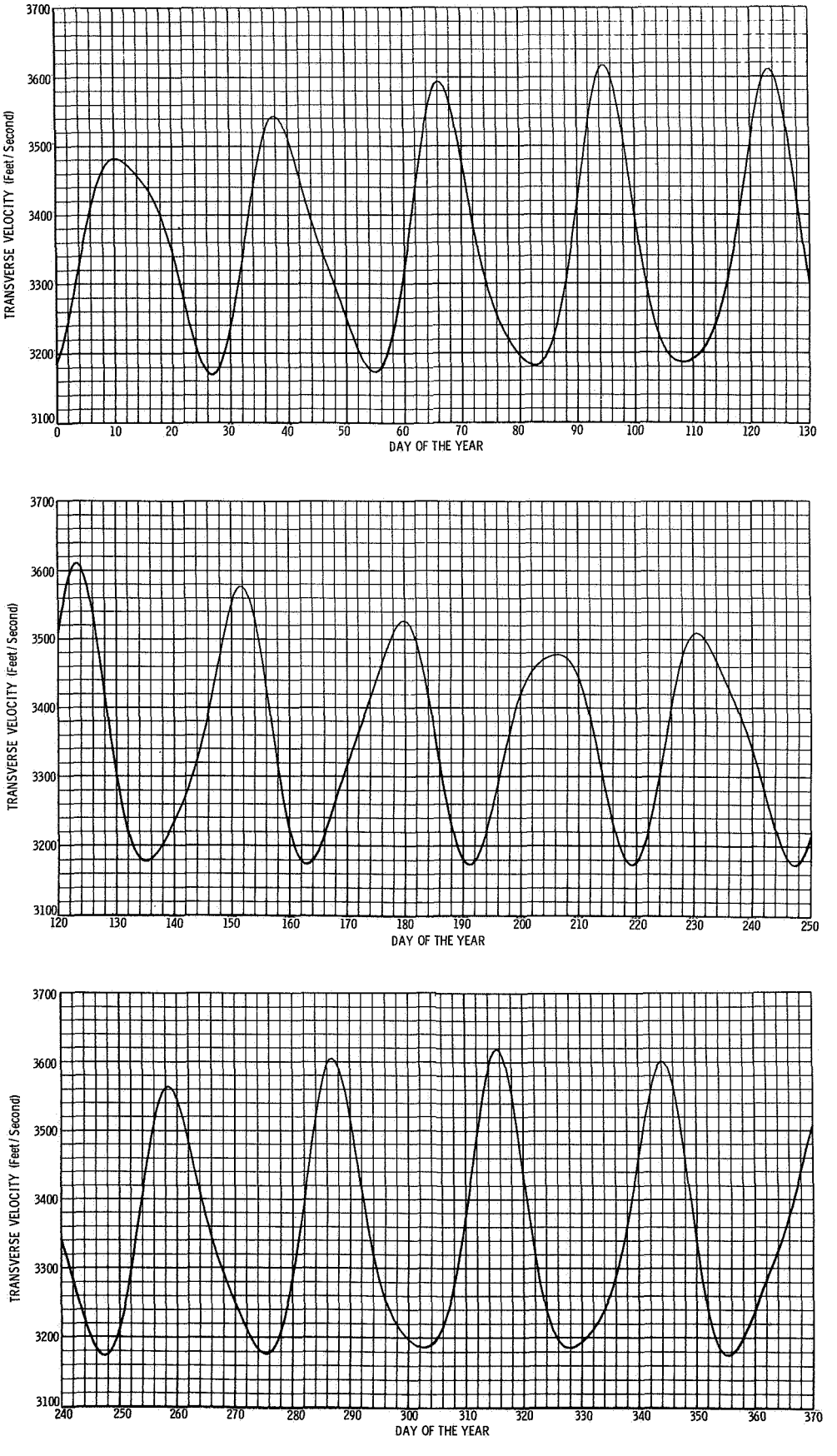
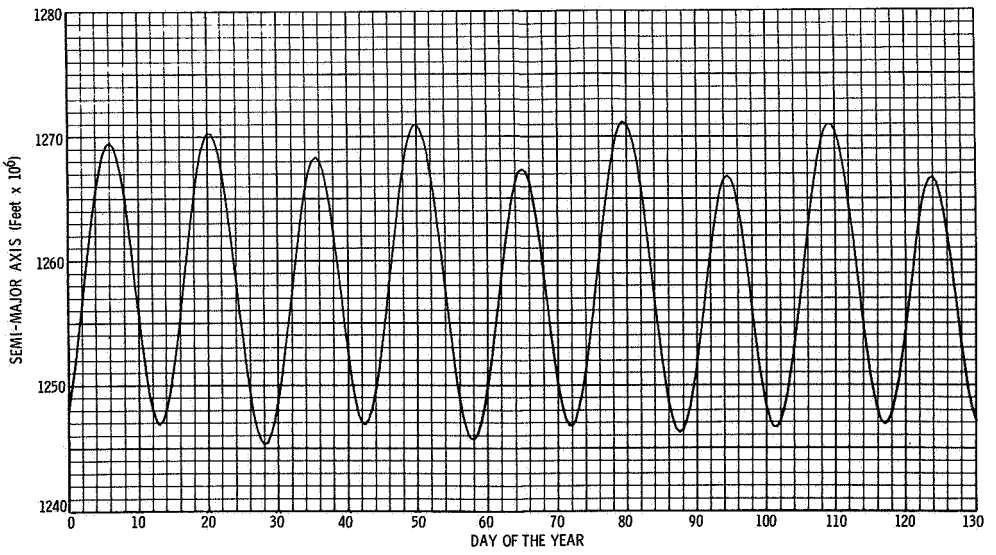
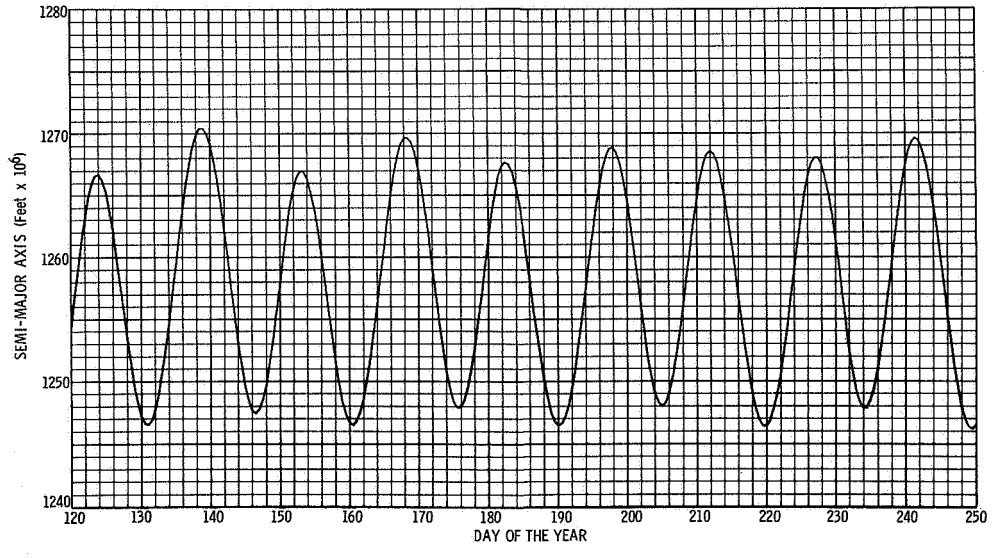


FIGURE B1985-5 TRANSVERSE VELOCITY OF THE MOON





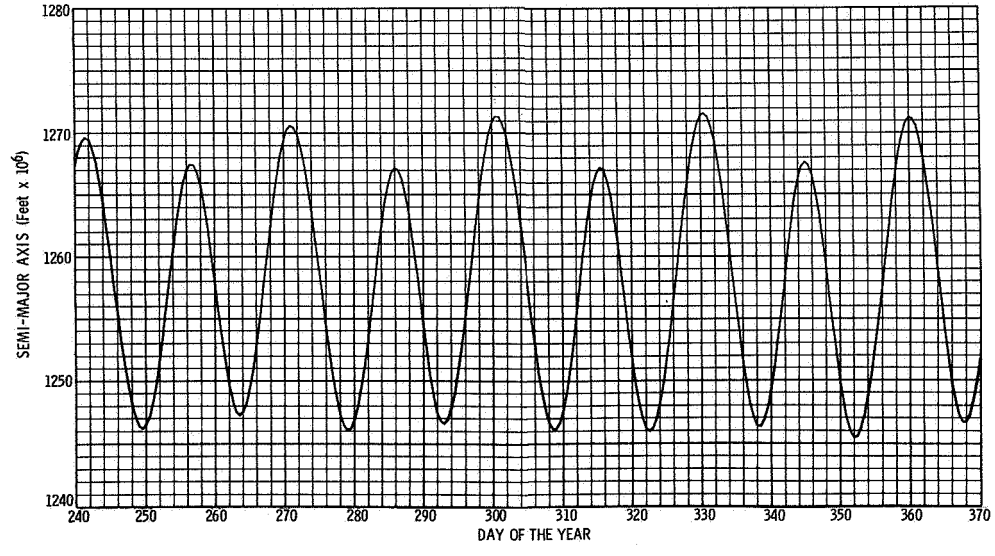


FIGURE B1985-6 SEMIMAJOR AXIS OF THE MOON'S OSCULATING ORBIT

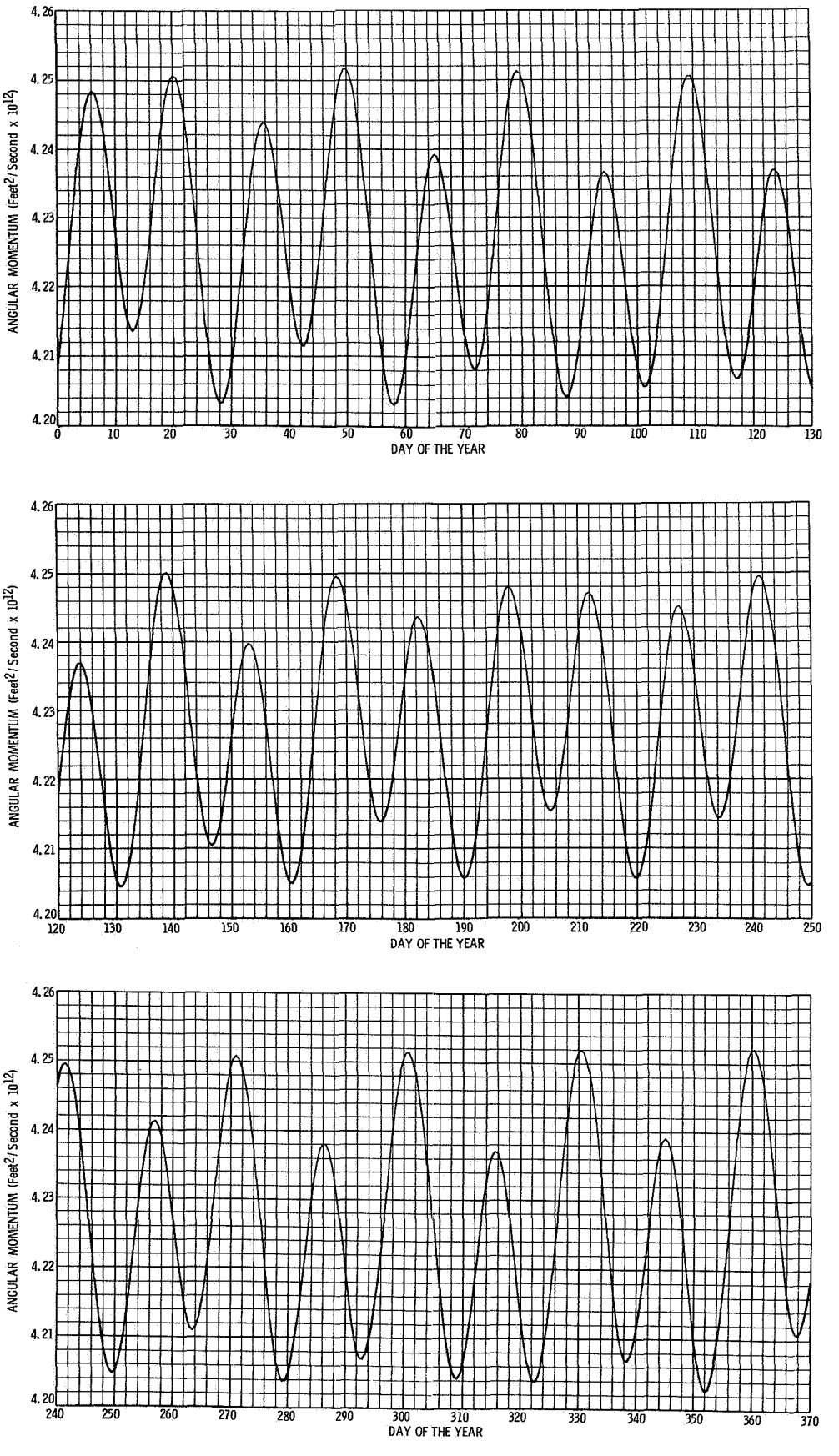


FIGURE B1985-7 GEOCENTRIC ANGULAR MOMENTUM OF THE MOON

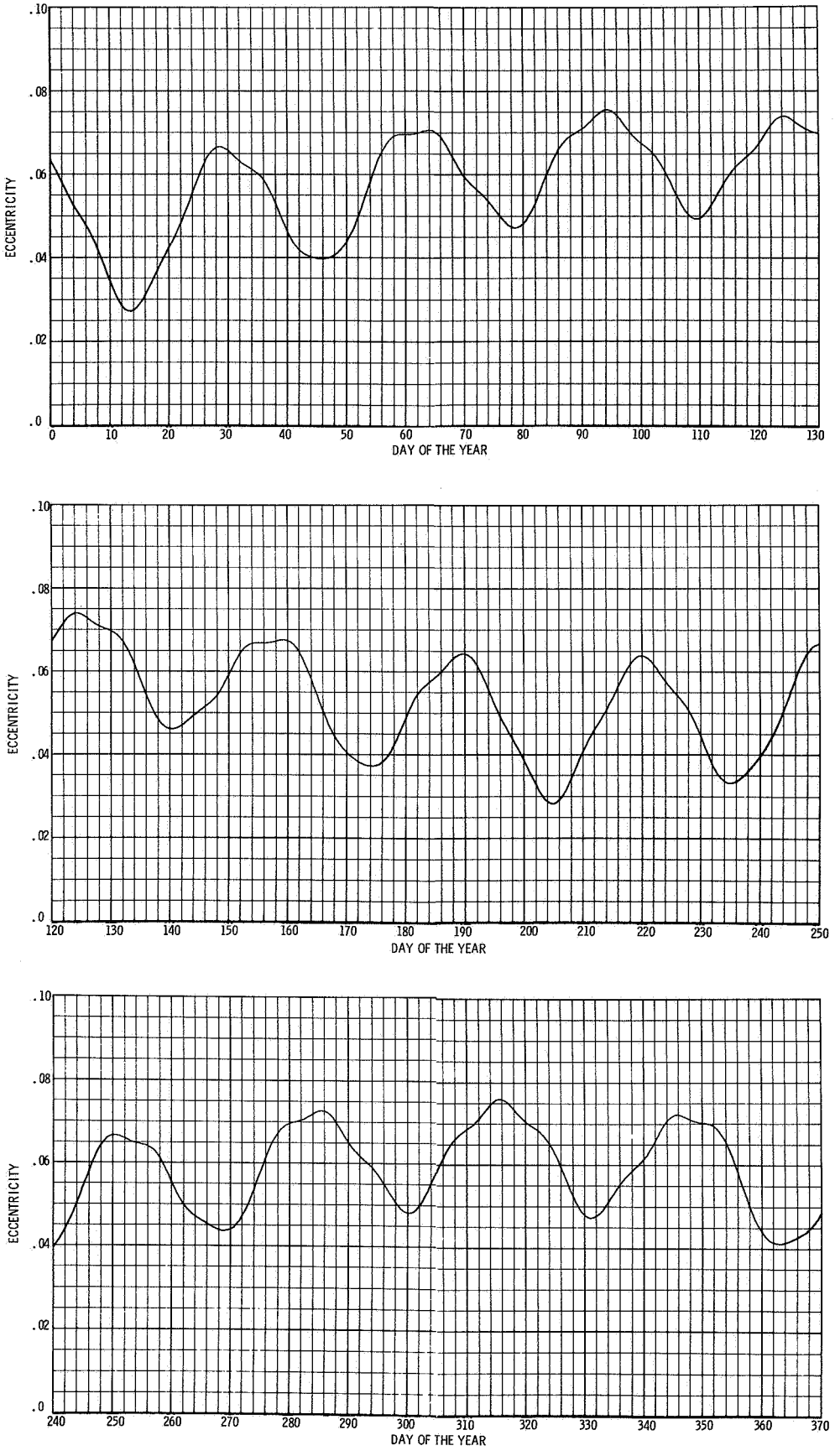


FIGURE B1985-8 ECCENTRICITY OF THE MOON'S OSCULATING ORBIT

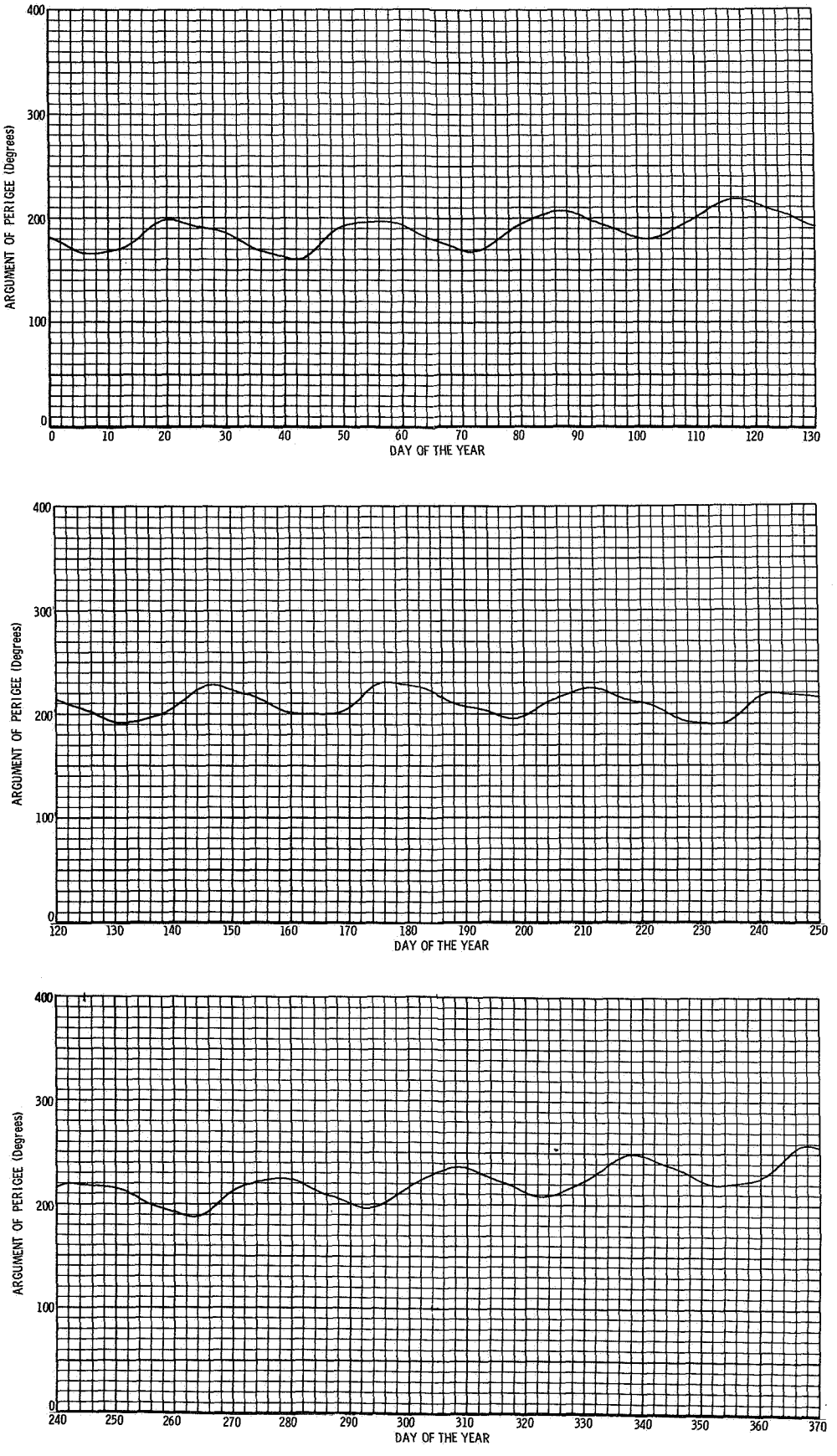
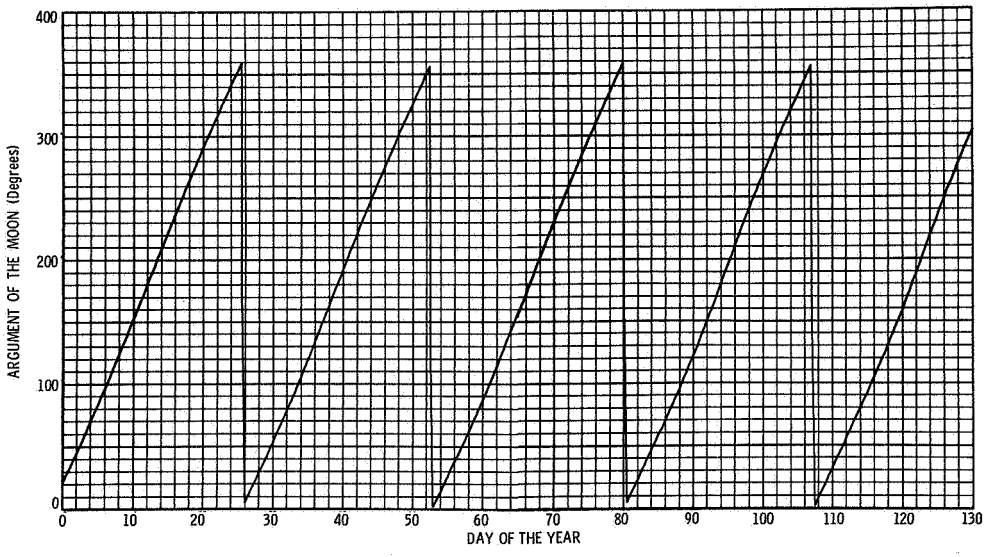
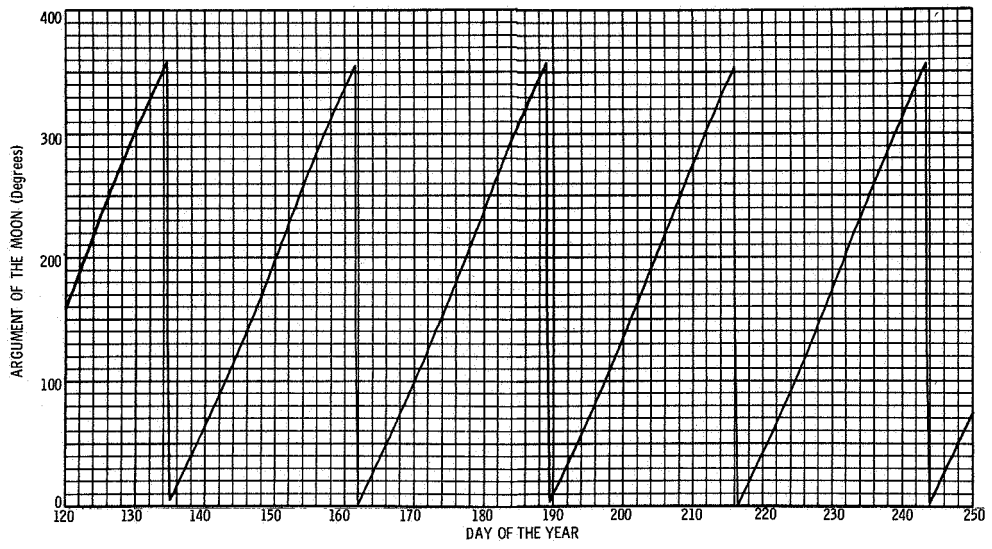


FIGURE B1985-9 OSCULATING ARGUMENT OF THE MOON'S PERIGEE





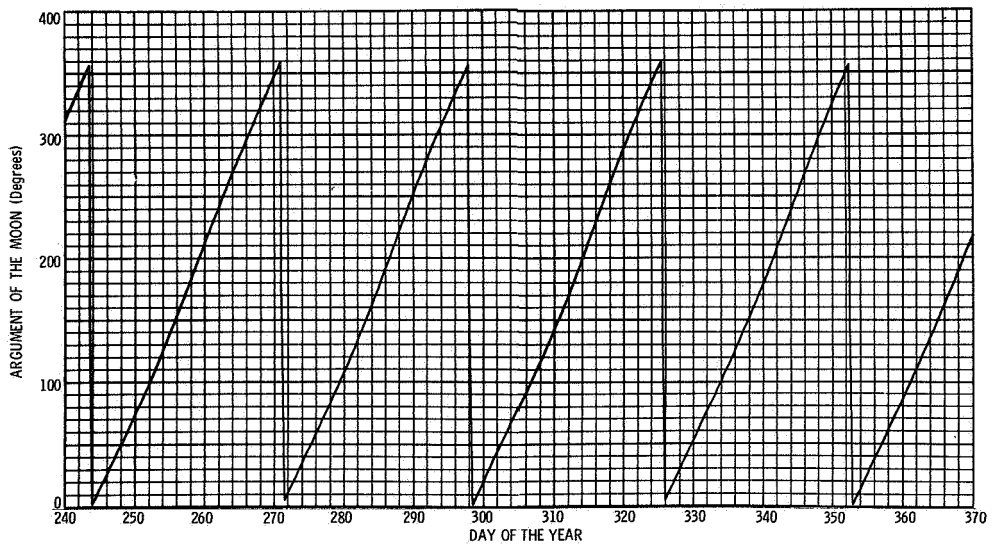


FIGURE B1985-10 ARGUMENT OF THE MOON'S POSITION

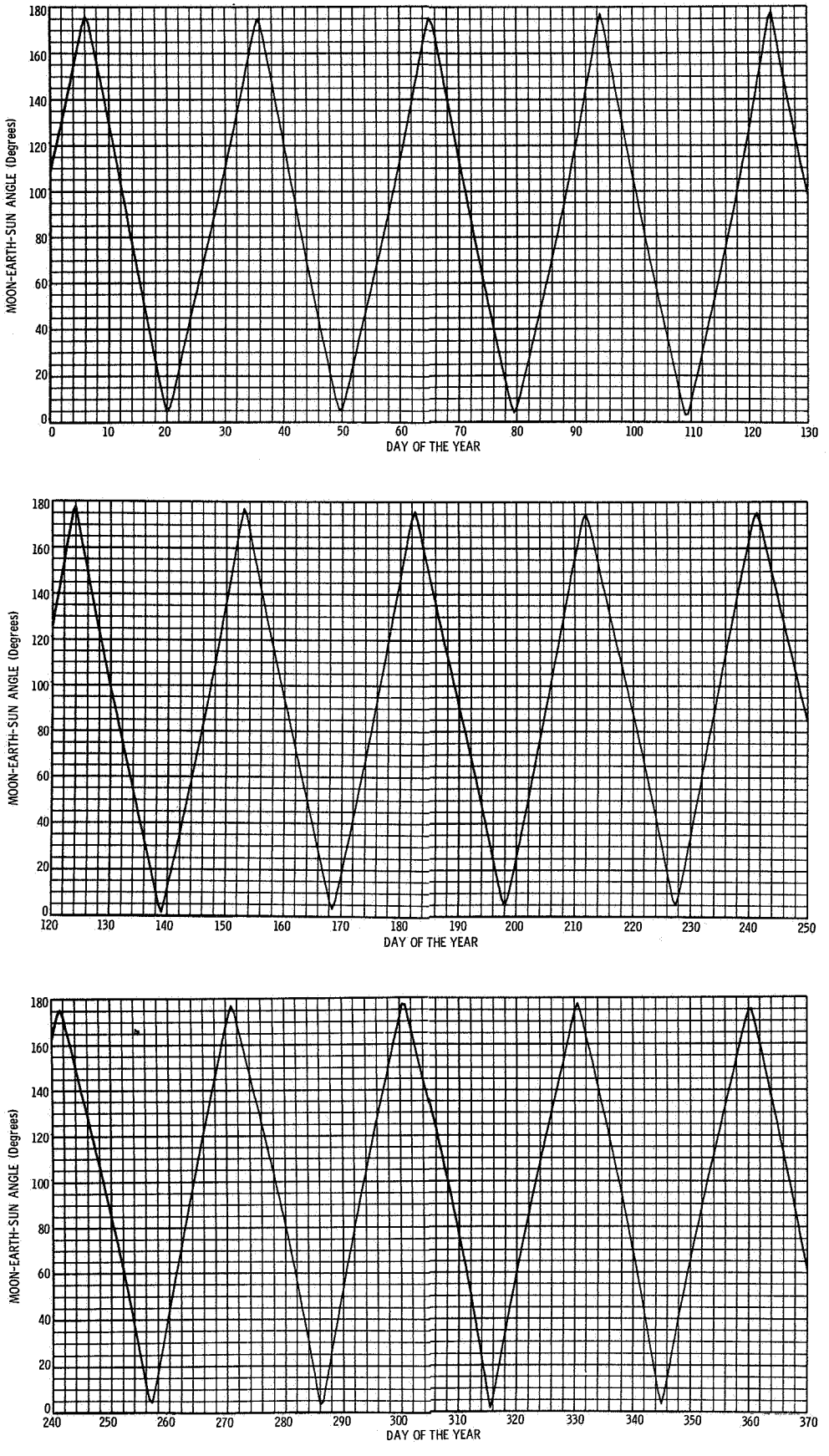
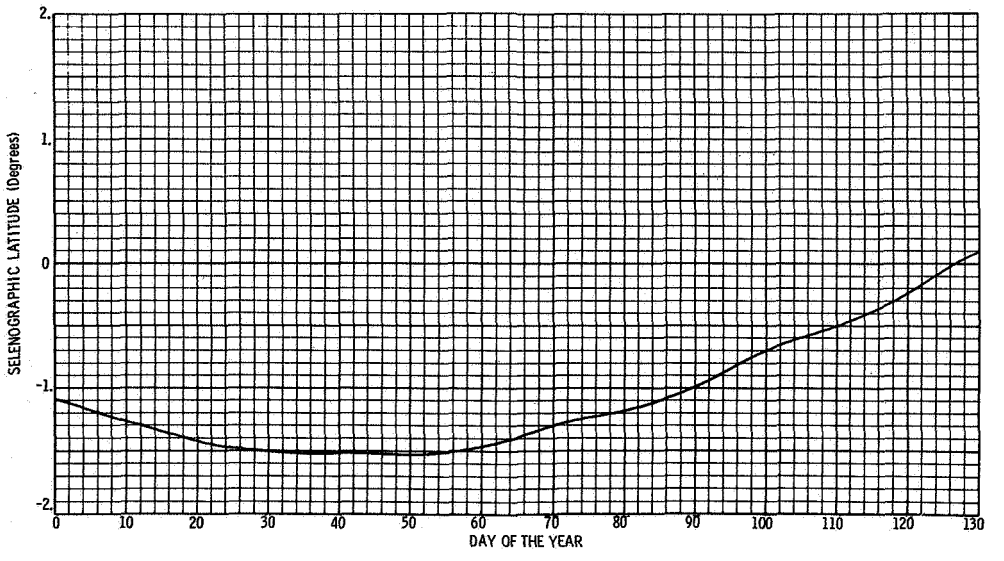
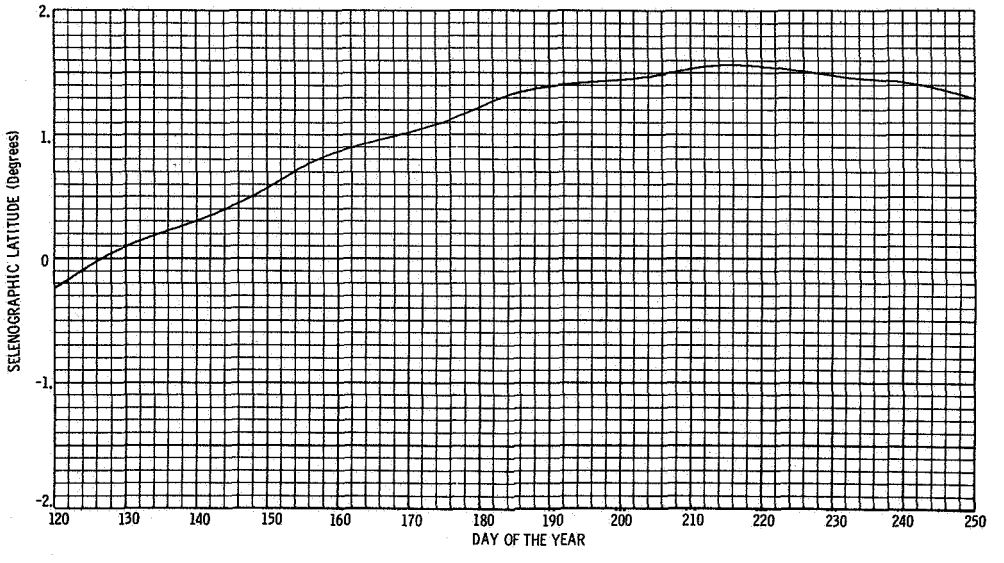


FIGURE B1985-11 MOON-EARTH-SUN ANGLE





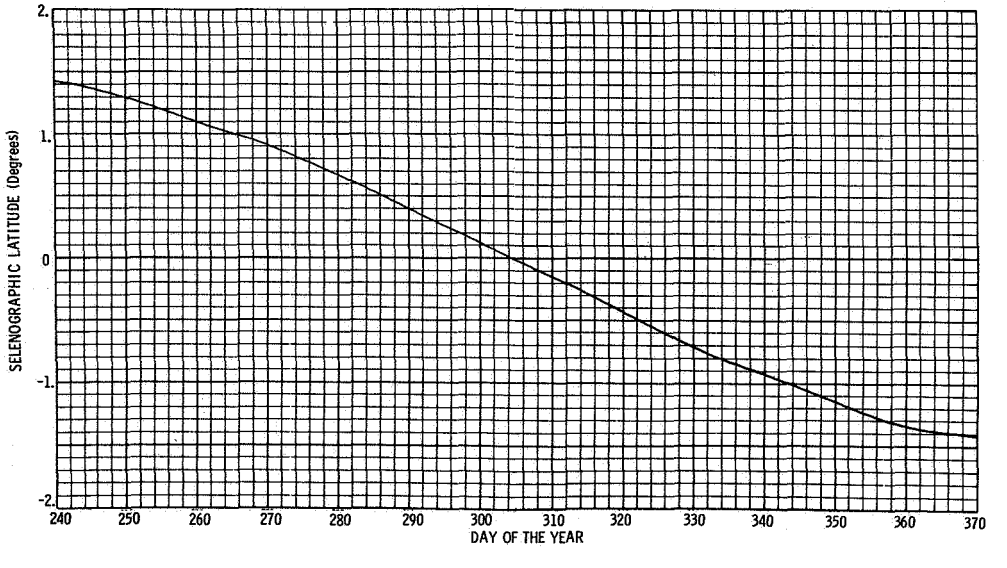
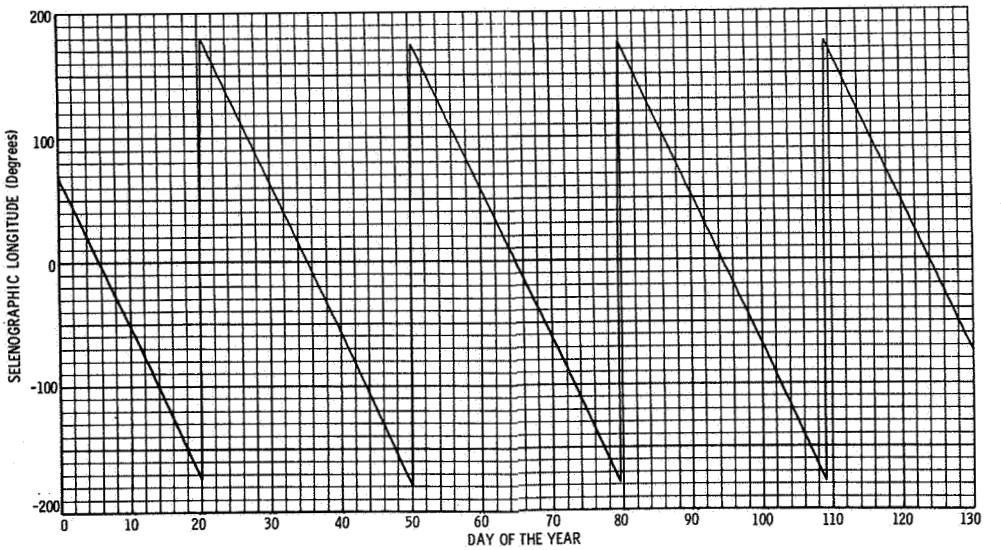
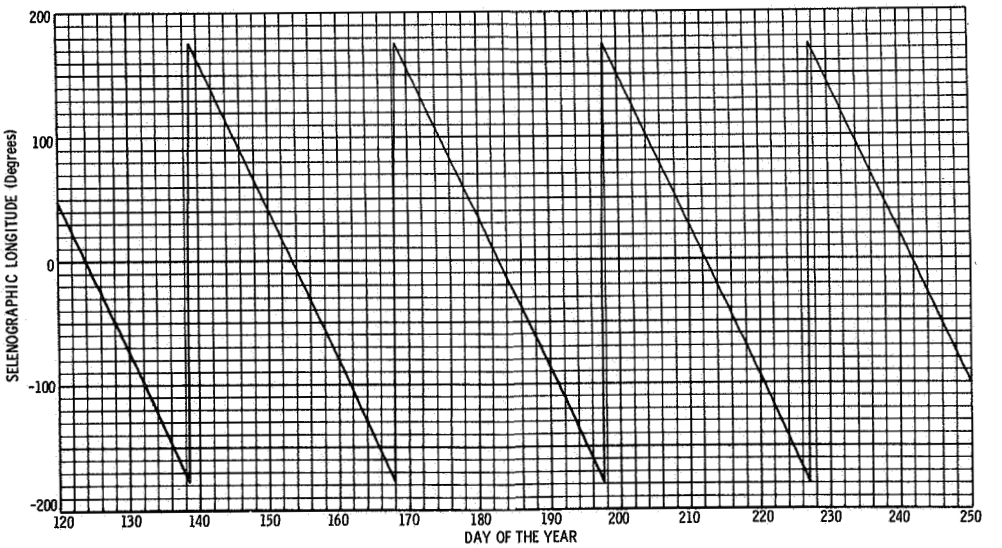


FIGURE B1985-12 SELENOGRAPHIC LATITUDE OF THE SUN





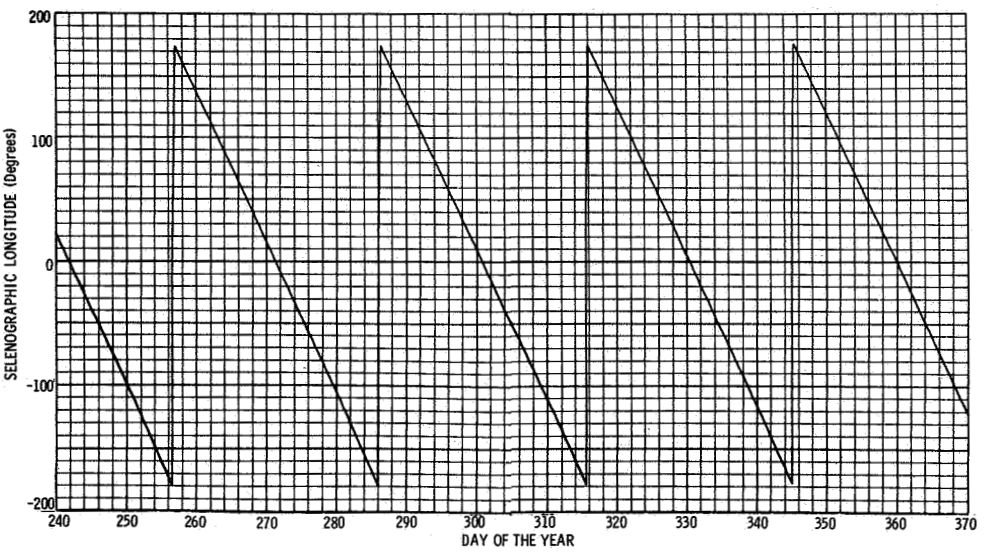


FIGURE B1985-13 SELENOGRAPHIC LONGITUDE OF THE SUN

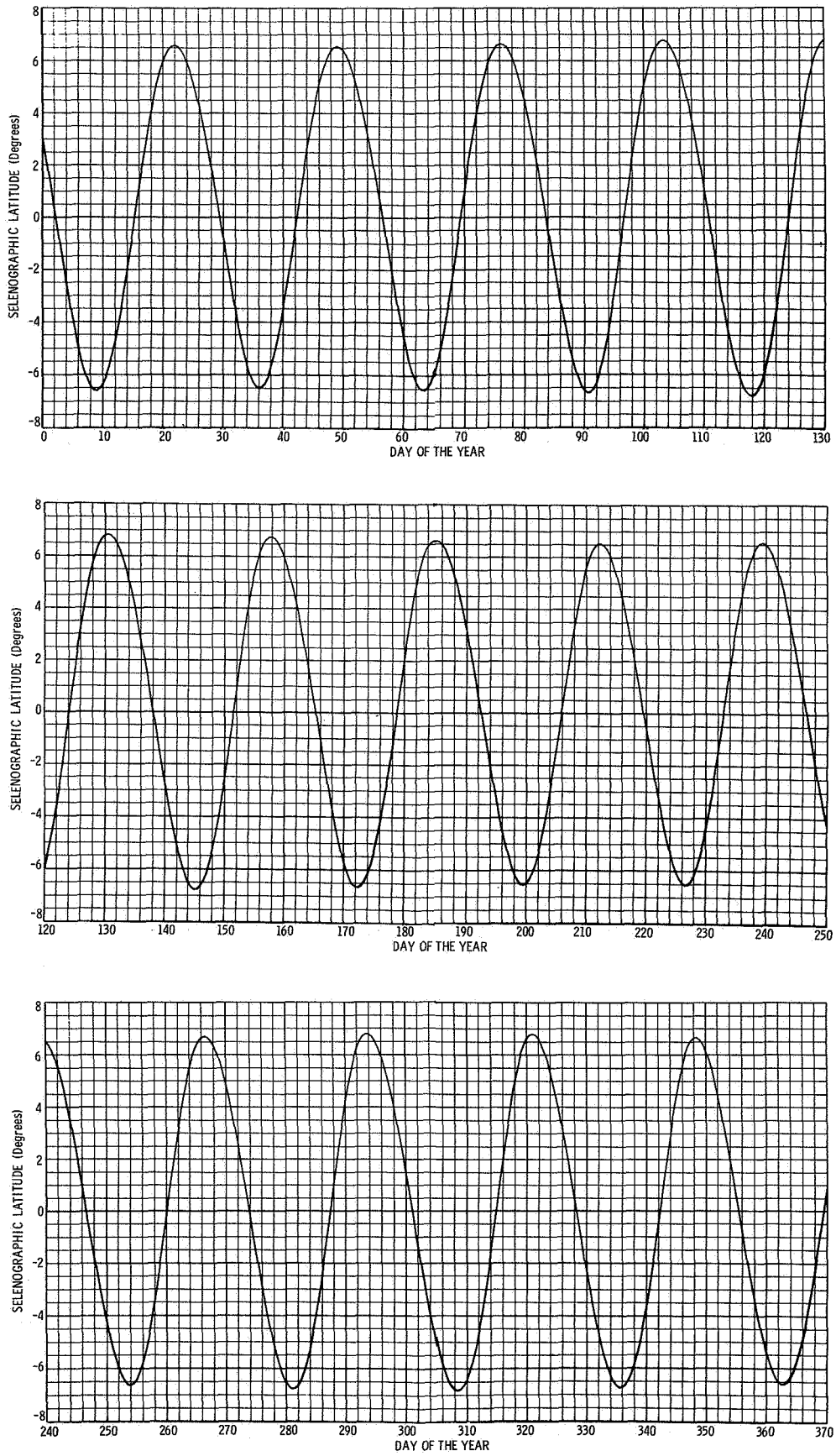


FIGURE B1985-14 SELENOGRAPHIC LATITUDE OF THE EARTH

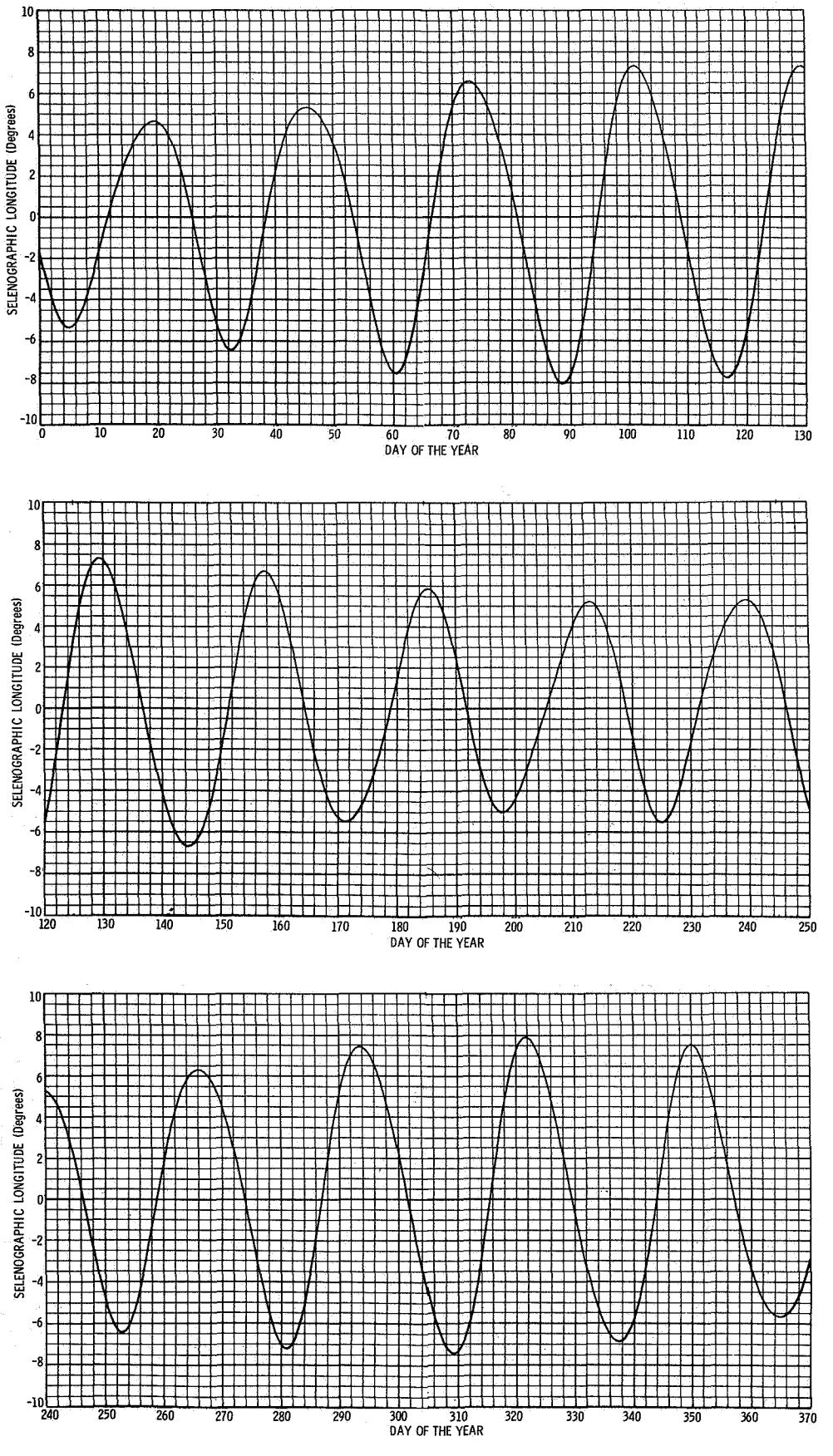


FIGURE B1985-15 SELENOGRAPHIC LONGITUDE OF THE EARTH

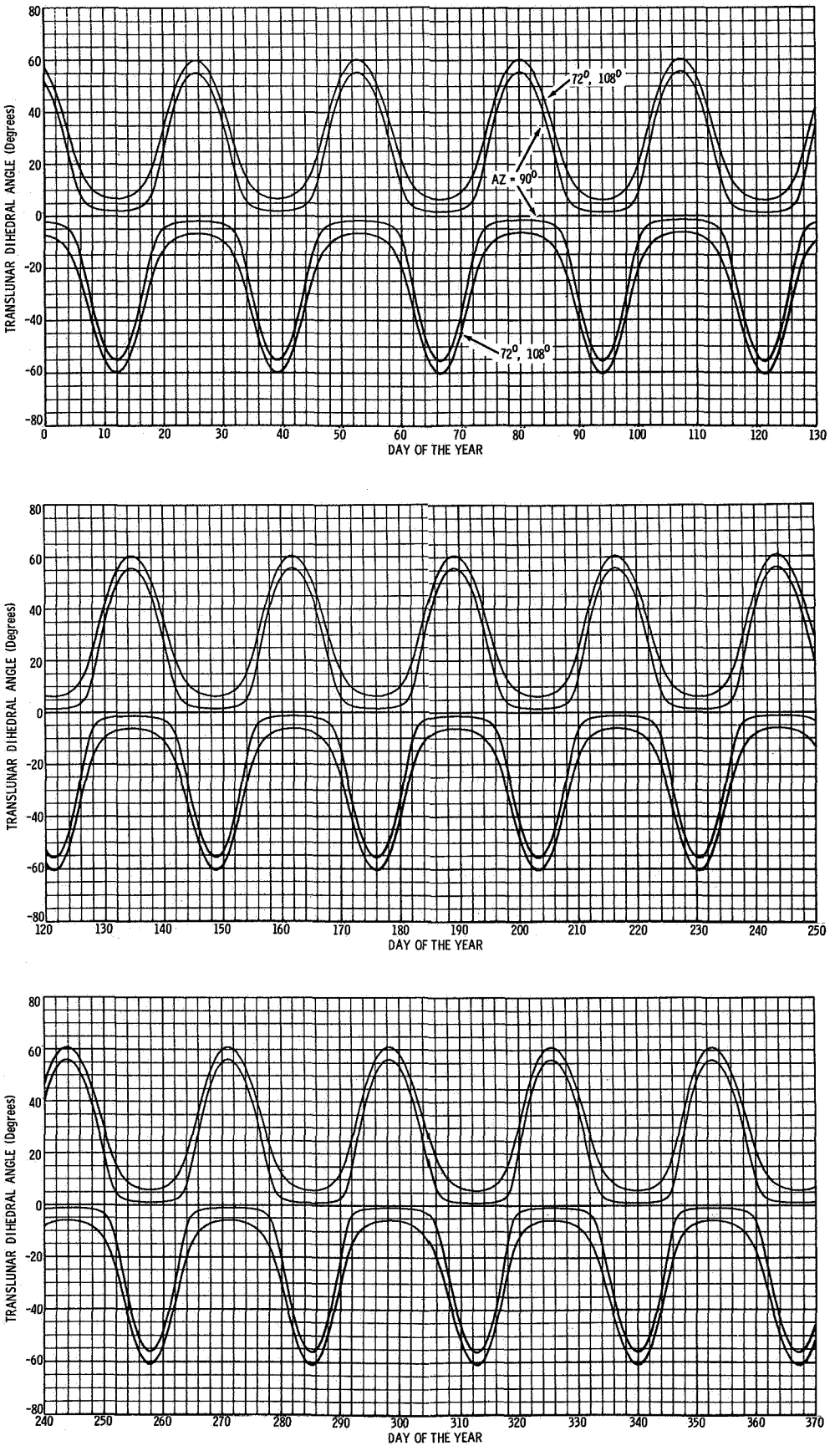


FIGURE B1985-16 TRANSLUNAR DIHEDRAL ANGLES

APPENDIX C

SELENOGRAPHIC COORDINATES OF THE SUN AND THE EARTH (LUNAR LIBRATION)

As seen from the Earth, the libration of the Moon, which is a periodic oscillation of the Moon's attitude with respect to its mean value, consists of two components, the so-called optical libration and the physical libration. The optical libration is an apparent libration which results from the fact that (a) due to the ellipticity of its orbit, the Moon's rate of revolution about the Earth differs from its rate of rotation about its spin axis except at two points during each revolution, giving rise to an apparent libration in longitude (b) due to the inclination of its axis of rotation with respect to the normal to its plane of motion, the Moon seems to wobble in a generally North-South direction, giving rise to an apparent libration in latitude, and (c) due to the Earth's rotation an observer will be moving in space, giving rise to an apparent diurnal libration. The physical libration, on the other hand, is a true libration in that it consists of small periodic variations from a mean rotational motion. The physical librations of the Moon are considerably smaller than the optical libration, with the maximum value of the former being of the order of 0.04° and the latter $6\text{-}7^\circ$.

By definition (Reference 6), the condition that the center of the apparent disk of the Moon be coincident with the mean center, and thus that the geocentric optical librations in latitude and longitude simultaneously vanish, is that $\lambda = \mathbb{C} = \Omega$, where

λ = the geocentric ecliptic longitude of the Moon, measured from the mean equinox of date

$\mathbb{C}$ = the mean longitude of the Moon, measured in the ecliptic from the mean equinox of date to the mean ascending node of the lunar orbit, and then along the lunar orbit

Ω = the longitude of the mean ascending node of the lunar orbit on the ecliptic, measured from the mean equinox of date

This set of conditions is equivalent to requiring the Moon to be at its ascending node with respect to the ecliptic (nulling the optical libration in latitude) when the node coincides with either mean perigee or mean apogee (nulling the optical libration in longitude).

Then, if the physical libration is neglected, the selenographic prime meridian (the meridian passing through the mean center of the apparent disk), which rotates at a rate equal to the mean orbital motion of the Moon, is displaced $\mathbb{C} - \Omega$ from the descending node of the lunar orbit on the ecliptic, measured on the selenocentric celestial sphere (Figure C-1), where $\mathbb{C} - \Omega$ is simply the angular displacement of the mean Moon from the ascending node of the lunar orbit. On the same sphere the Earth is at ecliptic longitude $180^\circ + \lambda$ and ecliptic latitude $-\beta$, where β is the geocentric ecliptic latitude of the Moon. The corresponding selenographic coordinates of the Earth represent the optical librations.

According to Cassini's third law (Reference 8), the mean lunar orbit plane and the mean lunar equatorial plane both intersect the ecliptic plane along the same node line, with the mean descending node of the lunar orbit on the ecliptic coinciding with the mean ascending node of the lunar equator on the ecliptic. Thus, the Euler angles which define the transformation from ecliptic coordinates to selenographic coordinates are simply (Figure C-1)

$$180^\circ + \Omega, I, \text{☾} - \Omega$$

where $180^\circ + \Omega$ is the longitude of the mean ascending node of the lunar equator on the ecliptic measured from the mean equinox of date, I is the mean inclination of the Moon's equatorial plane with respect to the plane of the ecliptic, and ☾ $- \Omega$ is defined above.

However, because of the physical libration of the Moon, the actual inclination and ascending node of the lunar equator on the ecliptic are $I + \rho$ and $180^\circ + (\Omega + \sigma)$, and the angular distance from the descending node of the lunar equator to the prime meridian is (☾ $- \Omega$) + $(\tau - \sigma)$ where ρ, σ and τ are determined from the dynamical theory of the rotation of the Moon.

Thus, when the physical libration is included, the Euler angles become

$$180^\circ + (\Omega + \sigma), I + \rho, (\text{☾} - \Omega) + (\tau - \sigma)$$

The algorithm adopted for the generation of the physical libration for this compendium is based on the development by Roth and Escobal (Reference 6). The adopted value of I is Hayn's 1923 determination (Reference 7), i.e., $I = 1^\circ 32' 20'' = 1^\circ.53889$.

Once the transformation Euler angles have been determined, the actual librations in latitude and longitude are obtained by transforming the Moon-Earth position vector in ecliptic coordinates into selenographic coordinates via these transformation angles, i.e., the librations are given by the selenographic coordinates of the Earth. Likewise the selenographic subsolar point is determined by transforming the Moon-Sun position vector in ecliptic coordinates, via the same transformation angles, into selenographic coordinates.

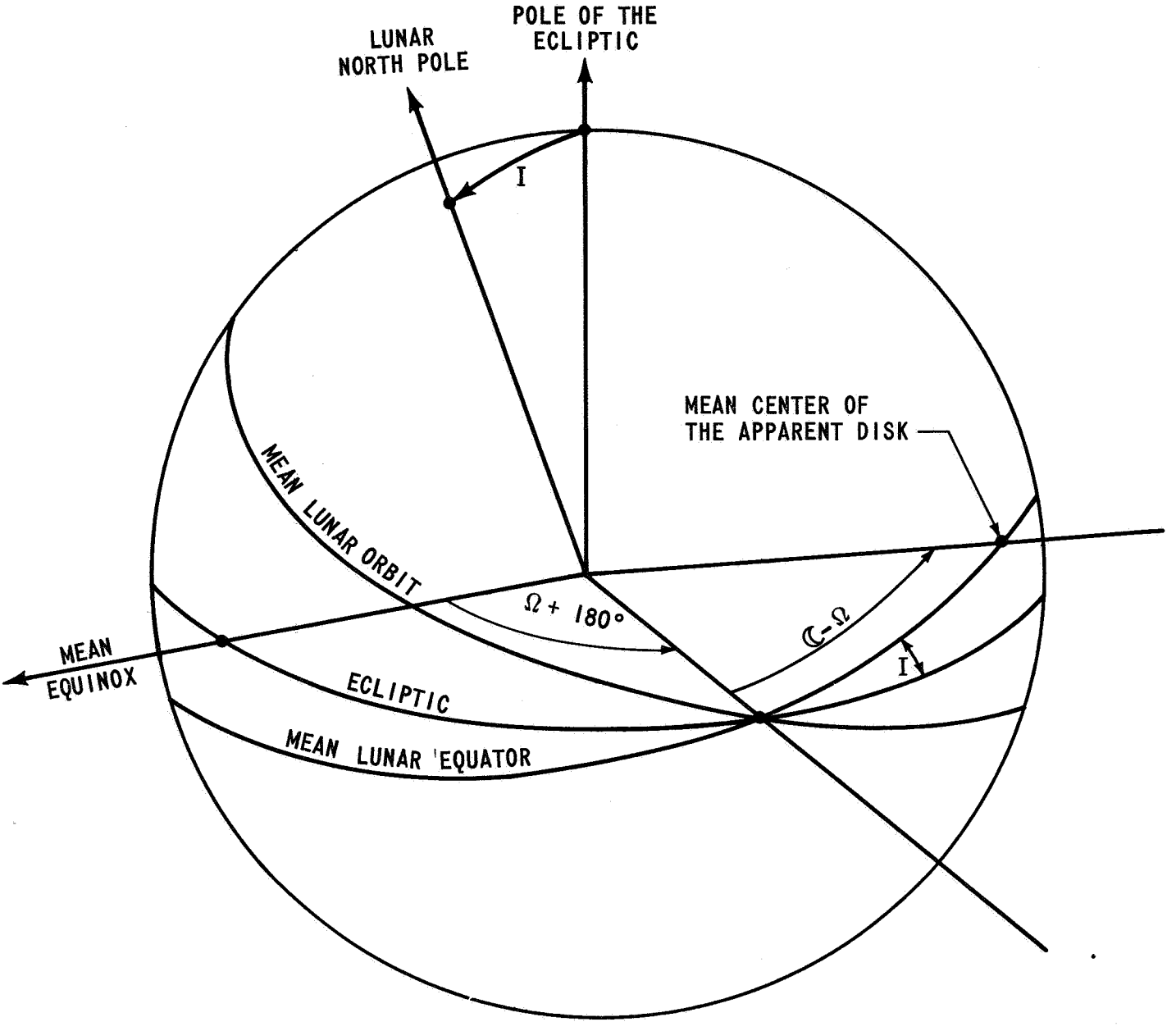


FIGURE C-1 - RELATIONSHIP BETWEEN ECLIPTIC AND SELENOGRAPHIC COORDINATE SYSTEMS

APPENDIX D

GENERATION OF THE DATA

The first step required in the generation of the data presented in this report is to extract the position of the Sun and the position and velocity of the Moon from the JPL ephemeris tapes. This is accomplished by a straightforward computer subroutine, which also converts this position and velocity information from the mean equator and equinox of 1950.0 to the mean equator and equinox of date.* The position and velocity of the Moon are then introduced into an orbit calculation subroutine which calculates the geocentric osculating orbital elements corresponding to the input position and velocity vectors. The quantities computed by this subroutine include Earth-Moon distance, geocentric angular momentum, eccentricity, semi-major axis, right ascension and declination, radial and transverse velocity, right ascension of the ascending node, inclination, and argument of perigee.

The orientation, including physical libration, of the Moon's rigid body axes is then calculated and the selenocentric position vector of the Earth (the negative of the geocentric position vector of the Moon) is transformed into this system in order to determine the selenocentric latitude and longitude of the Earth (libration in latitude and longitude). Similarly, the selenocentric position vector of the Sun is calculated from the geocentric Moon and Sun position vectors, and then transformed to obtain the selenocentric latitude and longitude of the subsolar point.

The final step is the determination of the translunar dihedral angles associated with 72°, 90° and 108° launch azimuths from Cape Kennedy. The translunar dihedral angle is the angle between the osculating plane of the Moon's motion and the geocentric translunar trajectory plane, treated as a conic section (see Figure B-1d). If the translunar trajectory lies above the plane of the Moon's motion, the translunar dihedral angle is considered to be positive and the translunar injection is termed a north injection; similarly, if the translunar trajectory lies below the plane of the Moon's motion, the translunar dihedral angle is considered to be negative and the corresponding translunar injection is termed a south injection. In all cases, the dihedral angles correspond to the translunar trajectories which arrive at the Moon at the time in question.

It should be noted (Figure B-2) that during 1968, 1969, and 1970 the declination of the Moon will sometimes be greater than the latitude of the launch site, rendering a 90° launch unuseable. When this occurs, the launch azimuth has been adjusted so as to give an inclination of the Earth parking orbit equal to the declination of the Moon. This results in a minimum deviation of the azimuth from 90°.

---

*It should be pointed out that no correction is made for the motion of the mean equator and equinox of date, i.e., they are treated as instantaneously inertially fixed.